# LJP : TE 16. Solutions.

## Exercice 1.

$f(x) = 3e^{-2x}$ $\quad f'(x) = -6e^{-2x}$ $\quad x = -1.$

$f(-1) = 3e^2$ $\quad f'(-1) = -6e^2$

$y = m\,x + h$

$f(-1) = f'(-1)(-1) + h \Rightarrow 3e^2 = +6e^2 + h \Rightarrow h = -3e^2$

$t: \; y = -6e^2 x - 3e^2$

## Exercice 2

$C(t) = C_0\left(1 + \frac{1}{2}\right)^t$ $\quad$ t en heures

$C(t) = 4C_0 \Leftrightarrow \cancel{C_0}\left(1+\frac{1}{2}\right)^t = 4\cancel{C_0} \Leftrightarrow \left(1+\frac{1}{2}\right)^t = 4 \Leftrightarrow$

$\Leftrightarrow \left(\frac{3}{2}\right)^t = 4 \Rightarrow t\ln\left(\frac{3}{2}\right) = \ln 4 \Rightarrow t = \frac{\ln 4}{\ln(1.5)} = 3.42\,h$

$3.42\,h = 3\,h \;\; 25\,min \;\; 12\,sec$

## Exercice 3

$f(x) = (2x+m)e^{3x+1}$ $\quad g(x) = 6xe^{3x+1}$

$f'(x) = 2e^{3x+1} + (2x+m)\cdot 3e^{3x+1} = e^{3x+1}\left(2 + (2x+m)3\right) =$

$= e^{3x+1}\left(2 + 6x + 3m\right) = g(x)$

Donc $2 + 3m = 0 \Leftrightarrow m = -2/3$

## Exercice 4

a) $\int 4e^{-5x}\,dx = 4\frac{1}{-5}e^{-5x} = -\frac{4}{5}e^{-5x} + c.$

b) $\int 2\cos(3x-1)\,dx = \frac{2}{3}\sin(3x-1) + c.$

c) $\int \frac{x^4 - 3x^2 + 5}{x^3}\,dx = \int\left(x - \frac{3}{x} + \frac{5}{x^3}\right)dx =$

$= \frac{1}{2}x^2 - 3\ln x - \frac{5}{2}x^{-2} + c$

d) $\int \frac{x^4-4x^3-5x^2+18x+4}{x^2-4x}\,dx \overset{*}{=}$

$$\begin{array}{l|l} x^4-4x^3-5x^2+18x+4 & x^2-4x \\ \hline \underline{-x^4+4x^3} & x^2-5 \\ \quad -5x^2+18x+4 & \\ \quad \underline{+5x^2-20x} & \\ \qquad -2x+4 & \end{array}$$

$* = \int\left(x^2-5+\frac{-2x+4}{x^2-4x}\right)dx = \frac{1}{3}x^3-5x+\int\frac{-2x+4}{x^2-4x}\,dx$ $**$

$t=x^2-4x \Rightarrow dt=(2x-4)dx \Rightarrow (-2x+4)dx=-dt$

$\int\frac{-2x+4}{x^2-4x}\,dx=\int\frac{-dt}{t}=-\ln|t|=-\ln|x^2-4x|$

$** = \frac{1}{3}x^3-5x-\ln|x^2-4x|+c$