FONCTIONS EXP-LOG 2

EXERCICE 1

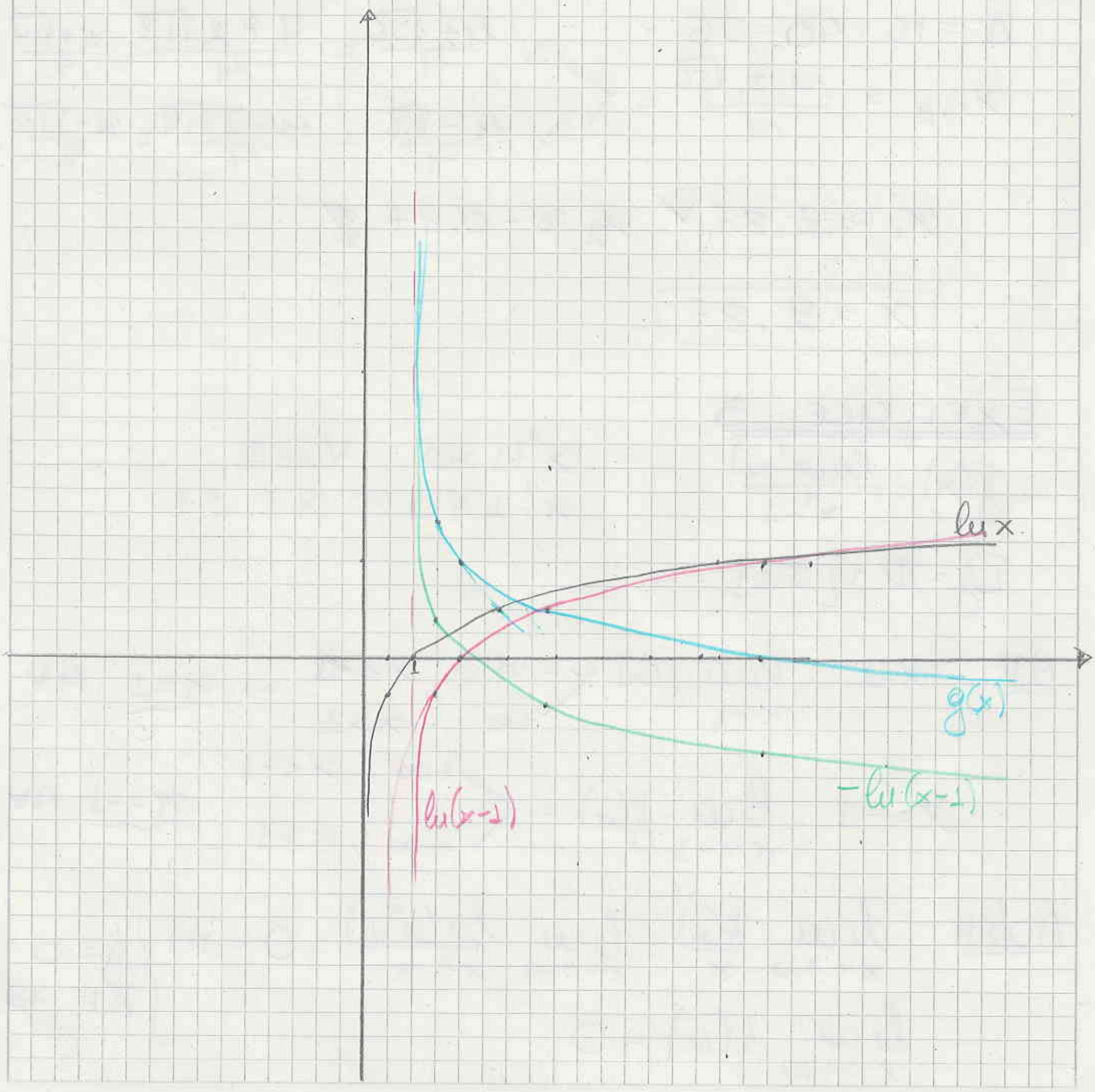
$f(x) = \ln x$

a) $x = 0.5 = \frac{1}{2}$ $\quad f\left(\frac{1}{2}\right) = \ln\frac{1}{2} = -\ln 2$ $\qquad (0.5\,;\, \overset{-0.7}{\overset{\shortparallel}{-\ln 2}})$

$x = 1$ $\quad f(1) = \ln 1 = 0$ $\qquad (1\,;0)$

$x = e$ $\quad f(e) = \ln e = 1$ $\qquad (e\,;1) \approx (2.7\,;1)$

$x = e^2$ $\quad f(e^2) = \ln e^2 = 2$ $\qquad (e^2;2) \approx (7.4\,;2)$



EXERCICE 2

$\ln(x^2+1) - \ln(3x+2) = \ln(x-2) \Leftrightarrow$

$\Leftrightarrow \ln \dfrac{x^2+1}{3x+2} = \ln(x-2) \Leftrightarrow$

$\Leftrightarrow \dfrac{x^2+1}{3x+2} = x-2 \Leftrightarrow$

$\Leftrightarrow x^2+1 = (x-2)(3x+2) \Leftrightarrow$

$\Leftrightarrow x^2+1 = 3x^2-6x+2x-4 \Leftrightarrow$

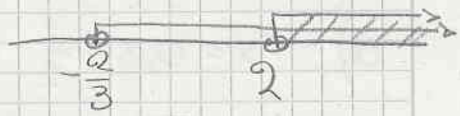
$\Leftrightarrow 2x^2-4x-5=0$

$\cdot\ x^2+1>0 \quad \forall x \in \mathbb{R}$

$\cdot\ 3x+2>0 \Leftrightarrow$

$x > -2/3$

$\cdot\ x-2>0 \Rightarrow x>2$



$x \in]2,+\infty[$

$\Delta = 16+40=56$

$$x_{1,2} = \frac{4 \pm \sqrt{56}}{4} = \begin{cases} \approx \dfrac{4+\sqrt{56}}{4} = \dfrac{4+2\sqrt{14}}{4} = \dfrac{2+\sqrt{14}}{2} \\ \approx \dfrac{4-\sqrt{56}}{4} = \dfrac{4-2\sqrt{14}}{4} = \dfrac{2-\sqrt{14}}{2} \end{cases}$$

$x_1 \approx 2,87$ ✓ $\quad x_2 \approx -0,87$ ↯

$$\boxed{x = 2,87}$$

EXERCICE 3

$f(x) = \dfrac{\ln(x^2+1)}{x^2-1}$

$x^2+1>0 \quad \forall x \in \mathbb{R}$

$x^2-1 \neq 0 \Leftrightarrow x \neq \pm 1$

$D_f = \mathbb{R} \setminus \{\pm 1\}$

AV $\quad x=1 \quad \lim\limits_{x \to 1} f(x) = \begin{cases} -\infty & x<1 \\ +\infty & x>1 \end{cases} \Rightarrow \underline{x=1}$ AV

$x=-1 \quad \lim\limits_{x \to -1} f(x) = \begin{cases} +\infty & x<-1 \\ -\infty & x>1 \end{cases} \quad \underline{x=-1}$ AV

AO/AH $\quad \lim\limits_{x \to +\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to +\infty} \dfrac{\ln(x^2+1)}{x^3-x} = 0 = m$

$\lim\limits_{x \to +\infty} f(x) = 0$

$\left.\right\} y=0$ AH $+\infty$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\ln(x^2+1)}{x^3-x} = 0 = m$$
$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\ln(x^2+1)}{x^2-1} = 0 = h$$
$y=0$ AH $x \to -\infty$

EXERCICE 4

$$f(x) = \ln\left(\frac{2x-1}{x^2+1}\right)$$

$x^2+1 \neq 0 \quad \forall x \in \mathbb{R}$

$\frac{2x-1}{x^2+1} > 0 \Leftrightarrow 2x-1>0 \Leftrightarrow x > 1/2$

$D_f =]\frac{1}{2}; +\infty[$

$$f'(x) = \frac{x^2+1}{2x-1} \cdot \frac{2(x^2+1)-(2x-1)\cdot 2x}{(x^2+1)^2} =$$
$$= \frac{\cancel{x^2+1}}{2x-1} \cdot \frac{2x^2+2-4x^2+2x}{(x^2+1)^{\cancel{2}}} = \frac{-2x^2+2x+2}{(2x-1)(x^2+1)} =$$
$$= \frac{-2(x^2-x-1)}{(2x-1)(x^2+1)}$$

EXERCICE 5

1e partie

$f(x) = x\cdot \ln^2(x) + kx \qquad D_f =]0, +\infty[$

a) $\lim_{x\to 0} f(x) = 0$

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x(\ln^2(x)+k) = +\infty$

b) $f'(x) = \ln^2(x) + x\, 2\ln(x)\frac{1}{x} + k =$
$= \ln^2(x) + 2\ln(x) + k = 0$

On pose $\ln(x) = t$
$t^2 + 2t + k = 0$

$\Delta = 4 - 4k = 4(1-k)$
- $k=1 \Rightarrow \Delta = 0$ 1 point
- $1>k \Rightarrow \Delta>0$ 2 points
- $1<k \Rightarrow \Delta<0$ 0 points

2e partie

$k=0 \qquad f(x) = x\ln^2(x)$

a) $f \cap Ox \qquad f(x)=0 \Leftrightarrow x=0$ ou $\ln(x)=0 \Leftrightarrow x=1$

La pente de la tangente en $x=1$

$f'(1)$

$f'(x) = \ln^2(x) + x\,\frac{2\ln(x)}{x} = \ln^2(x) + 2\ln(x)$

$f'(1) = 0 \Rightarrow$ tangente $/\!/ Ox \Rightarrow \alpha = 0°$

b) $x=e$

$f(e) = e\,\underbrace{\ln^2(e)}_{1} = e \qquad (e;e)$

$f'(e) = \ln^2(e) + 2\ln(e) = 1+2 = 3$

$\rightarrow y = 3x+h$

$(e,e) \quad e = 3e + h \Leftrightarrow h = -2e$

Donc $\underline{y = 3x - 2e}$