DDR - Niveau 2: TE 11 Calcul Integral.

Solutions

Problème 1

$f(x) = \left(\frac{1}{3}x - 1\right)^6 + 4 \qquad f(6) = 1 + 4 = 5 \qquad T\begin{pmatrix} x & y \\ 6 & 5 \end{pmatrix}$

$f'(x) = 6\left(\frac{1}{3}x - 1\right)^5 \cdot \frac{1}{3} = 2\left(\frac{1}{3}x - 1\right)^5 \qquad f'(6) = 2 = m$

$y = m \cdot x + h$

$5 = 2 \cdot 6 + h \Leftrightarrow h = -7$ $\Big\}$ $t(x) = 2x - 7$

$t(x) = 0 \Leftrightarrow 2x - 7 = 0 \Leftrightarrow x = \frac{7}{2}$

$F(x) = \frac{1}{7} \cdot 3\left(\frac{1}{3}x - 1\right)^7 + 4x = \frac{3}{7}\left(\frac{1}{3}x - 1\right)^7 + 4x$

$T(x) = \frac{2}{2}x^2 - 7x = x^2 - 7x$

$A = \int_{7/2}^{6} (f(x) - t(x))\,dx = (F(6) - T(6)) - \left(F\left(\frac{7}{2}\right) - T\left(\frac{7}{2}\right)\right) \approx 4,1786$

Problème 2

$f(x) = -2\cos(wx) = 0 \Leftrightarrow \cos(wx) = 0 \Leftrightarrow wx = \frac{\pi}{2} + k\pi$

$k = 0 \qquad wx = \frac{\pi}{2} \Leftrightarrow x = \frac{\pi}{2w} \qquad k = 1 \cdot wx = \frac{\pi}{2} + \pi = \frac{3\pi}{2} \Leftrightarrow x = \frac{3\pi}{2w}$

$A = \int_{\frac{\pi}{2w}}^{\frac{3\pi}{2w}} f(x)\,dx = 48 \Leftrightarrow -\frac{2}{w}\sin(wx)\Big|_{\frac{\pi}{2w}}^{\frac{3\pi}{2w}} = 48 \Leftrightarrow$

$\Leftrightarrow -\frac{2}{w}\sin\left(w \cdot \frac{3\pi}{2w}\right) + \frac{2}{w}\sin\left(w \cdot \frac{\pi}{2w}\right)\Big) = 48 \Leftrightarrow$

$\Leftrightarrow -\frac{2}{w}\sin\frac{3\pi}{2} + \frac{2}{w}\sin\frac{\pi}{2} = 48 \Leftrightarrow +\frac{2}{w} + \frac{2}{w} = 48 \Leftrightarrow$

$\Leftrightarrow \frac{4}{w} = 48 \Leftrightarrow w = \frac{4}{48} \Leftrightarrow w = \frac{1}{12}$

Problème 3

$f(x) = e^{4x-16} \qquad f(x) = 1 \Leftrightarrow e^{4x-16} = 1 \Leftrightarrow 4x - 16 = 0 \Leftrightarrow x = 4$

$A(4;1) \qquad F(x) = \frac{1}{4}e^{4x-16}$

$A = \int_4^m f(x)\,dx = 10.75 \Leftrightarrow \frac{1}{4}e^{4x-16}\Big|_4^m = 10.75 \Leftrightarrow \frac{1}{4}e^{4m-16} - \frac{1}{4}e^{16-16} = 10.75$

$\Leftrightarrow e^{4m-16} - 1 = 43 \Leftrightarrow e^{4m-16} = 44 \Leftrightarrow 4m - 16 = \ln 44 \Leftrightarrow$

$\Leftrightarrow 4m = \ln 44 + 16 \Leftrightarrow m = \frac{\ln 44}{4} + 4$

2- **Problème 4**

$f(x) = \sqrt{(ax+b)^3} = (ax+b)^{3/2}$ $\quad a \neq 0$

$F(x) = \frac{1}{\frac{3}{2}+1} \frac{1}{a} (ax+b)^{\frac{3}{2}+1} = \frac{2}{5a} (ax+b)^{5/2} = \frac{2}{5a} \sqrt{(ax+b)^5}$