LJP: TE 17.

Exercice 1

1) $\int x^3 \ln(2x)\,dx =$ $\qquad u' = x^3 \Rightarrow u = \frac{1}{4}x^4$

$\qquad v = \ln(2x) \Rightarrow v' = \frac{1}{2x}\cdot 2 = \frac{1}{x}$

$= \frac{1}{4}x^4\ln(2x) - \frac{1}{4}\int x^{\cancel{4}\,3}\cdot\frac{1}{\cancel{x}}\,dx = \frac{1}{4}x^4\ln(2x) - \frac{1}{4}\int x^3\,dx =$

$= \frac{1}{4}x^4\ln(2x) - \frac{1}{4}\cdot\frac{1}{4}x^4 + c = \frac{1}{4}x^4\ln(2x) - \frac{1}{16}x^4 + c$

2) $\int 12\,x\,e^{1+2x}\,dx =$ $\qquad u' = e^{1+2x} \Rightarrow u = \frac{1}{2}e^{1+2x}$

$\qquad v = x \Rightarrow v' = 1$

$= 12\cdot\frac{1}{2}e^{1+2x}\cdot x - \frac{1}{2}\int e^{1+2x}\,dx$

$= 6e^{1+2x}\cdot x - \frac{1}{2}\cdot\frac{1}{2}e^{1+2x} = 6e^{1+2x}\cdot x - \frac{1}{4}e^{1+2x} + c$

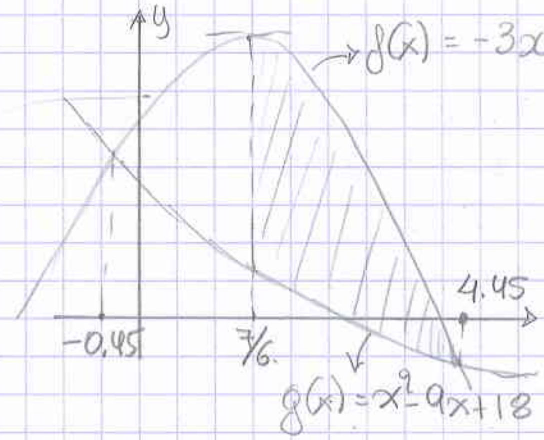
3) $\int 3x\cos(x^2+1)\,dx$ $\qquad t = x^2+1 \Rightarrow dt = 2x\,dx \Rightarrow$

$\qquad dx = \frac{1}{2x}\,dt$

$= 3\int \cancel{x}\cos t\,\frac{1}{2\cancel{x}}\,dt =$

$= \frac{3}{2}\int\cos t\,dt = \frac{3}{2}\sin t = \frac{3}{2}\sin(x^2+1) + c$

Exercice 2



$f(x) = g(x) \Leftrightarrow -3x^2+7x+26 = x^2-9x+18 \Leftrightarrow$

$\Leftrightarrow 4x^2 - 16x - 8 = 0 \Leftrightarrow$

$x^2 - 4x - 2 = 0$

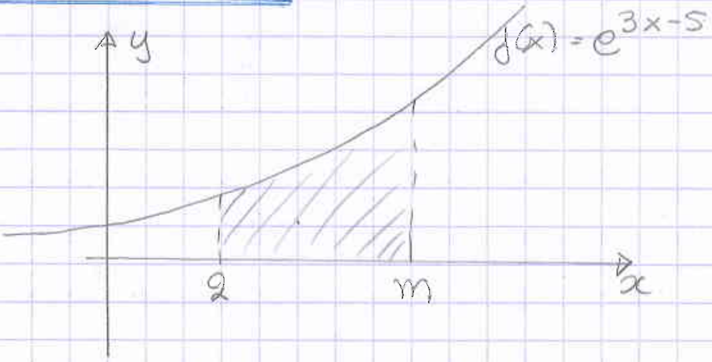
$\Delta = 16+8 = 24 \qquad x_{1,2} = \frac{4\pm\sqrt{24}}{2} = \begin{cases} 4.45 \\ -0.45 \end{cases}$

$f'(x) = -6x+7 = 0 \Leftrightarrow -6x = -7 \Leftrightarrow x = \frac{7}{6}$

$A = \int_{7/6}^{4.45}(f(x)-g(x))\,dx = \big(F(4.45)-G(4.45)\big) - \big(F(7/6)-G(7/6)\big) \simeq 58.4202$

$F(x) = -x^3 + \frac{7}{2}x^2 + 26x \qquad G(x) = \frac{1}{3}x^3 - \frac{9}{2}x^2 + 18x$

-2- EXERCICE 3



$A = 4$

$F(x) = \frac{1}{3} e^{3x-5}$

$$A = \int_2^m f(x)dx = F(m) - F(2) = \frac{1}{3} e^{3m-5} - \frac{1}{3} e^{1} = 4 \overset{\cdot 3}{\Leftrightarrow}$$

$$e^{3m-5} - e = 12 \Leftrightarrow e^{3m-5} = 12 + e \Leftrightarrow 3m-5 = \ln(12+e) \Leftrightarrow$$

$$\Leftrightarrow 3m = \ln(12+e) + 5 \Leftrightarrow m = \frac{1}{3} \ln(12+e) + 5$$