LDDR – Niveau 2: TE 10

3MG Level 2	INDUCTION COMBINATORICS	TEST#1A
2018/10/02	PROBABILITIES CALCULUS	3MG01

With formulaire /42 pts Name: _____ 90'

Exercise 1. [/8pts]

1) Use induction to show that for $n \geq 1$: $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$

2) Verify the inductive step for the assertion " $4^n + 1$ is divisible by 3 for $n \in \mathbb{N}$ " and determine whether the assertion is true or not.

Exercise 2. [/8pts]

1) How many different anagrams of the word « *SENTENCE* » are there ?

2) How many different ways are there to select two letters from « *SENTENCE* » ?

3) When forming at random an anagram of the word « *SENTENCE* » what's the probability that the three "*E*" are "together" ?

4) A bunch of 10 roses is to be formed from 6 different possible colors. How many different such bunches are there ? (*the order of the flower not being important, only the colors matter*)

5) Determine the number $\frac{100!}{96! \cdot 3!}$

Exercise 3. [/16pts]

1) In a sample space we consider two events A and B such that $p(A) = 0.75, P(\bar{B}) = 0.4$ and $p(A \cap B) = 0.4$. Determine $p(A \cup B)$, $p(\bar{A} \cap B)$ and $p(A|\bar{B})$.

2) Someone invites you to play that game

   "Let's roll two fair six-sided dice. If the product of the points is (strictly) smaller than 6 points, I win 10.- ; if the sum is (strictly) larger than 10 points I win 20.- Else you win x.-"

   For what possible amounts *x* (in swiss francs) do you accept to play ? Justify your answer.

3) When selecting 2 cards from a 36 cards deck. What's the probability that
   - the cards have different colors ? (*colors : hearth, diamond, club, spade*)
   - the cards have same symbol or same color ? (*symbols : 6,7,8,9,10, jack, queen, king, ace*)

4) Two types of confetti bags have been sold last week-end.

   60% of them were of "Type A" that contains 20% of red, 30% of yellow and 50% of blue confettis.

   The other were of "Type B" that contains 35% of red, 50% of yellow and 15% of green confettis.

   - Draw the tree diagram of the situation.
   - Determine the probability for a randomly found piece of confetti to be red.
   - A randomly found piece of confetti is green. What's its probability to come from a "Type A" bag ?
   - A randomly found piece of confetti is red. What's its probability to come from a "Type B" bag ?

Exercise 4. [/10pts]

1) What is the geometrical meaning of the number $f'(a)$ for a given function f and a real number a ? Give a precise answer.

2) Determine the derivative of $f(x) = \cos(3x) + \frac{4x^2 - x}{2x+1}$

3) Determine the equation of the tangent to the graph of $f(x) = x^2 \cdot \sin(x)$ at its point with abscissa $x = 2$.

4) We consider the graph of the function $f(x) = x^4 - 8x + 10$. Determine the coordinates of the point on that curve that is the closest to the line $y = 2$. What's the measure of that shortest distance ?