LJP : Exponentielle - Logarithme

Exercice 1

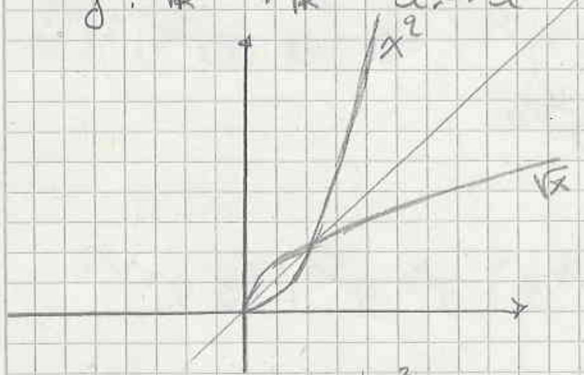
5 au carré	2 au cube	1 à la puiss 4	(-2) à la puis. 5	-5 au cube	-3 à la puissance 4
5^2	2^3	1^4	$(-2)^5$	$(-5)^3$	$(-3)^4$
5×5	$2\times2\times2$	$1\times1\times1\times1$	$(-2)\times(-2)\times(-2)\times(-2)\times(-2)$	$(-5)\times(-5)\times(-5)$	$(-3)\times(-3)(-3)(-3)$
25	8	1	-32	-125	81

Exercice 2

a. $5^2\times5^4=5^6$	b. $4^{-3}\cdot4^8=4^5$	c. $(-6)^{-7}\cdot(-6)^2=-6^{-5}$	d. $(-3)^7\times(-3)^{-4}=-3^3$
e. $5^{-3}\times5^{-1}\times5^8=5^4$	f. $7^9\,7^{-8}\,7^{-3}=7^{-2}$	g. $(-8)^9(-8)^{-5}(-8)^{-1}=8^{-4}$	h. $9^2\cdot9^{-1}\cdot9^{-7}\cdot9^{-4}=9^{-10}$
i. $\frac{5^7}{5^3}=5^4$	j. $\frac{7^{-4}}{7^3}=7^{-7}$	k. $\frac{(-6)^{-6}}{(-6)^{-1}}=-6^{-5}$	l. $\frac{(-5)^6}{(-5)^{-16}}=5^{22}$
m. $\frac{(-1)^{-12}}{(-1)^{-8}}=1$	n. $\frac{23^{-14}}{23^{-21}}=23^7$	o. $\frac{(-3)^{-9}}{(-3)^6}=-3^{-15}$	p. $\frac{2^{-3}}{2^3}=2^{-6}$
q. $(3^{-2})^7=3^{-14}$	r. $((-5)^{-7})^{-1}=-5^7$	s. $((-2)^4)^{-3}=2^{-12}$	t. $(12^7)^3=12^{21}$
u. $(8^{-8})^8=8^{-64}$	v. $((-9)^{-7})^{-2}=9^{14}$	w. $((-0.6)^{-11})^{-3}=-0.6^{33}$	x. $(7^{-8})^0=1$

Exercice 3

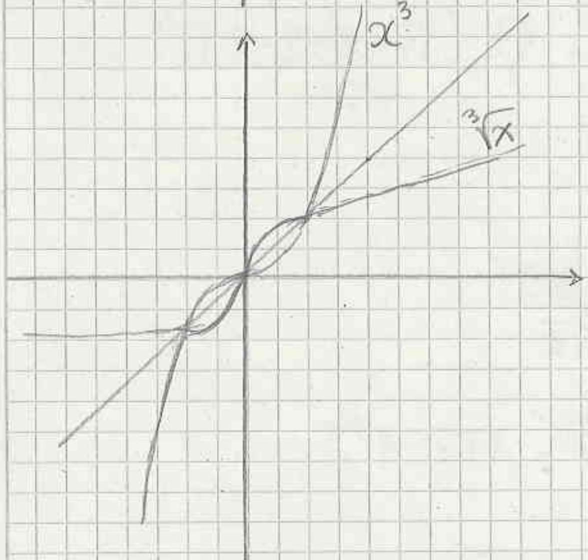
$f:\mathbb{R}^+\to\mathbb{R}^+ \quad x\mapsto x^2$ $\qquad$ $g:\mathbb{R}^+\to\mathbb{R}^+ \quad g(x)=\sqrt{x}$



2) Ses graphes sont symétriques par rapport la droite $y=x$ car $f^{-1}(x)=g(x)$

3) $f(g(x))=(\sqrt{x})^2=x$

4) $f(x)=x^3 \quad g(x)=\sqrt[3]{x}$



EXERCICE 4

a) $2^{-1} = \frac{1}{2} = 0.5$ b) $4^{-2} = \frac{1}{16}$ c) $\left(\frac{1}{8}\right)^{-2} = 8^2 = 64$

d) $\left(\frac{64}{27}\right)^{1/3} = \sqrt[3]{\frac{64}{27}} = \frac{4}{3}$ e) $125^{-1/3} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$

f) $0.125^{-1/3} = (2^{-3})^{-1/3} = 2$ g) $0.0001^{-3/4} = (10^{-4})^{-3/4} = 10^3 = 1000$

h) $16^{-\frac{3}{4}} = (2^4)^{-3/4} = 2^{-3} = \frac{1}{8}$ j) $3^{-2} \cdot 3^5 = 3^3 = 27$

k) $\frac{5^{-3}}{5^{-5}} = 5^2 = 25$ l) $(3^{1/2}\, 3^{1/4})^2 = (3^{1/2})^2 \cdot (3^{1/4})^2 = 3 \cdot 3^{1/2} = 3\sqrt{3}$

EXERCICE 5

a) $\frac{1}{a} = a^{-1}$ b) $\sqrt[3]{a} = a^{1/3}$ c) $\sqrt{a^3} = a^{3/2}$ d) $\sqrt{\frac{a^2}{a^3}} = (a^{-1})^{1/2} = a^{-1/2}$

e) $\frac{(a^2)^3}{\sqrt{a}} = \frac{a^6}{a^{1/2}} = a^{6-\frac{1}{2}} = a^{11/2}$ f) $\sqrt{(a^3)^2} = (a^6)^{1/2} = a^3$

g) $\frac{1}{a^3 \cdot a^5} = \frac{1}{a^8}$ h) $\frac{\sqrt{a^3}}{a^{1/2}} = \frac{a^{3/2}}{a^{1/2}} = a^{2/2} = a$

i) $\left(\frac{a^5}{a}\right)^{1/2} = (a^4)^{1/2} = a^2$ j) $(a+a^2)^2 = a^2 + 2a^2 + a^4 = 3a^2 + a^4$

k) $(a^2 \cdot a^{-3})^{-1/2} = (a^{-1})^{-1/2} = a^{1/2}$

l) $a\sqrt{a\sqrt{a\sqrt{a}}} = a\left(a\left(a\,(a)^{1/2}\right)^{1/2}\right)^{1/2} = a\left(a\,(a^{3/2})^{1/2}\right)^{1/2} =$

$= a\left(a^{5/2}\right)^{1/2} = a\, a^{5/4} = a^{9/4}$

EXERCICE 6

a) $\frac{\sqrt{18} \cdot 2^3}{3^{1/2}} = \frac{3\sqrt{2} \cdot 2^3}{3^{1/2}} = 3^{1/2}\, 2^{1/2}\, 2^3 = 3^{1/2}\, 2^{7/2} = (3 \cdot 2^7)^{1/2}$

b) $\frac{\sqrt{2}(1+\sqrt{2})}{2^{1/2}+2} = \frac{\sqrt{2}(1+\sqrt{2})(\sqrt{2}-2)}{2-4} = \frac{\sqrt{2}(\sqrt{2}+\cancel{2}-\cancel{2}-2\sqrt{2})}{-2} =$

$= \frac{\sqrt{2}(-\sqrt{2})}{-2} = 1$

c) $\frac{2^{1/3}+2^{5/3}}{2^{2/3}} = \frac{\sqrt[3]{2}\,(1+2^5)}{(\sqrt[3]{2})^2} = \frac{33}{\sqrt[3]{2}}$

d) $\frac{\sqrt{252}+\sqrt{7}}{8-1} = \frac{6\sqrt{7}+\sqrt{7}}{7} = \frac{7\sqrt{7}}{7} = \sqrt{7} = 7^{1/2}$

Exercice 7

a) $\sqrt{2} - \sqrt{8} + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{2 - 2\sqrt{8} + 3 - 1}{\sqrt{2}} = \frac{4 - 4\sqrt{2}}{\sqrt{2}} =$

$= \frac{4\sqrt{2} - 8}{2} = 2\sqrt{2} - 4$

b) $(3-\sqrt{5})^2 - (3+\sqrt{5})^2 + \sqrt{720} = \cancel{9} - 6\sqrt{5} + 5 - \cancel{9} - \cancel{6\sqrt{5}} - 5 + 12\sqrt{5} = 0$

c) $(5\sqrt{3})^3 - 375\sqrt{3} = 125 \cdot 3 \cdot \sqrt{3} - 375\sqrt{3} = 0$

d) $(\sqrt{2}+\sqrt{3})^2 - (2+\sqrt{3})(\sqrt{3}-2) + (1-\sqrt{6})(\sqrt{2}+\sqrt{27}) =$

$= 2 + 2\sqrt{6} + 3 - (3-4) + \sqrt{2} + \sqrt{27} - \sqrt{12} - \sqrt{6 \cdot 27} =$

$= 5 + 2\sqrt{6} + 1 + \sqrt{2} + 3\sqrt{3} - 2\sqrt{3} - 9\sqrt{2} =$

$= 6 + 2\sqrt{6} - 8\sqrt{2} + \sqrt{3}$

Exercice 8

- $a^{m-2} \cdot a^{3-m} = a$
- $\frac{(16x^4y^2)^n}{(4x^2y)^{2n-1}} = \frac{16^n x^{4n} y^{2n}}{4^{2n-1} x^{4n-2} y^{2n-1}} = \frac{4^{2n} x^2 y}{4^{2n-1}} =$

  $= 4x^2y$
- $\sqrt[3]{a\sqrt{a^4}} = (a(a^{4/2}))^{1/3} = a^{1/3}(a^2)^{1/3} = a^{1/3} \cdot a^{2/3} = a^{3/3} = a$
- $\sqrt[3]{a\sqrt[3]{a^4 \cdot \sqrt[3]{a^6}}} = \sqrt[3]{a\sqrt[3]{a^4 a^2}} = \sqrt[3]{a\sqrt[3]{a^6}} = \sqrt[3]{a a^2} = \sqrt[3]{a^3} = a$
- $a^{2n-9} \cdot a^{8-n} = a^{n-1}$
- $\frac{(27a^6b^3)^{n+1}}{(3a^2b)^{3n+3}} = \frac{3^{3n+3} a^{6n+6} b^{3n+3}}{3^{3n+3} b^{6n+6} b^{3n+3}} = 1$
- $\sqrt{a}\,\sqrt[5]{a^3} \cdot (\sqrt[10]{a})^4 = a^{1/2} a^{3/5} a^{2/5} = a^{\frac{5}{10} + \frac{6}{10} + \frac{4}{10}} = a^{\frac{15}{10}} = a^{3/2}$

Exercice 9

a. $3^3 \cdot 3^x = 243 \Rightarrow 3^{x+3} = 3^5 \Leftrightarrow x + 3 = 5 \Leftrightarrow x = 2$

b. $10^7 \cdot 10^{-6} = 10$

c. $10 : 0{,}01 = 10^1$ $f = 3$

d. $1^5 = 1$

e. $5^2 \cdot 5^0 = 5^2$

f. $6^3 : 6^0 = 216$

g. $4^{-3} : 4^{-5} = 4^{2}$

h. $(3^{2} \cdot 3^{1})^{2} = 3^{6}$

EXERCICE 10

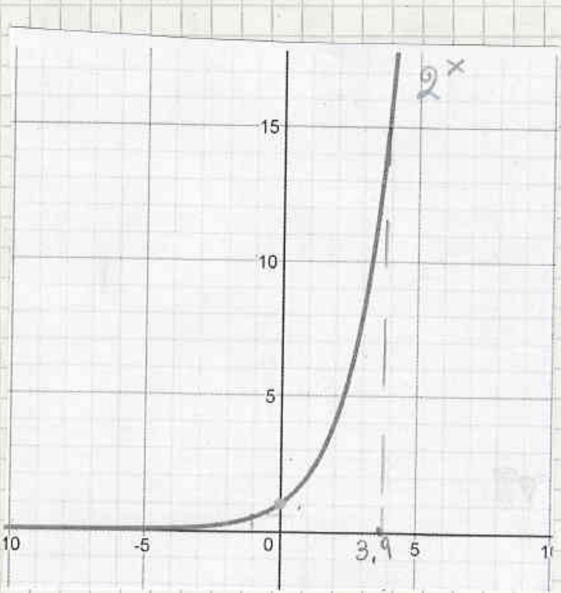
$f(x) = 2^{x}$

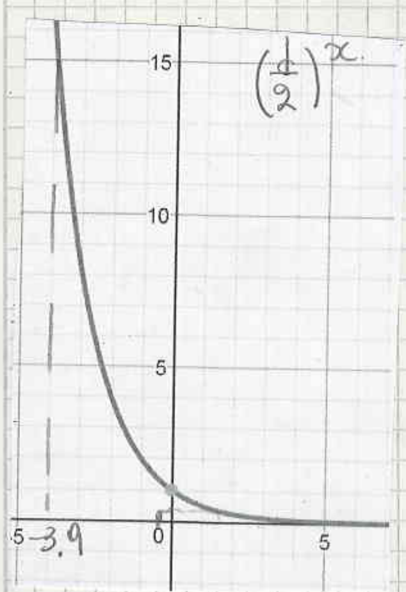
a) $2^{x} = 15 \Rightarrow x \approx 3.9$

b) $2^{x} = \frac{1}{2} = 2^{-1} \Rightarrow x = -1$

c) $2^{x} = -3$ impossible

d) $\left(\frac{1}{2}\right)^{x} = 15 \Rightarrow x \approx -3.9$





e) $\left(\frac{1}{2}\right)^{x} = 8 \Leftrightarrow 2^{-x} = 2^{3} \Leftrightarrow -x = 3 \Leftrightarrow x = -3$

f) $\left(\frac{1}{2}\right)^{x} = 0.4 \Rightarrow x \approx 1.3$

g) $3^{x} = 10 \Rightarrow x \approx 2.1$

h) $3^{x} = \frac{1}{9} = 3^{-2} \Leftrightarrow x = -2$

i) $3^{x} = \frac{1}{3} = 3^{-1} \Leftrightarrow x = -1$

EXERCICE 11

a) $2^{x} = 4\sqrt{2} = 2^{2}\,2^{1/2} = 2^{5/2} \Rightarrow x = 5/2$

b) $2^{x} = \sqrt[3]{\frac{\sqrt{2}}{64}} \Rightarrow 2^{x} = \sqrt[3]{\frac{2^{1/2}}{2^{6}}} = \sqrt[3]{2^{-11/2}} = \left(2^{-11/2}\right)^{1/3} = 2^{-11/6}$

c) $5^{2x+1} = \sqrt{5\sqrt{125}} = \left(5\,(125)^{1/2}\right)^{1/2} = 5^{1/2}\left((5^{3})^{1/2}\right)^{1/2} = 5^{1/2}\,5^{3/4} = 5^{5/4}$

$\Rightarrow 2x + 1 = \frac{5}{4} \Leftrightarrow 2x = \frac{5}{4} - 1 \Leftrightarrow 2x = \frac{1}{4} \Leftrightarrow x = \frac{1}{8}$

d) $3^{x}\sqrt{3} = 9 \Leftrightarrow 3^{x} = \frac{3^{2}}{3^{1/2}} \Rightarrow 3^{x} = 3^{3/2} \Leftrightarrow x = 3/2$

$x \neq 0$

e) $2^{x} \cdot \sqrt[x]{64} = 32 \Rightarrow 2^{x} \cdot 2^{6/x} = 2^{5} \Leftrightarrow 2^{x + \frac{6}{x}} = 2^{5} \Leftrightarrow$

$\Leftrightarrow x+\frac{6}{x}=2^5 \Leftrightarrow x^2+6-32x=0 \Leftrightarrow x^2-32x+6=0$

$\Delta=1000 \quad x_{1,2}=\frac{32\pm\sqrt{1000}}{2}$ $\begin{cases} x_1 = 31.8 \\ x_2 = 0.189 \end{cases}$

EXERCICE 12

$P(4,9) \qquad 9=a^4 \Leftrightarrow a=\sqrt[4]{9}$

EXERCICE 13

a) $15625=5^6 \Rightarrow \log_5 15625=6$

b) $\log_3 9=2 \Leftrightarrow 3^2=9$

c) $\log_5\left(\frac{4}{100}\right)=-2 \Leftrightarrow 5^{-2}=\frac{4}{100}=\frac{1}{25}$

d) $5=25^{1/2} \Leftrightarrow \log_{25}5=\frac{1}{2}$

e) $\log_2 8=3 \Leftrightarrow 2^3=8$

f) $16=2^4 \Leftrightarrow \log_2 16=4$

g) $\log_4(16)=2 \Leftrightarrow 4^2=16.$

EXERCICE 14

$7^x=343 \Leftrightarrow 7^x=7^3 \Leftrightarrow x=3$

EXERCICE 15

a) $\log_2 8=\log_2 2^3=3$

b) $\log_3 27=\log_3 3^3=3$

c) $\log_{12}144=\log_{12}12^2=2$

d) $\log 0.01=\log 10^{-2}=-2$

e) $\log 10^{-3}=-3$

f) $\log_3(81)=\log_3 3^4=4$

g) $\log_3\left(\frac{1}{81}\right)=\log_3 3^{-4}=-4$

h) $\log_4(0.25)=\log_4 4^{-1}=-1$

i) $\log_4(0.125)=\log_4 4^{-1.5}=-1.5$

j) $\log_a(a^2\cdot a^n)=\log_a(a^{n+2})=n+2.$

k) $\log_a\left(\sqrt{\sqrt{\sqrt{a}}}\right)=\log_a a^{1/8}=\frac{1}{8}$

l) $\log_{a^2}\left(\frac{1}{a^3}\right) = \frac{\log_a a^{-3}}{\log_a a^2} = -\frac{3}{2}$

m) $\log_a\left(\frac{\sqrt[3]{a}}{\sqrt{a}\cdot a^{-2}}\right) = \log_a\left(\frac{a^{1/3}}{a^{1/2}\cdot a^{-2}}\right) = \log_a a^{\frac{1}{3}-\frac{1}{2}+2} \Rightarrow$

$= \frac{11}{6}$

n) $\log 10 = 1$

o) $\log_{13}(1) = 0$

p) $\log(0.001) = \log 10^{-3} = -3$

q) $\log_{1/2} 2^5 = x \Leftrightarrow \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-5} \Leftrightarrow x = -5$.

EXERCICE 16

a) $x = \log_2 5 = \frac{\log 5}{\log 2} \approx 2.32$

b) $x = \log_5 2 = \frac{\log 2}{\log 5} \approx 0.43$

c) $\log_x(512) = 9 \Leftrightarrow x^9 = 512 \Leftrightarrow x = \sqrt[9]{512} \Leftrightarrow x = 2$.

d) $\log_x 1 = 0 \quad x \in]0,1[\cup]1,+\infty[$

e) $\log_8(x) = 2 \Leftrightarrow 8^2 = x = 64$

f) $\log_2(x) = 3 \Leftrightarrow 2^3 = x = 8$

g) $4 = \log_x(81) \Leftrightarrow x^4 = 81 \Leftrightarrow x = 3$.

h) $2 = \log_6(x) \Leftrightarrow 6^2 = x = 36$

i) $x = \log_2(64) = \frac{\log 64}{\log 2} = 6$

j) $5 = \log_x(32) \Rightarrow x^5 = 32 \Rightarrow x = 2$.

k) $3 = \log_3(x) \Leftrightarrow 3^3 = x \Rightarrow x = 27$

l) $x = \log_6(6^2) \Rightarrow x = 2$

Exercice 17

1) $\log(0.1\,a^3bc^2) = \log 10^{-1} + 3\log a + \log b + 2\log c$

3) $\log \sqrt[3]{a^5b^3c} = \log(a^5b^3c)^{1/3} = \frac{1}{3}(5\log a + 3\log b + \log c)$

2) $\log \frac{a^2\sqrt{b}}{c} = 2\log a + \frac{1}{2}\log b - \log c$

4) $\log(10a^2b^3\sqrt{c^2})^2 = 2(\log 10 + 2\log a + 3\log b + \log c)$
$= 2 + 4\log a + 6\log b + 2\log c$

5) $\log(\sqrt[3]{a^4b\sqrt{c}}) = \frac{1}{3}(4\log 4 + \log 5 + \frac{1}{2}\log c)$

Exercice 18

$\log_3 4 = 1.26185 \quad \log_3 5 = 1.46497 \quad \log_3 7 = 1.77124$

1. $\log_3 20 = \log_3(4\cdot 5) = \log_3 4 + \log_3 5 \simeq 2.72682$
2. $\log_3 16 = \log_3 4^2 = 2\log_3 4 = 2.5237$
3. $\log_3 28 = \log_3(4\cdot 7) = \log_3 4 + \log_3 7 = 3.03309$
4. $\log_3(25) - \log_3(35) = 2\log_3 5 - \log_3 5 - \log_3 7 =$
$= \log_3 5 - \log_3 7 \approx -0.30627$

Exercice 19

$\log 2 = 0.30103 \quad \log 3 = 0.47712$

1) $\log 6 = \log 2 + \log 3$
2) $\log 4 = 2\log 2$
3) $\log 9 = \log 3^2 = 2\log 3$
4) $\log 8 = \log 2^3 = 3\log 2$
5) $\log\sqrt{3} = \log 3^{1/2} = \frac{1}{2}\log 3$
6) $\log\sqrt{2} = \log 2^{1/2} = \frac{1}{2}\log 2$
7) $\log 30 = \log 10 + \log 3 = 1 + \log 3$
8) $\log 300 = \log(3\cdot 100) = \log 3 + \log 10^2 = \log 3 + 2$
9) $\log 30000 = \log 3 + \log 10^4 = 4 + \log 3$
10) $\log 54 = \log 18\cdot 3 = \log 18 + \log 3 = \log 9 + \log 2 + \log 3$
$= \log 3^2 + \log 2 + \log 3 = 3\log 3 + \log 2$

11) $\log(4.8) = \log\frac{48}{10} = \log 48 - \log 10 = \log 16\cdot 3 - 1 =$
$= \log 2^4 + \log 3 - 1 = 4\log 2 + \log 3 - 1$

12) $\log\left(\frac{4}{3}\right) = \log 4 - \log 3 = \log 2^2 - \log 3 =$
$2\log 2 - \log 3$

13) $\log 4.5 = \log\frac{9}{2} = \log 3^2 - \log 2 = 2\log 3 - \log 2$

14) $\log 45 = \log\frac{90}{2} = \log(9\cdot 10) - \log 2 =$
$= \log 3^2 + \log 10 - \log 2 = 2\log 3 + 1 - \log 2$

15) $\log\left(\frac{4}{9}\right) = \log 2^2 - \log 3^2 = 2\log 2 - 2\log 3$

EXERCICE 20

$\log_2 3 = 1.58496 \quad \log_2 7 = 2.80735 \quad \log_2 11 = 3.45943$

1. $\log_2(21) = \log_2 3 + \log_2 7$
2. $\log_2(33) = \log_2 3 + \log_2 11$
3. $\log_2 49 = \log_2 7^2 = 2\log_2 7$
4. $\log_2 9 - \log_2 6 = \log_2 3^2 - \log_2(2\cdot 3) =$
$= 2\log_2 3 - \log_2 2 - \log_2 3 = \log_2 3 - 1$

EXERCICE 21

a) $\log_5 x = 3 \Leftrightarrow 5^3 = x$ ✓ $\qquad x>0$

b) $\log_2 x = 0 \Leftrightarrow 2^0 = x \Leftrightarrow x = 1$ $\qquad x>0$

c) $\log_3 x = -3 \Leftrightarrow 3^{-3} = x = \frac{1}{27}$ ✓ $\qquad x>0$

d) $\log_{1/2} x = -5 \Leftrightarrow \left(\frac{1}{2}\right)^{-5} = x = \frac{1}{32}$ ✓ $\qquad x>0$

e) $\log_x 16 = 2 \Leftrightarrow x^2 = 16 \Leftrightarrow x = 4$ $\qquad x>0 \; x\neq 1$

f) $\log_x 1 = 0 \Leftrightarrow x^0 = 1 \quad x>0 \; x\neq 1$ $\qquad x>0 \; x\neq 1$

g) $\log_2(\log_2 x) = 0 \Leftrightarrow$
$\Leftrightarrow 2^0 = \log_2 x \Leftrightarrow 2^1 = x$ ✓
$x>0$
$\log_2 x > 0$
$\Leftrightarrow x > 1$

h) $\log_3(\log_3 x) = 1$
$\Leftrightarrow 3^1 = \log_3 x \Leftrightarrow 3^3 = x = 27$ ✓
$x>0$
$\log_3 x > 0 \Leftrightarrow x > 1$

EXERCICE 22

1) $400 = 300 \cdot 1.075^{n+2} \Leftrightarrow \frac{4}{3} = 1.075^{n+2} \Leftrightarrow \log\frac{4}{3} = (n+2)\log 1.075 \Leftrightarrow$
$\Leftrightarrow \frac{\log\frac{4}{3}}{\log 1.075} - 2 = n \Leftrightarrow n \approx 1.978$

2) $25 \cdot 4^{x} = 15 \cdot 2^{-x} \Leftrightarrow 25 \cdot 2^{2x} = 15 \cdot 2^{-x} \Leftrightarrow 25 \cdot 2^{3x} = 15 \Leftrightarrow$
$\Leftrightarrow 2^{3x} = \frac{3}{5} \Leftrightarrow 3x \log 2 = \log\frac{3}{5} \Leftrightarrow x = \frac{\log 3/5}{3\log 2} \approx -0.24$

3) $\frac{10}{5^{x-1}} = 2.75 \Leftrightarrow \frac{10}{2.75} = 5^{x-1} \Leftrightarrow (x-1)\log 5 = \log\frac{10}{2.75} \Leftrightarrow$
$x = \frac{1 - \log 2.75}{\log 5} + 1 \Leftrightarrow x \approx 1.8$

4) $\sqrt{8^{x}} = 2 \cdot 0.5^{x} \Leftrightarrow \sqrt{2^{3x}} = 2 \cdot \frac{1}{2^{x}} \Leftrightarrow 2^{\frac{3x}{2}} = 2^{1-x} \Leftrightarrow$
$\Leftrightarrow \frac{3x}{2} = 1 - x \Leftrightarrow 3x = 2 - 2x \Leftrightarrow 5x = 2 \Leftrightarrow x = 2/5$

EXERCICE 23

1) $\forall x \in \mathbb{R}_+^* \quad \log(x) + \log(1/x) = 0$
$\log(x) + \log(1/x) = \log x + \log x^{-1} = \log x - \log x = 0$

2) $\forall x \in \mathbb{R}_+^* : \log(x^3) - \log(x^4) = -\log(x)$
$\log(x^3) - \log(x^4) = 3\log x - 4\log x = -\log x$

EXERCICE 24

$\log(u) < 0 \Leftrightarrow u \in]0, 1[$
$\log(u) = 0 \Leftrightarrow u = 1$
$0 < \log u < 1 \Rightarrow 10^0 < 10^{\log u} < 10^1 \Leftrightarrow 1 < u < 10$
$\log(u) = 1 \Rightarrow u = 10$
$\log(u) > 1 \Rightarrow 10^{\log u} > 10^1 \Leftrightarrow u > 10$

EXERCICE 25

a) $\log(x+2) - \log(3) = \log(2x-1) + \log 7$
$\Leftrightarrow \log\frac{x+2}{3} = \log((2x-1) \cdot 7) \Leftrightarrow$
$\Leftrightarrow \frac{x+2}{3} = 14x - 7 \Leftrightarrow x + 2 = 42x - 21 \Leftrightarrow$
$\Leftrightarrow 41x = 23 \Leftrightarrow x = \frac{23}{41} \approx 0.56 > 1/2$

$\cdot\ x + 2 > 0 \Leftrightarrow x > -2$
$\cdot\ 2x - 1 > 0 \Leftrightarrow x > 1/2$

$\boxed{x > 1/2}$

b) $\log(x+2)+\log(x-1)=\log 18 \Leftrightarrow$

$\Leftrightarrow \cancel{\log}(x+2)(x-1)=\cancel{\log}18 \Leftrightarrow$

$x^2+2x-x-2=18 \Leftrightarrow x^2+x-20=0$

$\Delta=81 \quad x_{1,2}=\frac{-1\pm 9}{2}$; $x_1=-5$ ✗ ; $x_2=4$ ✓

- $x+2>0 \Leftrightarrow x>-2$
- $x-1>0 \Leftrightarrow x>1$

$-2 \quad 1$

$\boxed{x>1}$

c) $\log(2x-3)+\log(3x+10)=4\log 2$

$\Leftrightarrow \cancel{\log}(2x-3)(3x+10)=\cancel{\log}2^4 \Leftrightarrow$

$\Leftrightarrow 6x^2-9x+20x-30=16 \Leftrightarrow$

$\Leftrightarrow 6x^2+11x-46=0 \qquad \Delta=1225$

$x_{1,2}=\frac{-11\pm 35}{12}$; $x_1=-\frac{46}{12}=-\frac{23}{6}\approx-3.8$ ✗ ; $x_2=2$ ✓

- $2x-3>0 \Leftrightarrow x>\frac{3}{2}$
- $3x+10>0 \Leftrightarrow x>-\frac{10}{3}$

$-10/3 \quad 3/2$

$\boxed{x>3/2}$

d) $\log_2(x^2-4)=2\log_2(x+3) \Leftrightarrow$

$\Leftrightarrow \log_2(x^2-4)=\log_2(x+3)^2 \Leftrightarrow$

$\Leftrightarrow x^2-4=x^2+6x+9 \Leftrightarrow$

$6x=-13 \Leftrightarrow x=-2.1\overline{6} \in]-3,-2[$ ✓

- $x^2-4>0$; $-2 \quad 2$
- $x+3>0 \Leftrightarrow x>-3$

$-3 \quad -2 \quad 2$

$\boxed{-3<x<-2 \text{ ou } x>2}$

e) $\log_3(35-x^3)=3\log_3(5-x) \Leftrightarrow$

$\Leftrightarrow 35-x^3=(5-x)^3 \Leftrightarrow$

$\Leftrightarrow 35-\cancel{x^3}=5^3-75x+15x^2-\cancel{x^3} \Leftrightarrow$

$\Leftrightarrow 15x^2-75x+90=0 \Leftrightarrow$

$3x^2-25x+30=0 \qquad \Delta=265$

$x_{1,2}=\frac{25\pm\sqrt{265}}{6}=$; $x_1=1.45$ ✓ ; $x_2=6.9$ ✗

- $35-x^3>0 \Leftrightarrow \sqrt[3]{35}>x \Rightarrow x<3.27$
- $5-x>0 \Leftrightarrow x<5$

$3.27 \quad 5$

$\boxed{x<3.27}$

EXERCICE 26

1) $e^{2x}=5 \Leftrightarrow 2x=\ln 5 \Leftrightarrow x=\frac{\ln 5}{2}$

2) $\ln x=\frac{1}{2} \Leftrightarrow x=e^{1/2}$ ✓ $\quad \boxed{x>0}$

3) $\log_a 8=3/4 \Leftrightarrow a^{3/4}=8=2^3 \Leftrightarrow$ $\quad a>0$, $a\neq 1$

$\Leftrightarrow a^{1/4}=2 \Leftrightarrow \sqrt[4]{a}=2 \Leftrightarrow a=2^4$ ✓

4) $\log(4x-3) = -1 \Leftrightarrow 4x-3 = 10^{-1} \Leftrightarrow$

$\Leftrightarrow 4x = 0.1+3 \Rightarrow x = \frac{3.1}{4} \approx 0.775$ ✓

$4x-3>0$

$\boxed{x > 3/4} \approx 0.75$

5) $e^{\ln 7} = 3x+1 \Leftrightarrow 7 = 3x+1 \Leftrightarrow 3x = 6 \Leftrightarrow x = 2$

EXERCICE 27

1) $\ln(x+2) - \ln(x-1) = 1 \Leftrightarrow$

$\Leftrightarrow \ln\left(\frac{x+2}{x-1}\right) = 1 \Leftrightarrow \frac{x+2}{x-1} = e \Leftrightarrow$

$\Leftrightarrow x+1 = ex - e \Leftrightarrow (e-1)x = 1+e \Leftrightarrow$

$\Leftrightarrow x = \frac{1+e}{e-1} > 1$ ✓

• $x+2>0 \Leftrightarrow x>-2$

• $x-1>0 \Leftrightarrow x>1$

$-2 \qquad 1$

$\boxed{x>1}$

2) $e^{3x} + e^{2x} - 12e^{x} = 0 \Leftrightarrow e^{x}(e^{2x}+e^{x}-12) = 0$

$e^{x} = t > 0 \qquad t^2+t-12 = 0 \qquad \Delta = 49 \qquad t_{1,2} = \frac{-1\pm 7}{2} < \begin{matrix} -4 \\ 3 \end{matrix}$

$t = -4$ × $\qquad t = 3 = e^{x} \Leftrightarrow \underline{x = \ln 3}$

3) $2\cdot 3^{n+4} = 5\cdot 6^{n-7} \Leftrightarrow \ln(2\cdot 3^{n+4}) = \ln(5\cdot 6^{n-7}) \Leftrightarrow$

$\ln 2 + (n+4)\ln 3 = \ln 5 + (n-7)\ln 6 \Leftrightarrow$

$\Leftrightarrow \ln 2 + n\ln 3 + 4\ln 3 = \ln 5 + n\ln 6 - 7\ln 6 \Leftrightarrow$

$\Leftrightarrow n\ln 3 - n\ln 6 = \ln 5 - 7\ln 6 - \ln 2 - 4\ln 3 \Leftrightarrow$

$\Leftrightarrow n(\ln 3 - \ln 6) = \ln 5 - 7\ln 6 - \ln 2 - 4\ln 3 \Leftrightarrow$

$\Leftrightarrow n = \frac{\ln 5 - 7\ln 6 - \ln 2 - 4\ln 3}{\ln 3 - \ln 6} \approx 23.1$

4) $\frac{\log(4x)}{\log(x+1)} = 2 \Leftrightarrow \log 4x = \log(x+1)^2 \Leftrightarrow$

$4x = x^2+2x+1 \Leftrightarrow x^2-2x+1 = 0 \Leftrightarrow$

$(x-1)^2 = 0 \Leftrightarrow x = 1$ ✓

• $4x>0 \Leftrightarrow x>0$

• $x+1>0 \Leftrightarrow x>-1$

$-1 \qquad 0$

$\boxed{x>0}$

EXERCICE 28

a) $(3^{x-1})^3 = 9\cdot 3^{x-2} \Leftrightarrow 3^{3x-3} = 3^2\cdot 3^{x-2} \Leftrightarrow 3^{x-3} = 3^{x} \Leftrightarrow$

$\Leftrightarrow x-3 = x \Rightarrow -3 = 0$ impossible

b) $27^{x+2} = 3^{5x+8} \Leftrightarrow 3^{3x+6} = 3^{5x+8} \Leftrightarrow 3x+6 = 5x+8 \Leftrightarrow$

$\Leftrightarrow 2x = -2 \Leftrightarrow x = -1$

c) $e^{6/x} = e^{x-1} \Leftrightarrow \frac{6}{x} = x-1 \Leftrightarrow$

$x \neq 0$

$\Leftrightarrow 6 = x^2 - x \Leftrightarrow x^2 - x - 6 = 0 \qquad \Delta = 25$

$x_{1,2} = \frac{1\pm 5}{2} < \begin{matrix} x_1 = 3 \text{ ✓} \\ x_2 = -2 \text{ ✓} \end{matrix}$

d) $7^{2x-3} = 16807 \Leftrightarrow 7^{2x-3} = 7^5 \Leftrightarrow 2x-3 = 5 \Leftrightarrow 2x = 8 \Leftrightarrow x = 4$

e) $3^{5x+4} = 9\sqrt{3} \Leftrightarrow 3^{5x+4} = 3^{5/2} \Leftrightarrow 5x+4 = \frac{5}{2} \Leftrightarrow 5x = -\frac{3}{2} \Leftrightarrow$
$x = -\frac{3}{10}$

f) $4 \cdot 2^{-3x} - 5 \cdot 2^{-x} + 2^x = 0 \Leftrightarrow \frac{4}{2^{3x}} - 5\frac{1}{2^x} + 2^x = 0 \overset{\cdot 2^{3x}}{\Leftrightarrow}$

$\Leftrightarrow 4 - 5 \cdot 2^{2x} + 2^{4x} = 0 \Leftrightarrow (2^{2x})^2 - 5 \cdot 2^{2x} + 4 = 0$

$2^{2x} = t > 0 \qquad t^2 - 5t + 4 = 0 < \begin{matrix} t_1 = 4 \\ t_2 = 1 \end{matrix}$

$2^{2x} = 4 \Leftrightarrow 2^{2x} = 2^2 \Leftrightarrow 2x = 2 \Leftrightarrow x = 1$

$2^{2x} = 1 \Leftrightarrow x = 0$

g) $3^x + 9^x = 90 \Leftrightarrow 3^{2x} + 3^x - 90 = 0$

$3^x = t > 0 \qquad t^2 + t - 90 = 0 \qquad \Delta = 361$

$t_{1,2} = \frac{-1 \pm 19}{2} < \begin{matrix} t_1 = -10 \ \times \\ t_2 = 9 \ \checkmark \end{matrix}$

$3^x = 9 = 3^2 \Leftrightarrow x = 2$

h) $5 \cdot 5^{2x} + 14 \cdot 5^x - 3 = 0 \qquad t = 5^x > 0$

$5 \cdot t^2 + 14 \cdot t - 3 = 0 \qquad \Delta = 256$

$t_{1,2} = \frac{-14 \pm 16}{10} < \begin{matrix} t_1 = \frac{1}{5} \ \checkmark \\ t_2 = -3 \ \times \end{matrix}$

$5^x = \frac{1}{5} = 5^{-1} \Leftrightarrow x = -1$

i) $e^x - 6e^{-x} = -1 \Leftrightarrow e^x - \frac{6}{e^x} + 1 = 0 \Leftrightarrow e^{2x} + e^x - 6 = 0$

$t = e^x = t > 0 \qquad t^2 + t - 6 = 0 \qquad \Delta = 25$

$t_{1,2} = \frac{-1 \pm 5}{2} < \begin{matrix} t_1 = -3 \ \times \\ t_2 = 2 \ \checkmark \end{matrix} \qquad e^x = 2 \Leftrightarrow x = \ln 2$

EXERCICE 29

a) $f(x) = \frac{2}{10^{x-9}} \qquad D_f = \mathbb{R} \setminus \{9\}$

$f(x) = 0$ impossible $\quad f(x) > 0 \ \forall x \in D_f$

b) $f(x) = \log_7 \left(\frac{x^2-1}{x+3} \right)$

$D_f =]-3; -1[\cup]1, +\infty[$

$\frac{x^2-1}{x+3} > 0$

x	$-\infty$		-3		-1		1		$+\infty$
x^2-1		+		+	0	−	0	+	
$x+3$		−	0	+		+		+	
Q		−	‖	+	0	−	0	+	

$f(x)=0 \Leftrightarrow \log_7\left(\frac{x^2-1}{x+3}\right)=0 \Leftrightarrow \frac{x^2-1}{x+3}=1 \Leftrightarrow x^2-1=x+3 \Leftrightarrow$

$\Leftrightarrow x^2-x-4=0$ $\quad \Delta=17$ $\quad x_{1,2}=\frac{1\pm\sqrt{17}}{2}$ $\quad x_1=2.56$ ✓ $\quad x_2=-1.56$ ✓

x	-3		-1.56		-1		1		2.56		$+\infty$
f		$+$	0	$-$		//////		$-$	0	$+$	

c) $f(x)=\log(x^3+2x^2-3)$

$x^3+2x^2-3>0$

$x^3+2x^2-3=0$

1	2	0	-3	1
↓	1	3	3	
1	3	3	0	

$\Leftrightarrow (x-1)(x^2+3x+3)=0$

$\Delta=9-12<0 \Rightarrow x^2+3x+3>0 \quad \forall x\in\mathbb{R}$

Alors $x^3+2x^2-3>0 \Leftrightarrow x-1>0 \Leftrightarrow x>1$

$D_f=]1,+\infty[$

d) $f(x)=\frac{1}{\log(x^2-1)}$

$\cdot\, x^2-1>0$

		-1		1	
	$+$	0	$-$	0	$+$

$\log(x^2-1)\neq 0 \Leftrightarrow x^2-1\neq 1 \Leftrightarrow x^2\neq 2 \Leftrightarrow x\neq\pm\sqrt{2}$

$-\sqrt{2} \quad -1 \quad 1 \quad \sqrt{2}$

$D_f=]-\infty,-\sqrt{2}[\,\cup\,]-\sqrt{2},-1[\,\cup\,]1,\sqrt{2}[\,\cup\,]\sqrt{2},+\infty[$

$f(x)=0$ impossible

$\log(x^2-1)>0 \Rightarrow x^2-1>1 \Leftrightarrow x^2-2>0$

		$-\sqrt{2}$		$\sqrt{2}$	
	$+$	0	$-$	0	$+$

	$-\infty$		$-\sqrt{2}$		-1		1		$\sqrt{2}$		$+\infty$
f		$+$	//	$-$		//////		$-$	//	$+$	

EXERCICE 30

a) $\lim\limits_{x\to+\infty} \ln(x^3)=+\infty$ $\qquad$ b) $\lim\limits_{x\to 0} \frac{\ln x}{x}=-\infty$

c) $\lim\limits_{x\to+\infty} \frac{\ln x}{x^2}=-\infty$ $\qquad$ d) $\lim\limits_{x\to+\infty} \ln\left(\frac{x+1}{x-1}\right)=0$

e) $\lim\limits_{x\to+\infty} xe^x=0$ $\qquad$ f) $\lim\limits_{x\to+\infty} \frac{e^x}{x}=+\infty$

g) $\lim\limits_{x\to+\infty} (e^{-x})^2=0$ $\qquad$ h) $\lim\limits_{x\to+\infty} x^2e^{-x}=0$

i) $\lim_{x \to 0} (x^2+1)\ln(x) = -\infty$ j) $\lim_{x \to -\infty} (x^2+2)e^x = 0$

k) $\lim_{x \to -\infty} \frac{2x-1}{x^3+2} \ln(x^2+1) = 0$ l) $\lim_{x \to +\infty} x e^{\frac{1}{x}} = +\infty$

EXERCICE 31

$Q = 2 \cdot 3^t$

1) $t = 0$ $Q(0) = 2 \cdot 3^0 = 2$

2) 10 min $\rightsquigarrow \frac{1}{6}$ h 30 min $\rightsquigarrow \frac{1}{2}$ h

$Q\left(\frac{1}{6}\right) = 2 \cdot 3^{1/6} \approx 2.4$ $Q\left(\frac{1}{2}\right) = 2 \cdot 3^{1/2} \approx 3,46$ $Q(1) = 6$

3) $Q(t) = 54 = 2 \cdot 3^t \Leftrightarrow 3^t = 27 \Leftrightarrow 3^t = 3^3 \Rightarrow t = 3$ h

EXERCICE 32

$Q = 5 \cdot 0.9^t$

1) $Q(t) = 1 \Leftrightarrow 5 \cdot 0,9^t = 1 \Rightarrow 0,9^t = 1/5 \Rightarrow t = \frac{\ln 1/5}{\ln 0.9} \approx 15.3$

2) 70 min $\sim \frac{7}{6}$ h

$Q(7/6) = 5 \cdot 0.9^{7/6} \approx 4.42$

EXERCICE 33

$V_t = 10,2 \cdot 2,36^t$

1) 30 min $\rightsquigarrow \frac{1}{2}$ h $V\left(\frac{1}{2}\right) = 10,2 \cdot 2,36^{1/2} = 15,67$

2) $V(t) = 46 \Leftrightarrow 10,2 \cdot 2,36^t = 46 \Leftrightarrow 2,36^t = 4,51 \Rightarrow$
$t = \frac{\ln 4.51}{\ln 2.36} \Rightarrow t \approx 1.75\ t$

EXERCICE 34

$N = 2 \cdot 10^4 \cdot 0,97^{n-1}$

1) $N(6) = 2 \cdot 10^4 \cdot 0.97^{6-1} \approx 17174.6$

2) $N(t) = 12\,000 = 20000 \cdot 0,97^{n-1} \Leftrightarrow$
$\Rightarrow \frac{12}{20} = 0,97^{n-1} \Rightarrow n-1 = \frac{\ln 3/5}{\ln 0,97} \Rightarrow n-1 = 16.771$

$\Rightarrow n = 17,771$

EXERCICE 35

$V = 2\cdot10^4\cdot0.8^{n-1} = 12\cdot10^3 \Leftrightarrow 0{,}8^{n-1} = \frac{12}{20} \Rightarrow$

$(n-1)\ln 0.8 = \ln\frac{12}{20} \Leftrightarrow n\ln 0.8 = \ln\frac{3}{5} + \ln 0.8 \Leftrightarrow$

$\Leftrightarrow n(-0.22) = -0.51 + (-0.22) \Leftrightarrow n = 3{,}32$ ans

EXERCICE 36

$P_0 = 10^5$

$t=0 \quad 10^5$

$t=20 \quad 2\cdot10^5 \qquad \Rightarrow P(t) = 8^t\cdot10^5$ t en heures.

$t=40 \quad 2^2\cdot10^5$

$t=60 \quad 2^3\cdot10^5$

a) $P(1) = 8\cdot10^5 \quad P(2) = 8^2\cdot10^5 \quad P(1.5) = 8^{1.5}10^5$

b) $P(x) = 8^x\cdot10^5$

c) $P(x) = 20\cdot\cancel{10^5} = 8^t\cdot\cancel{10^5} \Leftrightarrow t\ln 8 = \ln 20 \Leftrightarrow$

$\Leftrightarrow t = \frac{\ln 20}{\ln 8} \Rightarrow t = 1{,}44$ h $= 1$ h 26 min 26 sec

$P(x) = 10^3\cdot\cancel{10^5} = 8^t\cdot\cancel{10^5} \Leftrightarrow t\ln 8 = \ln 10^3 \Leftrightarrow$

$t = \frac{\ln 10^3}{\ln 8} \Rightarrow t = 3.32$ h $= 1$ h 19 min 19 sec

d) $P(t) = P_0 + \frac{15}{100}P_0 = P_0(1+0{,}15) \Rightarrow$

$P(t) = 1.15P_0 \Leftrightarrow P_0\cdot8^t = 1.15P_0 \Rightarrow 8^t = 1{,}15 \Leftrightarrow$

$t\ln 8 = \ln 1{,}15 \Rightarrow t = \frac{\ln 1.15}{\ln 8} \Rightarrow t = 0.0672$ h

4 min 2 sec

EXERCICE 37

$P(t) = 1800\cdot1{,}03^t =$

$P(123) = 1800\cdot1{,}03^{123} = 68273{,}34$

EXERCICE 38

$P_0 = 6700 \qquad P(25) = 9095 \qquad P(t) = P_0\cdot a^t$

$P(25) = 6700\cdot a^{25} = 9095 \Leftrightarrow a^{25} = 1{,}35 \Leftrightarrow$

$a \simeq \sqrt[25]{1{,}35} \Rightarrow a = 1{,}012$

EXERCICE 39

$P(t) = P_0\cdot\left(1+\frac{1}{4}\right)^t = P_0\left(\frac{5}{4}\right)^t = 2P_0 \Leftrightarrow \left(\frac{5}{4}\right)^t = 2 \Leftrightarrow$

$t\ln\frac{5}{4} = \ln 2 \Rightarrow t \simeq 3.1$ ans

EXERCICE 40

1) $P(t) = 6 \cdot 10^3 \left(1 + \frac{5}{100}\right)^t = 6 \cdot 10^3 \; 1.05^t$

$P(6) = 6 \cdot 10^3 \; 1.05^6 \approx 8 \cdot 10^3$

2. $P(t) = 15 \cdot 10^3 = 6 \cdot 10^3 \cdot 1.05^t \Leftrightarrow$

$\Leftrightarrow \frac{15}{6} = 1.05^t \Rightarrow t \ln 1.05 = \ln \frac{15}{6} \Leftrightarrow$

$t = 18.78$ ans

EXERCICE 41

1. $P(t) = 1 \cdot \left(1 + \frac{4}{100}\right)^t = \cdot 1.04^t$

$P(100) = \cdot 1.04^{100} = 50.5.-$

2) $P(t) = 1000 = \cdot 1.04^t \Leftrightarrow t = \frac{\log 1000}{\log 1.04} \approx 3$

EXERCICE 42

- $P(t) = P_0 \cdot 1.04^t = 2 P_0 \Leftrightarrow t \ln 1.04 = \ln 2 \Leftrightarrow$

  $t = 17.67$ ans
- $P(t) = P_0 \cdot 1.03^t = 2 P_0 \Leftrightarrow t \ln 1.03 = \ln 2 \Leftrightarrow$

  $t = 23.45$ ans

EXERCICE 43

$P(t) = 1200 \cdot \left(1 + \frac{a}{100}\right)^t$

- $P(12) = 2 \cdot 1200 = 1200 \cdot \left(1 + \frac{a}{100}\right)^{12} \Leftrightarrow$

  $\Leftrightarrow \left(1 + \frac{a}{100}\right)^{12} = 2 \Leftrightarrow 1 + \frac{a}{100} = \sqrt[12]{2} \Rightarrow$

  $\frac{a}{100} = \sqrt[12]{2} - 1 \Leftrightarrow a = (\sqrt[12]{2} - 1) \cdot 100 \Leftrightarrow a \approx 5.95\%$

- $P(12) = 3000 \left(1 + \frac{a}{100}\right)^{12} = 2 \cdot 3000 \Leftrightarrow 1 + \frac{a}{100} = \sqrt[12]{2} \Leftrightarrow$

  $\Leftrightarrow \frac{a}{100} = \sqrt[12]{2} - 1 \Rightarrow a = (\sqrt[12]{2} - 1) \cdot 100 \Rightarrow a = 5.95\%$

EXERCICE 44

$P(8) = P_0 \cdot 1.03^8 = 12345 \Rightarrow P_0 = \frac{12345}{1.03^8} \Rightarrow$

$P_0 \approx 9745.-$

EXERCICE 45

$\frac{P(6) - P(0)}{P(0)} = \frac{P(6) \cdot 1.035^6 - P(0)}{P(0)} = 0.23 \approx 23\%$

EXERCICE 46.

$P(t) = 1000\,(1+0{,}12)^t = 1000\,(1{,}12)^t$

$P(\frac{1}{12}) = 1000 \cdot 1{,}12^{1/12} = 1009{,}5$

$P(\frac{1}{2}) = 1000 \cdot 1{,}12^{1/2} = 1058{,}3$

$P(1) = 1000 \cdot 1{,}12 = \boxed{1120}$

$P(20) = 1000 \cdot 1{,}12^{20} = 9646{,}3$

EXERCICE 47

$P(t) = 24 \cdot (1+0{,}06)^t = 24 \cdot 1{,}06^t$

$P(370) = 24 \cdot 1{,}06^{370} = 5{,}54 \cdot 10^{10}$

EXERCICE 48

$P(t) = 150 \cdot 1{,}025^t$

$150 \cdot 1{,}025^t = 35000 \Leftrightarrow 1{,}025^t = 233{,}\bar{3} \Rightarrow t = \dfrac{\ln 233{,}\bar{3}}{\ln 1{,}025}$

$\Rightarrow t = 220{,}8137$ ans $= 220$ ans 10 mois

EXERCICE 49

$Q(t) = 40000 \cdot (1-0{,}09)^t = 40000 \cdot 0{,}91^t$

$Q(9) = 40000 \cdot 0{,}91^9 = 17117{,}2$

EXERCICE 51

1) $P(t) = 2400\left(1+\frac{a}{100}\right)^t$

5 mois: $P(\frac{5}{12}) - P(0) = 35 \Leftrightarrow$

$\Leftrightarrow 2400\left(1+\frac{a}{100}\right)^{5/12} - 2400 = 35 \Leftrightarrow$

$\Leftrightarrow \left(1+\frac{a}{100}\right)^{5/12} - 1 = 0{,}0146 \Rightarrow \left(1+\frac{a}{100}\right)^{5/12} = 1{,}0146$

$\Leftrightarrow 1 + \frac{a}{100} = 1{,}03536 \Leftrightarrow \frac{a}{100} = 0{,}03536 \Rightarrow a = 3{,}5\%$

2) $P(t) = 5000\,(1{,}045)^t$

$P(13) = 5000\,(1{,}045)^{13} = 8860{,}98.-$

3) $P(t) = 3P_0 = P_0\,(1{,}04)^t \Rightarrow t = \dfrac{\ln 3}{\ln 1{,}04} \Rightarrow t = 28$

EXERCICE 50

a) $P(t) = 10^5 \cdot 1{,}02^t \quad P(10) = 10^5\,1{,}02^{10} = 121899{,}44$

b) $P(t) = P_0 \cdot (1{,}05)^t = 3P_0 \Rightarrow t = \dfrac{\ln 3}{\ln 1{,}05} = 22{,}52$

c) $P(17) = 25000 \cdot \left(1+\frac{a}{100}\right)^{17} = 48697 \Leftrightarrow$

$\Leftrightarrow \left(1+\frac{a}{100}\right)^{17} = 1{,}948 \Leftrightarrow 1+\frac{a}{100} = 1{,}04 \Leftrightarrow a = 4\%$

EXERCICE 52

1) $P(20) = 40000 \cdot \left(1+\frac{a}{100}\right)^{20} = 2 \cdot 40000 \Leftrightarrow$
$1+\frac{a}{100} = \sqrt[20]{2} \Leftrightarrow \frac{a}{100} = 0.0353 \Leftrightarrow a = 3.5\%$

2) $P(15) = 40000\left(1+\frac{3.25}{100}\right)^{15} = 64626.54$

3) $P(t) = 40000\,(1.04)^{t} = 3 \cdot 40000 \Rightarrow t = \frac{\ln 3}{\ln 1.04} = 28$

EXERCICE 53

- $P(4.5) = 10^6 \cdot 1.043^{4.5} = 1208591.09$
- $P(t) = 1208591.09 \cdot 1,056^{t} = 1400000 \Leftrightarrow$
  $\Leftrightarrow 1.056^{t} = 1.158 \Leftrightarrow t = \frac{\ln 1,158}{\ln 1.056} \simeq 2.69$

EXERCICE 57

a.

x	$-\infty$		-7		$-4/3$		$3/2$		10		$+\infty$
f		$-$	$\|\|$	$-$	0	$+$	$\|\|$		0		

b.

x	$-\infty$		-7		$3/2$		
f'		$-$	$\|\|$	$+$	$\|\|$	$-$	
f		$\searrow$		$\nearrow$		$\searrow$	

c.

x	$-\infty$		-7		$-4/3$		$3/2$	
f''		$-$	$\|\|$	$-$	0	$+$	$\|\|$	$+$
f		$\cap$		$\cap$		$\cup$		$\cup$

EXERCICE 58

1) $f(x) = \ln(3x) \Rightarrow f'(x) = \frac{1}{3x}\cdot 3 = \frac{1}{x}$

2) $f(x) = e^x(1-x) \Rightarrow f'(x) = e^x(1-x) - e^x = e^x(\cancel{1}-x\cancel{-1}) = -xe^x$

3) $f(x) = e^{-x} \Rightarrow f'(x) = -e^{-x}$

4) $f(x) = \frac{\ln(x)}{x^2} \Rightarrow f'(x) = \frac{\frac{1}{x}\cdot x^2 - \ln x\cdot 2x}{x^4} = \frac{x - 2x\ln x}{x^2} =$

$= \frac{\cancel{x}(1-2\ln x)}{x^{\cancel{2}}} = \frac{1-2\ln x}{x}$

5) $f(x) = e^x\cos(2x) \Rightarrow f'(x) = e^x\cos 2x - 2e^x\sin(2x) =$

$= e^x(\cos(2x) - 2\sin(2x))$

6) $f(x) = e^{x^2-2x} \Rightarrow f'(x) = e^{x^2-2x}(2x-2) = 2(x-1)e^{x^2-2x}$

7) $f(x) = \frac{x^5}{e^{x-1}} \Rightarrow f'(x) = \frac{5x^4\cdot e^{x-1} - x^5 e^{x-1}}{(e^{x-1})^2} = \frac{x^4\cancel{e^{x-1}}(5-x)}{(e^{x-1})^{\cancel{2}}} = \frac{x^4(5-x)}{e^{x-1}}$

8) $f(x) = \cancel{\ln e}^x = x \Rightarrow f'(x) = 1$

EXERCICE 59

$f(x) = \frac{2-x}{x}e^{-x}$ $\qquad D_f = \mathbb{R}\setminus\{0\}$

$f(x) = 0 \Leftrightarrow 2-x = 0 \Leftrightarrow x = 2$

car $e^{-x} > 0 \ \forall x \in \mathbb{R}$!!

	$-\infty$		0		2		$+\infty$
x		$-$	0	$+$		$+$	
$2-x$		$+$		$+$	0	$-$	
f		$-$	$\|\|$	$+$	0	$-$	

EXERCICE 60

$f(x) = 3^x = e^{\ln 3^x} = e^{x\ln 3}$

$f'(x) = e^{x\ln 3} \cdot \ln 3 = 3^x \cdot \ln 3$

EXERCICE 61

$f(x) = x^2 e^{3x} \qquad D_f = \mathbb{R}$

$f'(x) = 2x e^{3x} + x^2 \cdot 3 e^{3x} = x e^{3x}(2+3x) = 0 \Leftrightarrow$

$\Leftrightarrow x = 0$ ou $2 + 3x = 0 \Leftrightarrow x = -2/3$

	$-\infty$		$-2/3$		0		$+\infty$
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$	Max	$\searrow$	Min	$\nearrow$	

$\text{Max} = f\left(-\frac{2}{3}\right) = \frac{4}{9} e^{-2}$

$\text{Min} = f(0) = 0$

EXERCICE 62

$f(x) = (2x-1)e^{-x} \qquad D_f = \mathbb{R}$

$f'(x) = 2e^{-x} - (2x-1)e^{-x} = e^{-x}(2-2x+1) = (3-2x)e^{-x}$

$f''(x) = -2e^{-x} - (3-2x)e^{-x} = -e^{-x}(2+3-2x) = -e^{-x}(5-2x) = 0$

$\Leftrightarrow 5 = 2x \Leftrightarrow x = 5/2$

x	$-\infty$		$5/2$		$+\infty$
f''		$+$	0	$-$	
f		$\cup$	PI	$\cap$	

PI $f\left(\frac{5}{2}\right) = (5-1)e^{-5/2} = 4e^{-5/2}$

EXERCICE 63

$f(x) = \ln x$

1) $D_f =]0, +\infty[$

2) $f(x) = 0 \Leftrightarrow \ln x = 0 \Leftrightarrow x = 1$

	0		1		$+\infty$
f		$-$	0	$+$	

3) AV: $\lim\limits_{x \to 0^+} f(x) = -\infty$ AV $x = 0$

AH: $\lim\limits_{x \to +\infty} f(x) = +\infty$ $\quad \emptyset$

4) O_x: $y = 0 \Leftrightarrow x = 1$ $\quad I_x(1;0)$ $\quad O_y$: $x = 0$ impossible

5) $f'(x) = \frac{1}{x} = 0$ imp

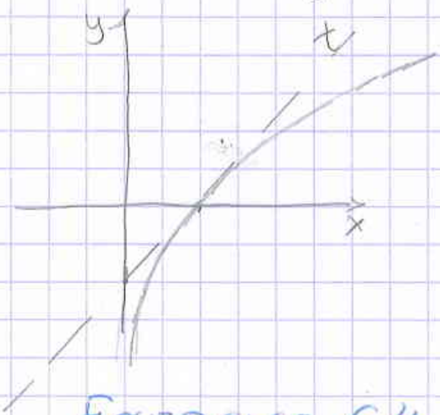
	0		$+\infty$
f'		$+$	
f		$\nearrow$	

6) $f''(x) = -\frac{1}{x^2} < 0 \quad \forall x \in D_f$

	0		$+\infty$
f''		$-$	
f		$\cap$	

7) $f(1)=0 \qquad f'(1)=1=m$

$$\left.\begin{aligned} y &= m\cdot x + h \\ 0 &= 1\cdot 1 + h \Rightarrow h=-1 \end{aligned}\right\} t: y = x-1$$



EXERCICE 64

1) $f(x)=\ln(5x) \Rightarrow f'(x)=\frac{1}{\cancel{5}x}\cdot\cancel{5}=\frac{1}{x}$

2) $f(x)=\frac{\ln x}{x^3} \Rightarrow f'(x)=\frac{\frac{1}{x}\cdot x^3-\ln x\cdot 3x^2}{x^6}=\frac{x^2-3x^2\ln x}{x^6}=$
$=\frac{\cancel{x^2}(1-3\ln x)}{x^{\cancel{6}4}}=\frac{1-3\ln x}{x^4}$

3) $f(x)=\cancel{\ln} e^{3x}=3x \Rightarrow f'(x)=3$

4) $f(x)=\ln(x^2-3x) \Rightarrow f'(x)=\frac{1}{x^2-3x}\cdot(2x-3)=\frac{2x-3}{x^2-3x}$

5) $f(x)=x\ln(7-x)$
$f'(x)=\ln(7-x)+x\cdot\frac{1}{7-x}\cdot(-1)=\ln(7-x)-\frac{x}{7-x}$

6) $f(x)=2\ln(\sqrt{x}) \Rightarrow f'(x)=\cancel{2}\cdot\frac{1}{\sqrt{x}}\cdot\frac{1}{\cancel{2}\sqrt{x}}=\frac{1}{\sqrt{x}^2}=\frac{1}{x}$

EXERCICE 65

1) $f(x)=x^3\ln x \rightarrow f'(x)=3x^2\ln x+x^3\cdot\frac{1}{x}=3x^2\ln x+x^2=$
$=x^2(3\ln x+1)$

2) $f(x)=\frac{3\ln x}{x^2} \Rightarrow f'(x)=\frac{3\cdot\frac{1}{\cancel{x}}\cdot x^{\cancel{2}}-3\ln x\cdot 2x}{x^4}=\frac{3x-3\cdot 2x\ln x}{x^4}=$
$=\frac{3\cancel{x}(1-2\ln x)}{x^{\cancel{4}3}}=\frac{3(1-2\ln x)}{x^3}$

3) $f(x)=e^{\frac{1}{2}x}\ln(4x) \Rightarrow f'(x)=\frac{1}{2}e^{\frac{1}{2}x}\ln(4x)+e^{\frac{1}{2}x}\frac{1}{\cancel{4}x}\cdot\cancel{4}=$
$=\frac{1}{2}e^{\frac{1}{2}x}\ln(4x)+\frac{1}{x}e^{\frac{1}{2}x}=e^{\frac{1}{2}x}\left(\frac{\ln(4x)}{2}+\frac{1}{x}\right)$

4) $f(x)=\log_5 x=\frac{\ln x}{\ln 5}$
$f'(x)=\frac{1}{\ln 5}(\ln x)'=\frac{1}{\ln 5}\cdot\frac{1}{x}=\frac{1}{x\ln 5}$

EXERCICE 66

$f(x) = (x-k)e^{-x} \qquad x=2$

1) PTH $\Rightarrow f'(x)=0$

$f'(x) = e^{-x} - (x-k)e^{-x} = e^{-x}(1-x+k)$

$f'(2) = 0 \Leftrightarrow e^{-2}(1+k-2)=0 \Leftrightarrow -1+k=0 \Leftrightarrow k=1$

2) $f(x) = (x-2)e^{-x} \qquad f'(x) = e^{-x}(2-x) = 0 \Leftrightarrow x=2$

	$-\infty$		2		$+\infty$
f'		+	0	−	
f		↗	Max	↘	

$Max = f(2) = 0$

EXERCICE 67

a) $f(x) = (2x^2+ax)e^{-x}$

$f'(x) = (4x+a)e^{-x} - (2x^2+ax)e^{-x} = e^{-x}(4x+a-2x^2-ax) =$

$= e^{-x}(-2x^2+(4-a)x+a)$

$f'(x)=0 \Leftrightarrow -2x^2+(4-a)x+a=0$

$\Delta = (4-a)^2+8a > 0 \quad \forall a \in \mathbb{R} \Rightarrow$ l'équation $f'(x)=0$ admet 2 solutions $\Rightarrow f$ 2 PTH

b) $f'(0) = 3 \Rightarrow e^0(a) = 3 \Rightarrow a=3$

EXERCICE 68

$f(x) = e^{-2x} \qquad g(x) = e^{x+3}$

1) $f(x) = g(x) \Leftrightarrow e^{-2x} = e^{x+3} \Leftrightarrow -2x = x+3 \Leftrightarrow +3x = -3 \Leftrightarrow x=-1$

$I(1 ; e^2)$

2) $f'(x) = -2e^{-2x} \qquad f'(-1) = -2e^2 \qquad \vec{a} = \begin{pmatrix} 1 \\ -2e^2 \end{pmatrix}$

$g'(x) = e^{x+3} \qquad g'(1) = e^2 \qquad \vec{b} = \begin{pmatrix} 1 \\ e^2 \end{pmatrix}$

$\cos\alpha = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|} = \dfrac{1-2e^4}{\sqrt{1+4e^4}\cdot\sqrt{1+e^4}} = \dfrac{-108.2}{14.81\cdot 7.46} = -0.979 \Rightarrow$

$\alpha = 168.32°$

$\vec{a}\cdot\vec{b} = 1 - 2e^2\cdot e^2 = 1-2e^4$

$\|\vec{a}\| = \sqrt{1+4e^4}$

$\|\vec{b}\| = \sqrt{1+e^4}$

EXERCICE 69

$f(x) = e^{2-\frac{1}{2}x}$

1) $D_f = \mathbb{R}$

2) $f(x) = 0$ impossible $f(x) > 0 \quad \forall x \in \mathbb{R}$

3) AV $\emptyset$ AH: $\lim\limits_{x\to+\infty} f(x) = 0 \quad y = 0 \quad (+\infty)$

$\lim\limits_{x\to-\infty} f(x) = +\infty \quad \emptyset \quad (-\infty)$

4) Ox: $y=0$ imp. Oy: $x=0 \quad y = e^2 \quad I_y(0, e^2)$

5) $f'(x) = e^{2-\frac{1}{2}x} \cdot \left(-\frac{1}{2}\right) = -\frac{e^{2-\frac{1}{2}x}}{2} = 0 \Rightarrow e^{2-\frac{1}{2}x} = 0$ imp.

$f'(x) < 0 \quad \forall x \in \mathbb{R} \qquad f \nearrow$

6) $f''(x) = -\frac{1}{2} e^{2-\frac{1}{2}x}\left(-\frac{1}{2}\right) = \frac{1}{4} e^{2-\frac{1}{2}x} > 0 \quad \forall x \in \mathbb{R} \Rightarrow f \cup$

EXERCICE 70

$f(x) = e^{-x^2}$

1) $D_f = \mathbb{R}$

2) AV $\emptyset$ AH. $\lim\limits_{x\to+\infty} e^{-x^2} = 0$ AH $y = 0 \quad (+\infty)$

$\lim\limits_{x\to-\infty} e^{-x^2} = +\infty \quad \emptyset \quad (-\infty)$

3) Ox: $y=0$ impossible Oy: $x = 0 \quad y = e^0 = 1 \quad I_y(0,1)$

4) $f'(x) = -2x\, e^{-x^2} = 0 \Leftrightarrow x = 0$

$Max = f(0) = 1$

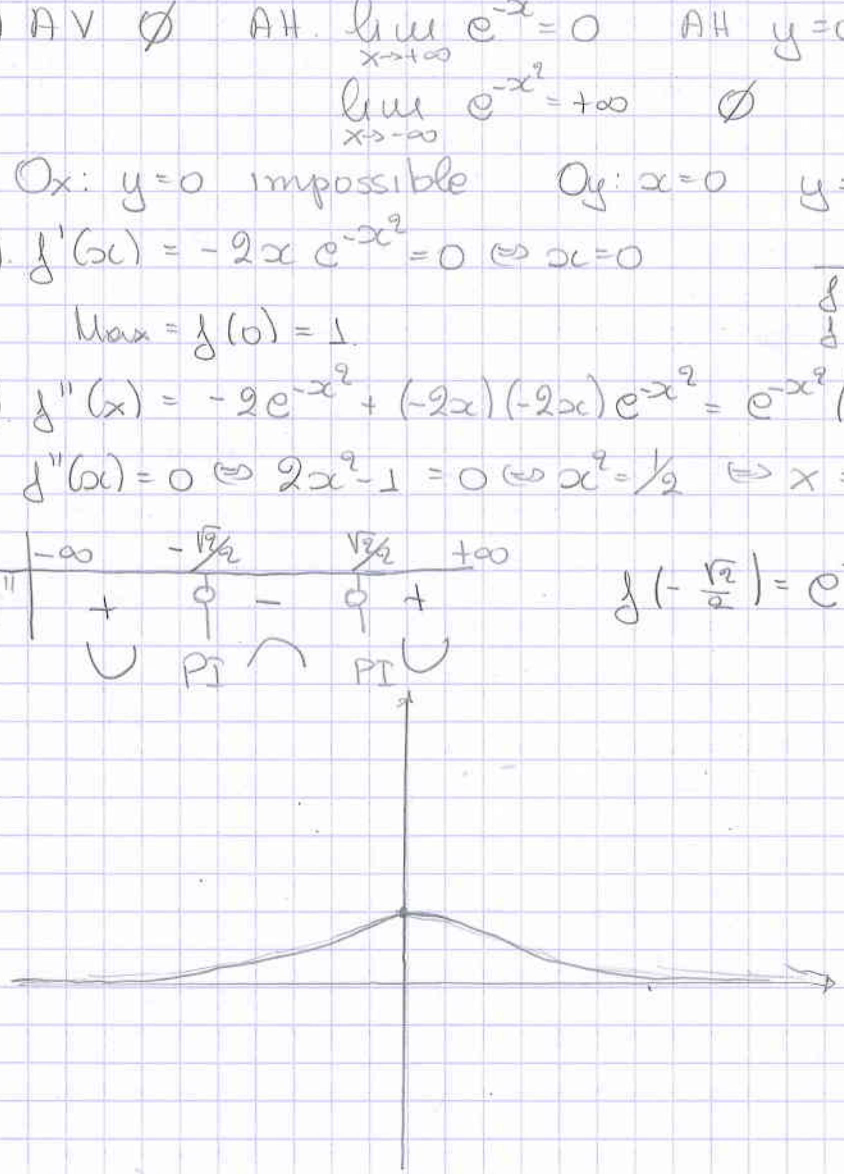
	$-\infty$		0		$+\infty$
f'		$+$	0	$-$	
f		$\nearrow$	Max	$\searrow$	

5) $f''(x) = -2e^{-x^2} + (-2x)(-2x)e^{-x^2} = e^{-x^2}(-2+4x^2) = 2e^{-x^2}(2x^2-1)$

$f''(x) = 0 \Leftrightarrow 2x^2 - 1 = 0 \Leftrightarrow x^2 = \frac{1}{2} \Leftrightarrow x = \pm\frac{\sqrt{2}}{2}$

	$-\infty$		$-\sqrt{2}/2$		$\sqrt{2}/2$		$+\infty$
f''		$+$	0	$-$	0	$+$	
f		$\cup$	PI	$\cap$	PI	$\cup$	

$f\left(-\frac{\sqrt{2}}{2}\right) = e^{-\frac{1}{2}} \qquad f\left(\frac{\sqrt{2}}{2}\right) = e^{-1/2}$



EXERCICE 71

$f(x) = 4x + 1 - 2e^x$

1) $D_f = \mathbb{R}$

2) AV $\emptyset$ AH: $\lim_{x \to +\infty} f(x) = -\infty$ $\emptyset$ $(+\infty)$

$\lim_{x \to -\infty} f(x) = -\infty$ $\emptyset$ $(-\infty)$

3) O_y: $x = 0$ $y = 1 - 2 \cdot 1 = -1$ $(0; -1)$

4) $f'(x) = 4 - 2e^x = 0 \Leftrightarrow e^x = 2 \Leftrightarrow x = \ln 2$

	$-\infty$	$\ln 2$	$+\infty$
f'	+	0	−
f	↗	Max	↘

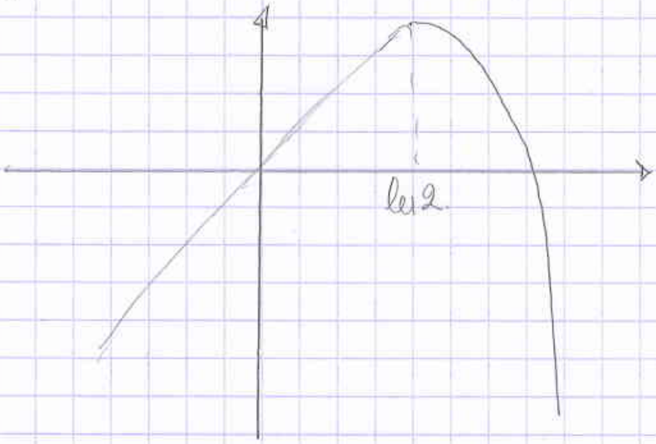
$Max = f(\ln 2) = 4\ln 2 + 1 - 2e^{\ln 2} =$

$= 4\ln 2 + 1 - 4 = 4\ln 2 - 3$

5) $f''(x) = -2e^x = 0$ impossible

	$-\infty$	$+\infty$
f''	−	
f	$\frown$	

6)



EXERCICE 72

$f(x) = x^n e^x$ $f'(x) = nx^{n-1}e^x + x^n e^x = e^x x^{n-1}(n+x) = 0 \Leftrightarrow$

$\Leftrightarrow x = 0$ $x = -n$

$f''(x) = e^x x^{n-1}(n+x) + e^x(n-1)x^{n-2}(n+x) + e^x \cancel{x}^{n-1} =$

$= e^x x^{n-2}\left(x(n+x) + (n-1)(n+x) + x\right) =$

$= e^x x^{n-2}\left(x^2 + nx + n(n-1) + (n-1)x + x\right) =$

$= e^x x^{n-2}\left(x^2 + (n + n - 1 + 1)x + n(n-1)\right) =$

$= e^x x^{n-2}\left(x^2 + 2nx + n(n-1)\right) = 0 \Leftrightarrow$ $x = 0$, $x^2 + 2nx + n(n-1) = 0$

$\Delta = 4n^2 - 4n(n-1) = 4n^2 - 4n^2 + 4n = 4n$

$x_{1,2} = \dfrac{-2n \pm \sqrt{4n}}{2} = -n \pm \sqrt{n}$

EXERCICE 73

$f(x) = e^{-kx^2}$ PI en $x=1$

$f'(x) = -2kx\,e^{-kx^2}$ $f''(x) = -2k\,e^{-kx^2} + (2kx)(2kx)e^{-kx^2} =$

$= e^{-kx^2}(-2k + 4k^2x^2) = 2k\,e^{-kx^2}(-1+2kx^2)$

$f''(x) = 0 \Leftrightarrow 2k = 0$ ou $-1 + 2kx^2 = 0$

Pour $x = 1$ $k = 0$ ou $-1 + 2k\cdot 1 = 0 \Leftrightarrow k = 1/2$

EXERCICE 74

$f(x) = (x+p)e^{x}$

- $f'(x) = e^{x} + (x+p)e^{x} = e^{x}(1+x+p) = 0 \Leftrightarrow x = -1-p$

$y = (-1-p+p)e^{-1-p} = -e^{-1-p} = -\frac{1}{e^{1+p}}$

Min $\left(-1-p \,;\, -\frac{1}{e^{1+p}}\right)$

$\left.\begin{array}{l} x = -1-p \\ y = -e^{-1-p} \end{array}\right\}$ $\underline{y = -e^{x}}$ $\rightarrow$ le lieu géométrique des minimums

- $f''(x) = e^{x}(1+x+p) + e^{x} = e^{x}(1+x+p+1) = e^{x}(2+x+p) = 0$

$\Leftrightarrow x = -2-p$ $y = (-2-p+p)e^{-2-p} = -2e^{-2-p}$

Donc $y = -2e^{x}$ $\rightarrow$ le lieu géométrique des PI

EXERCICE 75

$f(x) = 3x\ln(2x)$

- $f'(x) = 3\ln(2x) + 3\cancel{x}\frac{1}{\cancel{2x}}\cdot\cancel{2} = 3\ln(2x) + 3 = 3(\ln 2x + 1) = 0$

$\Leftrightarrow \ln 2x = -1 \Leftrightarrow 2x = e^{-1} \Leftrightarrow x = \frac{1}{2e}$

- $f(x) = x^4\ln x \Rightarrow f'(x) = 4x^3\ln x + x^4\frac{1}{x} = 4x^3\ln x + x^3 =$

$= x^3(4\ln x + 1) = 0 \Leftrightarrow$

$\Leftrightarrow x = 0$ imp car $D_f =]0, +\infty[$

$4\ln x = -1 \Leftrightarrow \ln x = -1/4 \Rightarrow x = e^{-1/4}$

EXERCICE 76

$f(x) = \ln\left(\frac{3x-2}{x-6}\right)$

a. $\frac{3x-2}{x-6} > 0$

	$-\infty$	$2/3$		6	$+\infty$
$3x-2$	$-$	0	$+$		$+$
$x-6$	$-$		$-$	0	$+$
Q	$+$	0	$-$	$\|\|$	$+$

$D_f =]-\infty ; 2/3[\cup]6, +\infty[$

b. $f(x) = \ln\left(\frac{3x-2}{x-6}\right) = 0 \Leftrightarrow \frac{3x-2}{x-6} = 1 \Leftrightarrow 3x-2 = x-6 \Leftrightarrow 2x = -4 \Leftrightarrow x = -2$

c. AV $\lim\limits_{x \to 2/3^-} f(x) = -\infty$ AV : $x = 2/3$

$\lim\limits_{x \to 6^+} f(x) = \lim\limits_{u \to +\infty} \ln u = +\infty$ AV : $x = 6$

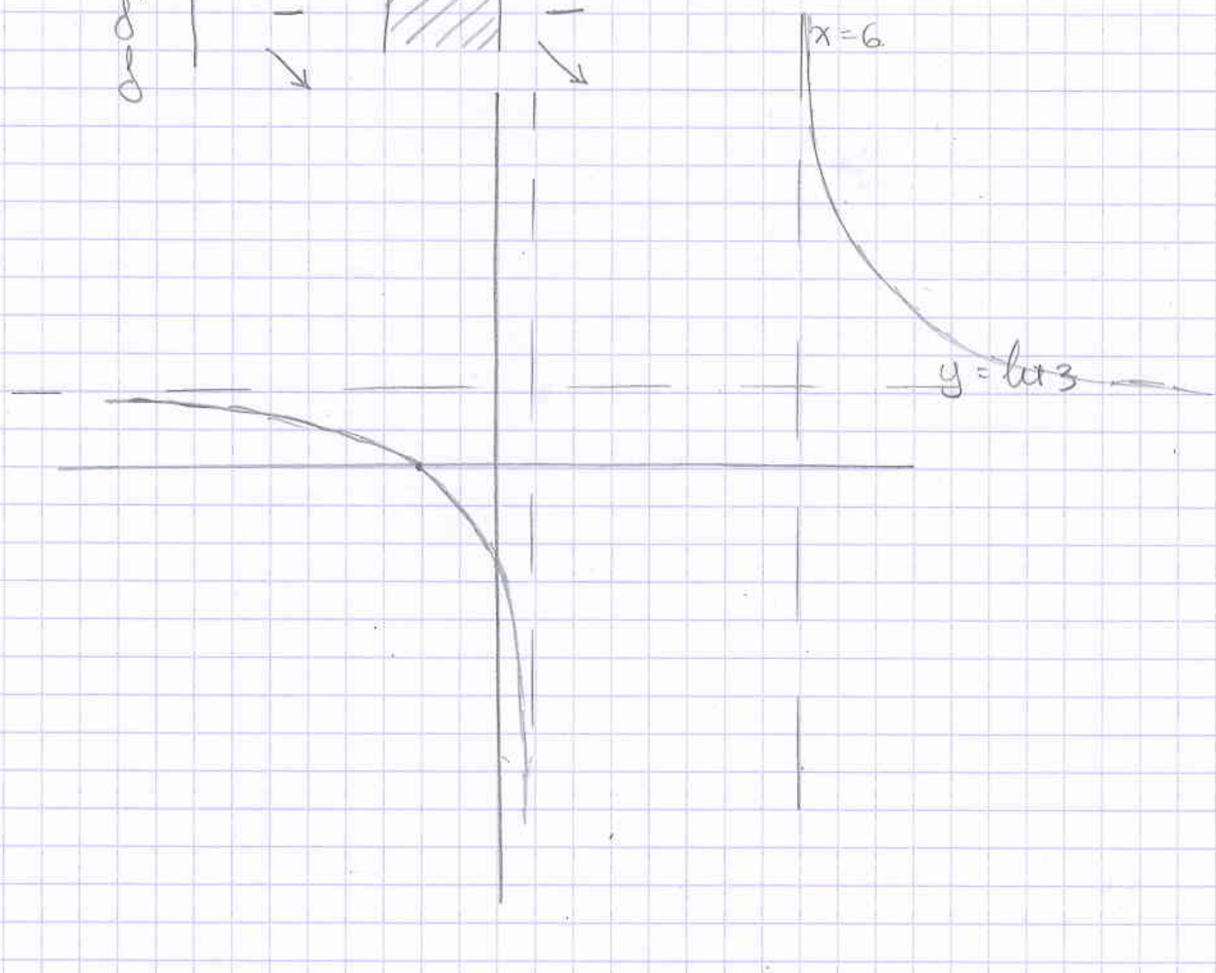
d. AH. $\lim\limits_{x \to +\infty} \ln\left(\frac{3x-2}{x-6}\right) = \ln 3$ AH : $y = \ln 3$ $(+\infty)$

$\lim\limits_{x \to -\infty} \ln\left(\frac{3x-2}{x-6}\right) = \ln 3$ AH : $y = \ln 3$ $(-\infty)$

e. $f'(x) = \frac{x-6}{3x-2} \cdot \frac{3(x-6)-(3x-2)}{(x-6)^{\cancel{2}}} = \frac{\cancel{3x}-18-\cancel{3x}+2}{(3x-2)(x-6)} = \frac{-16}{(3x-2)(x-6)}$

$f'(x) = 0 \Rightarrow -16 = 0$ impossible

	$-\infty$		$2/3$		6		$+\infty$
f'		$-$		//////		$-$	
f		↘		//////		↘	



EXERCICE 77

$f(x) = \ln \frac{4x+2}{x-1}$

$\frac{4x+2}{x-1} > 0$

x	$-\infty$		$-1/2$		1		$+\infty$
$4x+2$		$-$	0	$+$		$+$	
$x-1$		$-$		$-$	0	$+$	
Q		$+$	0	$-$	$\|\|$	$+$	

$D_f =]-\infty, -1/2[\cup]1, +\infty[$

$\lim_{x \to \pm\infty} f(x) = \ln 4 \implies$ AH $y = \ln 4$

EXERCICE 78

$f(x) = x \ln(x^2)$

1) • $x^2 > 0 \Rightarrow x \neq 0 \quad D_f = \mathbb{R} \setminus \{0\}$

• $f(x) = 0 \Leftrightarrow x = 0$ impossible

$\ln(x^2) = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$

	$-\infty$		-1		0		1		$+\infty$
x		$-$		$-$	0	$+$		$+$	
$\ln x^2$		$+$	0	$-$		$-$	0	$+$	
f		$-$	0	$+$	$\|\|$	$-$	0	$+$	

• $\lim_{x \to 0} x \ln(x^2) = 0 \cdot (-\infty) = 0$ Trou $(0;0)$

AV

AH: $\lim_{x \to +\infty} f(x) = +\infty \cdot (+\infty) \quad \emptyset$

$\lim_{x \to -\infty} f(x) = -\infty \cdot (+\infty) = -\infty \quad \emptyset$

• $f'(x) = \ln(x^2) + x \cdot \frac{1}{x^2} \cdot 2x = \ln(x^2) + 2 = 0 \Leftrightarrow$

$\Leftrightarrow \ln(x^2) = -2 \Leftrightarrow x^2 = e^{-2} \Leftrightarrow x = \pm \frac{1}{e}$

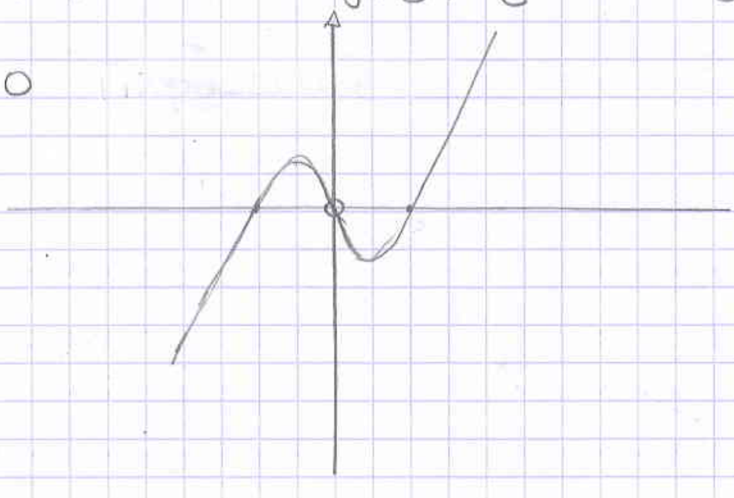
		$-1/e$		0		$1/e$		$+\infty$
f'	$+$	0	$-$	$\|\|$	$-$	0	$+$	
f	$\nearrow$	Max	$\searrow$		$\searrow$	Min	$\nearrow$	

Max $= f\left(-\frac{1}{e}\right) = -\frac{1}{e} \ln e^{-2} = \frac{2}{e}$

Min $= f\left(\frac{1}{e}\right) = \frac{1}{e} \ln e^{-2} = -\frac{2}{e}$

• $f''(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x} = 0$ imp

	$-\infty$		0		$+\infty$
f''		$-$	$\|\|$	$+$	
f		$\cap$	$\|\|$	$\cup$	



2. $f(e) = e \cdot \ln(e^2) = e \cdot 2 = 2e = y$

$f'(e) = \ln(e^2) + 2 = 4 = m$

$$\left.\begin{array}{l} y = m \cdot x + h \\ 2e = 4 \cdot e + h \Rightarrow h = -2e \end{array}\right\} t: y = 4x - 2e$$

Exercice 79

$f(x) = f(x) = \ln(x^2 - 2x)$

- D_f : $x^2 - 2x > 0 \Leftrightarrow x(x-2) > 0$

x	$-\infty$		0		2		$+\infty$
		$+$	0	$-$	0	$+$	

$D_f =]-\infty; 0[\cup]2, +\infty[$

- AV: $\lim_{x \to 0} f(x) = -\infty \qquad x = 0$

$\lim_{x \to 2} f(x) = -\infty \qquad x = 2$

AH: $\lim_{x \to \pm\infty} f(x) = +\infty \qquad \emptyset$

- O_x: $y = 0 \Leftrightarrow \ln(x^2 - 2x) = 0 \Leftrightarrow x^2 - 2x = 1 \Leftrightarrow x^2 - 2x - 1 = 0$

$\Delta = 4 + 4 = 8 \qquad x_{1,2} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$; $x_1 = 2.41$, $x_2 = -0.41$

$I_1(-0.41; 0) \quad I_2(2.41; 0)$

O_y: $x = 0$ impossible

- $f'(x) = \frac{2x-2}{x^2-2x} = \frac{2(x-1)}{x^2-2x} = 0 \Leftrightarrow x = 1$ impossible

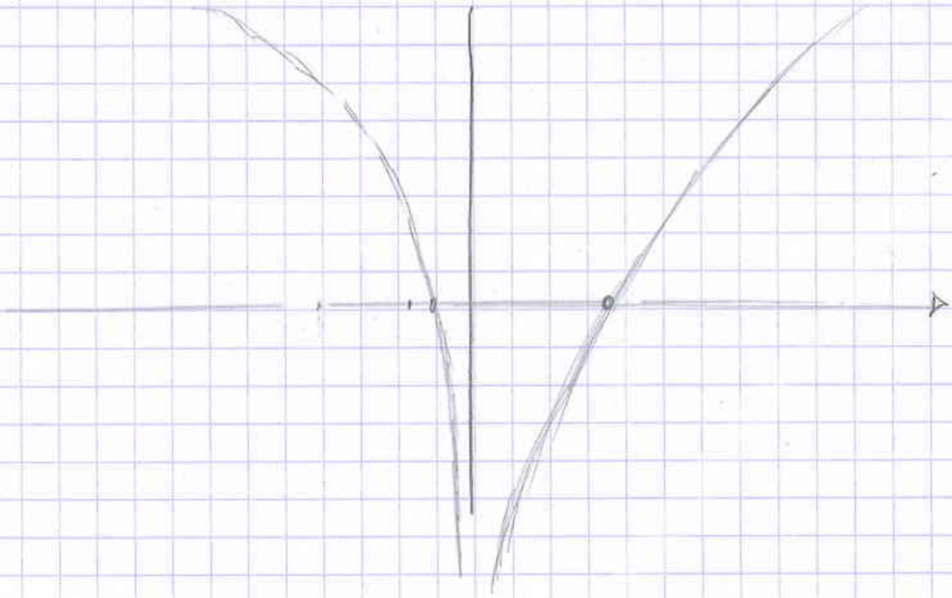
	$-\infty$	0	2	$+\infty$
f'	$-$			$+$
f	↘			↗

- $f''(x) = \frac{2(x^2-2x) - (2x-2)(2x-2)}{(x^2-2x)^2} = \frac{2x^2 - \cancel{4x} - 4x^2 + \cancel{4x} + 4x - 4}{(x^2-2x)^2} =$

$= \frac{-2x^2 + 4x - 4}{(x^2-2x)^2} = \frac{-2(x^2 - 2x + 2)}{(x^2-2x)^2} = 0 \Leftrightarrow x^2 - 2x + 2 = 0$

$\Delta = 4 - 8 < 0 \Rightarrow$

	$-\infty$	0	2	$+\infty$
f''	$-$			$-$
f	$\frown$			$\frown$



EXERCICE 80

$f(x) = \ln(x^2+bx+c) \qquad (-3;0)$

$f(-3) = \ln(9-3b+c) = 0 \Leftrightarrow 9-3b+c = 1 \Leftrightarrow -3b+c = -8$

$f'(x) = \dfrac{1}{x^2+bx+c} \cdot (2x+b) = \dfrac{2x+b}{x^2+bx+c}$

$f'(-3) = 0 \Leftrightarrow -6+b = 0 \Leftrightarrow b = 6 \Rightarrow c = 10$

EXERCICE 81

$f(x) = \ln(x^2-4x) \qquad x = 2+t \qquad x = 2-t$

$f'(x) = \dfrac{2x-4}{x^2-4x} \qquad f'(2+t) = \dfrac{2(2+t)-4}{(2+t)^2-4(2+t)} = \dfrac{4+2t-4}{(2+t)(2+t-4)} = \dfrac{2t}{(2+t)(t-2)} = m_1$

$f'(2-t) = \dfrac{2(2-t)-4}{(2-t)^2-4(2-t)} = \dfrac{4-2t-4}{(2-t)(2-t-4)} = \dfrac{-2t}{(2-t)(-2-t)} = \dfrac{-2t}{-(2-t)(2+t)}$

$m_1 = \dfrac{2t}{t^2-4} \qquad m_2 = \dfrac{-2t}{-(4-t^2)} = \dfrac{2t}{4-t^2}$

Donc $m_1 = -m_2$