

LJP TE 4: Log-Exp

EXERCICE 1

$$f_1(x) = 3^{\frac{4x}{2+x^2}}$$

$$2+x^2 \neq 0$$

$$D_f = \mathbb{R}$$

$$f_2(x) = \frac{x-3}{e^x}$$

$$e^x \neq 0$$

$$D_f = \mathbb{R}$$

$$f_3(x) = \log(x+10)$$

$$x+10 > 0 \Leftrightarrow x > -10$$

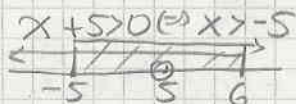
$$D_f =]-10; +\infty[$$

$$f_4(x) = \log_{6-x}(x+5)$$

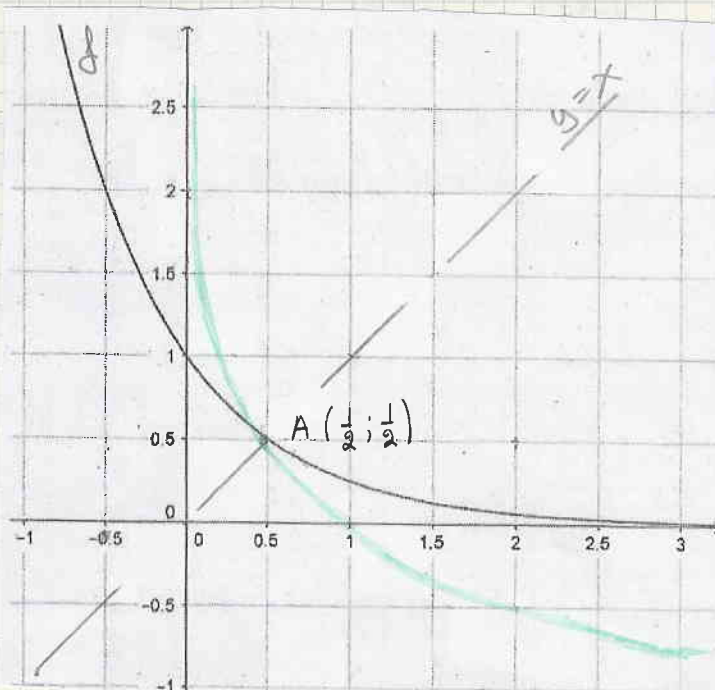
$$6-x > 0 \Leftrightarrow x < 6$$

$$6-x \neq 1 \Leftrightarrow x \neq 5$$

$$D_f =]-5; 5[\cup]5; 6[$$

$$x+5 > 0 \Leftrightarrow x > -5$$


EXERCICE 2



1. $f(x) = a^x$ A: $\frac{1}{2} = a^{1/2} \Leftrightarrow \frac{1}{2} = \sqrt{a} \Rightarrow a = \frac{1}{4} \Rightarrow f(x) = \left(\frac{1}{4}\right)^x$

3. $f^{-1}(x) = \log_{1/4} x$

EXERCICE BONUS

$$3^{2x} - 7 \cdot 3^x - 18 = 0 \quad t = 3^x > 0$$

$$t^2 - 7t - 18 = 0$$

$$\Delta = 121$$

$$t_{1,2} = \frac{7 \pm 11}{2} \begin{cases} t_1 = 9 \\ t_2 = -2 \end{cases}$$

$$t_1 = 9 = 3^x \Leftrightarrow x = 2 \quad \checkmark$$

$$t_2 = -2 = 3^x \quad \text{impossible}$$

EXERCISE 3

1) $\log_5(40-x) = 2 \Leftrightarrow$

$| 40-x > 0 \Leftrightarrow x < 40$

$5^2 = 40-x \Leftrightarrow x = 15 \checkmark$

2) $\log_x 1 = -1 \Leftrightarrow x^{-1} = 1 \Leftrightarrow \frac{1}{x} = 1 \Leftrightarrow$

$x > 0 \quad x \neq 1$

$x = 1 \quad \text{impossible}$

3) $7^{4x}, 7^x; 7^{3-x} = \sqrt{7} \Leftrightarrow 7^{4x+x-3+x} = 7^{1/2} \Leftrightarrow 7^{6x-3} = 7^{1/2} \Leftrightarrow$

$6x-3 = \frac{1}{2} \Leftrightarrow 6x = \frac{7}{2} \Leftrightarrow x = \frac{7}{12}$

4) $25^{(1-x)} = 5^{(x+7)} \Leftrightarrow 5^{2-2x} = 5^{x+7} \Leftrightarrow 2-2x = x+7 \Leftrightarrow$

$3x = -5 \Leftrightarrow x = -\frac{5}{3}$

5) $1 + \log(2) = \log(x) - \log(x+1)$

$\left. \begin{array}{l} x > 0 \\ x+1 > 0 \Leftrightarrow x > -1 \end{array} \right\} x > 0$

$\Leftrightarrow \log 10 + \log 2 = \log x - \log(x+1) \Leftrightarrow$

$\Leftrightarrow \log 20 = \log \frac{x}{x+1} \Leftrightarrow \frac{x}{x+1} = 20 \Leftrightarrow x = 20x + 20 \Leftrightarrow$

$19x = -20 \Leftrightarrow x = -\frac{20}{19} < 0 \quad \text{impossible}$

6) $\log_4(x+4) = \log_2(3x-1)$

$\left. \begin{array}{l} x > -4 \\ 3x-1 > 0 \Leftrightarrow x > \frac{1}{3} \end{array} \right\}$

$\Leftrightarrow \frac{\log_2(x+4)}{\log_2 2^2} = \log_2(3x-1) \Leftrightarrow$

$\frac{1}{2} \log_2(x+4) = \log_2(3x-1) \Leftrightarrow \log_2(x+4) = 2 \log_2(3x-1)$

$\Leftrightarrow \log_2(x+4) = \log_2(3x-1)^2 \Leftrightarrow$

$x+4 = 9x^2 - 6x + 1 \Leftrightarrow 9x^2 - 7x - 3 = 0$

$\Delta = 157 \quad x_{1,2} = \frac{7 \pm \sqrt{157}}{18} \Rightarrow x_1 = -0.31 \quad x_2 = 1.085 \checkmark$

7) $3^{x-5} > 20 \Leftrightarrow \log_3 3^{x-5} > \log_3 20 \Leftrightarrow$

$x-5 > \log_3 20 \Leftrightarrow x > 5 + \log_3 20$

8) $\left(\frac{1}{2}\right)^{x-5} > 20 \Leftrightarrow \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{x-5} < \log_{\frac{1}{2}} 20 \Leftrightarrow$

$x-5 < \log_{\frac{1}{2}} 20 \Leftrightarrow x < \log_{\frac{1}{2}} 20 + 5$

EXERCISE 4

1. $f(x) = \ln\left(\frac{x}{2}\right) \quad D_f =]0, +\infty[$

$f(x) = 0 \Leftrightarrow \ln\left(\frac{x}{2}\right) = 0 \Leftrightarrow \frac{x}{2} = 1 \Leftrightarrow x = 2$

0	2	+∞
-	0	+

$$2. \quad g(x) = \log_{\frac{1}{3}}(3x - x^2)$$

$$3x - x^2 > 0 \Leftrightarrow x(3 - x) > 0$$

$$\begin{array}{c} - \quad 0 \quad + \quad 0 \quad - \\ \hline 0 \quad \quad 3 \end{array}$$

$$D_g =]0; 3[$$

$$g(x) = 0 \Leftrightarrow \log_{\frac{1}{3}}(3x - x^2) = 0 \Leftrightarrow 3x - x^2 = 1 \Leftrightarrow$$

$$x^2 - 3x + 1 = 0 \quad \Delta = 5 \quad x_{1,2} = \frac{3 \pm \sqrt{5}}{2} \quad \begin{cases} x_1 = 0.382 \\ x_2 = 2.62 \end{cases}$$

$$\begin{array}{c|cccc} & 0 & 0.382 & 2.62 & 3 \\ \hline \delta & + & 0 & - & 0 & + \end{array}$$