

TE 7 Fonctions.

EXERCICE 1. $f(x) = x^2 - 1$ $g(x) = \frac{x-3}{3x-1}$

a) $(g \circ f)(x) = g(f(x)) = \frac{x^2-1-3}{3(x^2-1)-1} = \frac{x^2-4}{3x^2-4}$

b) $y = \frac{x-3}{3x-1} \Leftrightarrow 3xy - y = x - 3 \Leftrightarrow 3yx - x = y - 3 \Leftrightarrow$
 $\Leftrightarrow (3y-1)x = y-3 \Leftrightarrow x = \frac{y-3}{3y-1}$

$g^{-1}: \mathbb{R} \setminus \{1/3\} \rightarrow \mathbb{R} \setminus \{1/3\}$ $g^{-1}(x) = \frac{x-3}{3x-1} = g(x)$

c) On conclut $g^{-1} = g$. Alors $(g \circ g \circ g)(x) = (g^{-1} \circ g \circ g)(x) = g(x)$.

EXERCICE 2

$f: x \mapsto y = \frac{3x+3}{2x} + \frac{3}{2} = \frac{3x+3+3x}{2x} = \frac{6x+3}{2x}$

a.

• $D_f = \mathbb{R} \setminus \{0\}$

• $\text{NO}_x: y = 0 \Leftrightarrow \frac{3x+3}{2x} = -\frac{3}{2} \Leftrightarrow \frac{x+1}{x} = -1 \Leftrightarrow x+1 = -x \Leftrightarrow 2x = -1$
 $\underline{I_x(-1/2; 0)} \quad \Leftrightarrow x = -1/2$

$\text{NO}_y: \text{imp.}$

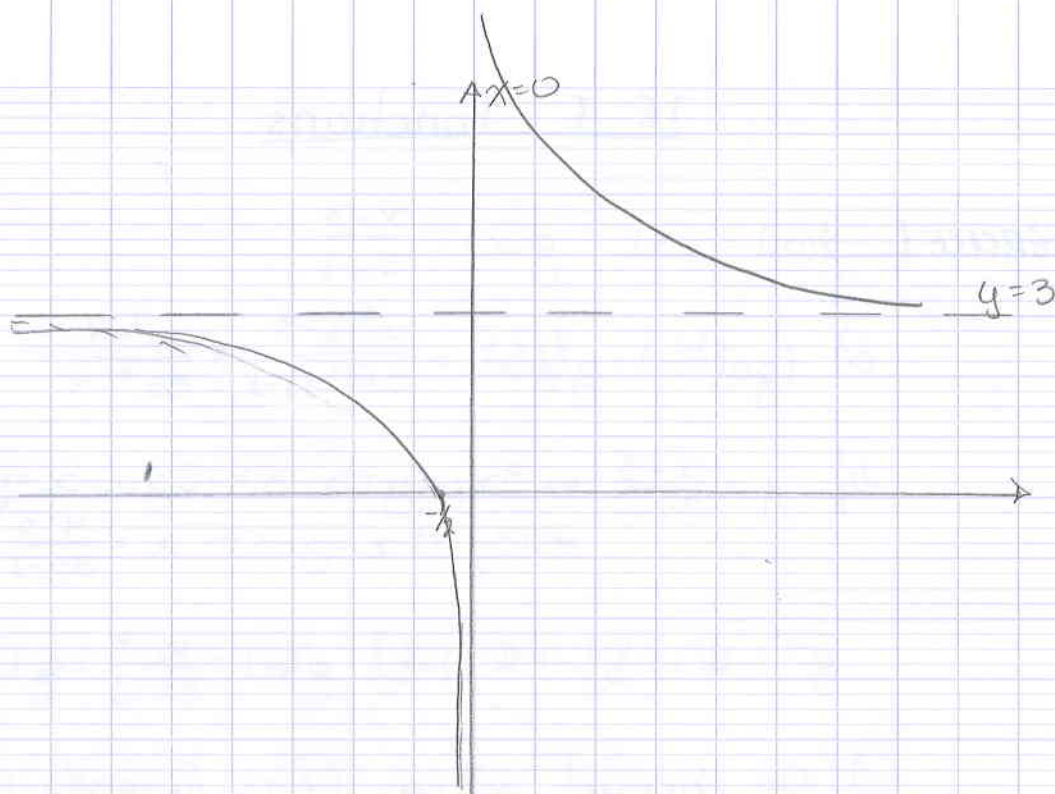
• tableau des signes:

x	$-\infty$	$-1/2$	0	$+\infty$
$6x+3$	-	0	+	+
$2x$	-	-	0	+
f	+	0	-	+

• A.V: $x = 0$

A.H: $y = 3$

$\lim_{x \rightarrow 0^-} f(x) = -\infty$ $\lim_{x \rightarrow 0^+} f(x) = +\infty$
 $\lim_{x \rightarrow \pm\infty} f(x) = +\infty$



$$b) \begin{cases} y = \frac{6x+3}{2x} \\ y = 12x \end{cases} \Leftrightarrow \frac{6x+3}{2x} = 12x \Leftrightarrow 6x+3 = 24x \Leftrightarrow 18x = 3 \Leftrightarrow x = \frac{1}{6} \quad y = 2$$

$$I\left(\frac{1}{6}; 2\right)$$

$$c) \begin{cases} y = kx \\ y = \frac{6x+3}{2x} \end{cases} \Rightarrow kx = \frac{6x+3}{2x} \Leftrightarrow 2kx^2 - 6x - 3 = 0$$

$$\Delta = 0 \Leftrightarrow 36 + 24k = 0 \Leftrightarrow k = -\frac{36}{24} \Leftrightarrow k = -\frac{6}{4} = \boxed{-\frac{3}{2} = k}$$

EXERCICE 3

$$a) 10^{3x^2-15} = 1 \Leftrightarrow 10^{3x^2-15} = 10^0 \Leftrightarrow 3x^2 - 15 = 0 \Leftrightarrow x^2 = 5 \Leftrightarrow x = \pm\sqrt{5}$$

$$b) \log(\sqrt{x-1}) = 1 \quad x > 1$$

$$\Leftrightarrow \sqrt{x-1} = 10^1 \Leftrightarrow x-1 = 10^2 \Leftrightarrow x = 101 \quad \checkmark$$

$$c) \log(\log(\log x)) = 0 \quad (1)$$

$$x > 0$$

$$\log x > 0 \Leftrightarrow x > 10^0 \Leftrightarrow x > 1$$

$$\log(\log x) > 0 \Leftrightarrow \log x > 1 \Leftrightarrow x > 10$$

$$(1) \Leftrightarrow \log(\log x) = 1 \Leftrightarrow \log x = 10 \Leftrightarrow x = 10^{10} > 10 \quad \checkmark$$

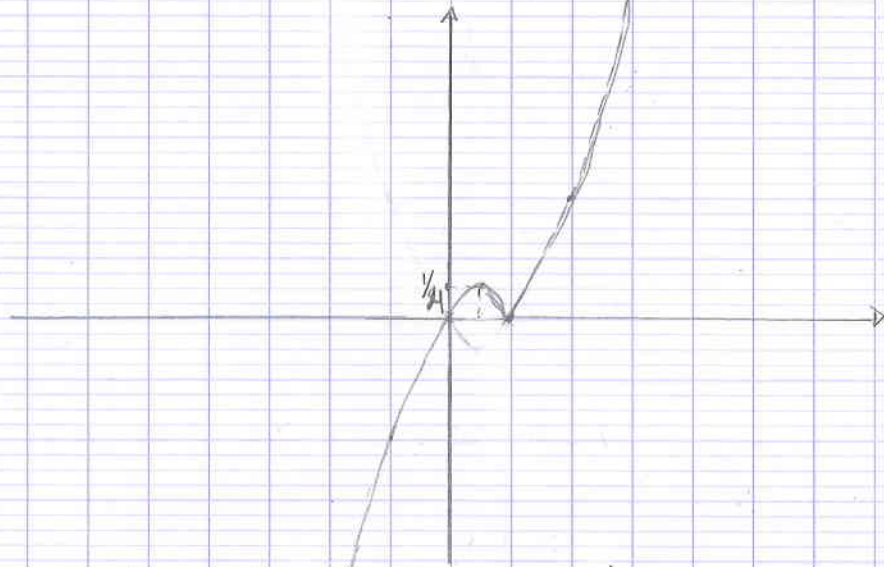
d) $10^{2x} + 2 \cdot 10^x + 1 = 0$ On pose $t = 10^x > 0$
 $t^2 + 2t + 1 = 0 \Leftrightarrow (t+1)^2 = 0 \Leftrightarrow t = -1 \Rightarrow 10^x = -1$ impossible

EXERCICE 4

$f(x) = |x \cdot (x-1)|$

a) $f(x) = \begin{cases} x(x-1) = x^2 - x & x \geq 1 \\ -x(x-1) = -x^2 + x & x < 1 \end{cases}$

b)



Sommets: $(0, 1)$ et $(\frac{1}{2}; \frac{1}{4})$

Intersections: $O_x: I_x(1, 0)$ et $I_x(0, 0)$

$O_y: I_y(0, 0)$