LJP: Exponentielles - Logarithmes

EXERCICE 1

$2^{-1}=\frac{1}{2}$	$4^{-2}=\frac{1}{4^2}=\frac{1}{16}$	$3^{-3}=\frac{1}{3^3}=\frac{1}{27}$	$\left(\frac{1}{8}\right)^{-2}=8^2=64$
$\left(\frac{64}{27}\right)^{1/3}=\left(\left(\frac{4}{3}\right)^3\right)^{1/3}=\frac{4}{3}$	$125^{-1/3}=\frac{1}{\sqrt[3]{5^3}}=\frac{1}{5}$	$0.125^{-1/3}=\frac{1}{\sqrt[3]{0.125}}=2$	$\left(\frac{1}{81}\right)^{-\frac{3}{4}}=\sqrt[4]{81^3}=3^3=27$
$0.0001^{-3/4}=(10^{-4})^{-3/4}=10^3=1000$	$16^{-3/4}=(2^4)^{-3/4}=2^{-3}=\frac{1}{8}$	$(7^2\cdot7^7)^{2/9}=(7^9)^{2/9}=7^2=49$	$(3^{1/2}\cdot3^{1/4})^2=(3^{3/4})^2=3^{3/2}=\sqrt{27}$

EXERCICE 2

$\frac{1}{a}=a^{-1}$ $\quad \sqrt{a}=a^{1/2}$ $\quad \sqrt[4]{b^3}=b^{3/4}$ $\quad \sqrt{\frac{x^2}{x^5}}=\sqrt{x^{-3}}=x^{-3/2}$

$\frac{(d^2)^3}{\sqrt{d}}=\frac{d^6}{d^{1/2}}=d^{6-1/2}=d^{11/2}$ $\quad \sqrt{(y^3)^6}=\sqrt{y^{18}}=y^{18/2}=y^9$

$\frac{1}{k^3\cdot k^4}=k^{-7}$ $\quad (3^2\cdot3^{-3})^{-1/2}=(3^{-1})^{-1/2}=3^{1/2}$

$$a\sqrt{a\sqrt{a\sqrt{a}}}=a\sqrt{a\sqrt{a\cdot a^{1/2}}}=a\sqrt{a\sqrt{a^{3/2}}}=a\sqrt{a(a^{3/2})^{1/2}}=$$
$$=a\sqrt{a\cdot a^{3/4}}=a\sqrt{a^{7/4}}=a\cdot(a^{7/4})^{1/2}=a\cdot a^{7/8}$$
$$=a^{15/8}$$

$$(a+a)^2\cdot a^4=(2a)^2\cdot a^4=4\cdot a^6$$

$$\sqrt{y}\sqrt{y^7\sqrt[3]{y^8}}=\sqrt{y}\sqrt{y^7y^{8/3}}=\sqrt{y}\sqrt{y^{29/3}}=y^{1/2}y^{29/6}=y^{\frac{33}{6}}=y^{11/2}$$

EXERCICE 3

- $\frac{\sqrt{18}\cdot2^3}{3^{1/2}}=\frac{3\sqrt{2}\cdot2^3}{3^{1/2}}=3^{1/2}\cdot2^{1/2}\cdot2^3=6^{1/2}\cdot8=8\sqrt{6}$
- $\frac{\sqrt{2}(1+\sqrt{2})}{2^{1/2}+2}=\frac{\sqrt{2}+2}{\sqrt{2}+2}=1$
- $\frac{2^{-1/3}+2^{5/3}}{2^{2/3}}=\frac{\frac{1}{\sqrt[3]{2}}+(\sqrt[3]{2})^5}{(\sqrt[3]{2})^2}=\frac{1+(\sqrt[3]{2})^6}{(\sqrt[3]{2})^3}=\frac{1+4}{2}=\frac{5}{2}$
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EXERCICE 5

$\log_2 35 = 5$	$2^5 = 32$
$\log_3 27 = 3$	$3^3 = 27$
$\log_4 2 = \frac{1}{2}$	$4^{1/2} = 2$
$\log_2 4 = 2$	$2^2 = 4$
$\log \frac{1}{100} = -2$	$10^{-2} = \frac{1}{100}$
$\log_3 1 = 0$	$3^0 = 1$
$\log_5 0.04 = -2$	$5^{-2} = 0.04$
$\log_{12} \frac{1}{144} = -2$	$12^{-2} = \frac{1}{144}$
$\log_3 3 = 1$	$3^1 = 3$

EXERCICE 6

a) $\log_3 81 = \log_3 3^4 = 4$ b) $\log_9 27 = \frac{\log_3 3^3}{\log_3 3^2} = \frac{3}{2}$

c) $\log_8 2 = \frac{\log_2 2}{\log_2 2^3} = \frac{1}{3}$ d) $\log_{13} 1 = 0$

e) $\log_{2/3}\left(\frac{16}{81}\right) = \log_{2/3}\left(\frac{2}{3}\right)^4 = 4$ f) $\log_5 \frac{1}{625} = \log_5 5^{-4} = -4$

g) $\log_5 (125) = \log_5 5^3 = 3$ h) $\log_2\left(\frac{1}{16}\right) = \log_2 2^{-4} = -4$

i) $\log_2 (0.125) = \log_2 2^{-3} = -3$ j) $\log 10 = 1$

k) $\log (0.001) = \log 10^{-3} = -3$ l) $\log_{1/2} 32 = \frac{\log_2 2^5}{\log_2 2^{-1}} = \frac{5}{-1} = -5$

m) $\log_4 8 = \frac{\log_2 2^3}{\log_2 2^2} = \frac{3}{2}$ n) $\log_a 1 = 0$

o) $\log_a (a) = 1$ p) $\log_a\left(\frac{1}{a}\right) = \log_a a^{-1} = -1$

q) $\log_{1/a} (a) = \frac{\log_a a}{\log_a a^{-1}} = \frac{1}{-1} = -1$ r) $\log_a \sqrt[n]{a} = \log_a a^{1/n} = \frac{1}{n}$

EXERCICE 7

$\log_3(2187) = \log_3 3^7 = 7$ $\quad \log_{10}(100) = \log 10^2 = 2$

$\log_{10} 50 = \log(5 \cdot 10) = \log 5 + \log 10 = \log 5 + 1 \approx 1.698$

$\log_{0.5}(0.125) = \log_{0.5} 0.5^3 = 3$ $\quad \log_5(0.2) = \log_5 5^{-1} = -1$

$\log_{0.25} 16 = \log_{1/4} 16 = \dfrac{\log_4 16}{\log_4 4^{-1}} = \dfrac{2}{-1} = -2$

EXERCICE 8

$f_1(x) = 4^{\frac{3-x}{x+1}}$ $\quad x+1 \neq 0 \Leftrightarrow x \neq -1 \quad D = \mathbb{R} \setminus \{-1\}$

$f_3(x) = (0.7)^{\sqrt{\frac{4}{3+x}}}$ $\quad \left.\begin{array}{l} \frac{4}{3+x} \geq 0 \Leftrightarrow 3+x \geq 0 \Leftrightarrow x \geq -3 \\ 3+x \neq 0 \Rightarrow x \neq -3 \end{array}\right\} x > -3$

$D_2 =]-3, +\infty[$

$f_2(x) = \log_4 \frac{3-x}{x+1}$ $\quad \frac{3-x}{x+1} > 0 \Leftrightarrow (3-x)(x+1) > 0$

$\begin{array}{ccccc} & -1 & & 3 & \\ - & 0 & + & 0 & - \end{array}$ $\quad D_3 =]-1 ; 3[$

$f_4(x) = \log_{(-x)} \frac{8}{3x+4}$ $\quad \left.\begin{array}{l} \cdot\ -x > 0 \Leftrightarrow x < 0 \\ \cdot\ -x \neq 1 \Leftrightarrow x \neq -1 \\ \cdot\ 3x+4 > 0 \Leftrightarrow x > -4/3 \end{array}\right\}$ $\quad -4/3 \quad -1 \quad 0$

$D_4 =]-4/3 ; -1[\cup]-1 ; 0[$

$f_5(x) = \log_1 x$ impossible

$f_6(x) = \pi^{\sin(2x)}$ $\quad D_5 = \mathbb{R}$

EXERCICE 9

- $f(x) = \dfrac{10}{5 - \log_5 x}$ $\quad \cdot\ x > 0$ $\quad \cdot\ 5 - \log_5 x \neq 0 \Leftrightarrow \log_5 x \neq 5 \Leftrightarrow x \neq 5^5$

  $D_f =]0, 5^5[\cup]5^5, +\infty[$

- $f(x) = \log_{(\pi - ex)} 10$ $\quad \cdot\ \pi - ex > 0 \Leftrightarrow \pi > ex \Leftrightarrow x < \frac{\pi}{e}$ $\quad \cdot\ \pi - ex \neq 1 \Leftrightarrow ex \neq \pi - 1 \Leftrightarrow x \neq \frac{\pi - 1}{e}$

  $D =]-\infty, \frac{\pi-1}{e}[\cup]\frac{\pi-1}{e}, \frac{\pi}{e}[$ $\quad \frac{\pi-1}{e} \quad \pi/e$

- $f(x) = \left(\frac{3}{4}\right)^{\frac{\sqrt{x}}{2-x}}$ $\quad \cdot\ x \geq 0$ $\quad \cdot\ 2-x \neq 0 \Leftrightarrow x \neq 2$

  $D = [0 ; 2[\cup]2, +\infty[$

- $f(x) = 7^x - \dfrac{\sqrt{x}}{2-x}$ $\quad \cdot\ x \geq 0$ $\quad \cdot\ 2-x \neq 0 \Leftrightarrow x \neq 2$

  $D = [0, 2[\cup]2, +\infty[$

- $f(x) = \sqrt[9]{\dfrac{\sqrt{7}}{x}} + \log_8 x$ $\quad \cdot\ x > 0$

  $D =]0, +\infty[$

- $f(x) = \sqrt{\frac{\sqrt[9]{-7}}{x}} + \log_8 x$ · $x > 0$

  $D_f =]0, +\infty[$

- $f(x) = 8^{\log_8 x} \sqrt[54]{-3x-12}$ · $x > 0$
  · $-3x-12 \geq 0 \Leftrightarrow -12 \geq 3x \Leftrightarrow x \leq -4$

  $D = \emptyset$

- $f(x) = 16^{x+7}$
  $D = \mathbb{R}$

- $f(x) = \log_{0.7} \frac{11+x^2}{2x-3}$ · $2x-3 > 0 \Leftrightarrow x > 3/2$

  $D =]\frac{3}{2}; +\infty[$

EXERCICE 10

1. $a^0 = 1 \Leftrightarrow \log_a 1 = 0$

2. $a^1 = a \Leftrightarrow \log_a a = 1$

3. $a^{x+y} = a^x \cdot a^y = u \cdot v \Rightarrow \log_a(u \cdot v) = x + y \Rightarrow$
   $\Rightarrow \log_a(u \cdot v) = \log_a u + \log_a v$

EXERCICE 11

a) $x - 1 = \log_2(3) \Leftrightarrow x = \frac{\log 3}{\log 2} + 1 \approx 2.58$

b) $5 \cdot 3^x = 20 \Leftrightarrow 3^x = 4 \Leftrightarrow x = \log_3 4 = \frac{\log 4}{\log 3} = 1,26$

c) $3^x = 2^{x+1} \Leftrightarrow \log 3^x = \log 2^{x+1} \Leftrightarrow x \log 3 = (x+1) \log 2 \Leftrightarrow$
$\Leftrightarrow x \log 3 = x \log 2 + \log 2 \Leftrightarrow x(\log 3 - \log 2) = \log 2$
$\Leftrightarrow x = \frac{\log 2}{\log 3 - \log 2}$

d) $\log_4(x-1) = 2 \Leftrightarrow 4^2 = x - 1 \Leftrightarrow$ $\quad x - 1 > 0 \Leftrightarrow x > 1$
$x = 16 + 1 \Leftrightarrow x = 17$ ✓

e) $\log_5(x^2) = 1 \Leftrightarrow x^2 = 5 \Leftrightarrow x = \pm\sqrt{5}$ ✓ $\quad x \neq 0.$

f) $4 \cdot 7^x = 3^{2x} \Leftrightarrow \log(4 \cdot 7^x) = \log 3^{2x} \Leftrightarrow$
$\Leftrightarrow \log 4 + x \log 7 = 2x \log 3 \Leftrightarrow x(2\log 3 - \log 7) = \log 4$
$\Leftrightarrow x = \frac{\log 4}{2\log 3 - \log 7}$

g) $9 = \log_x 7 \Leftrightarrow x^9 = 7 \Leftrightarrow x = \sqrt[9]{7}$ $\quad x>0 \quad x \neq 1$

h) $10 = \log_2(x) \Leftrightarrow 2^{10} = x$ ✓ $\quad x>0$

i) $2^{x+7} = 5 \cdot 3^{8-5x} \Leftrightarrow \log 2^{x+7} = \log 5 + \log 3^{8-5x} \Leftrightarrow$

$\Leftrightarrow (x+7)\log 2 = \log 5 + (8-5x)\log 3 \Leftrightarrow$

$\Leftrightarrow x\log 2 + 7\log 2 = \log 5 + 8\log 3 - 5x\log 3 \Leftrightarrow$

$\Leftrightarrow x(\log 2 + 5\log 3) = \log 5 + 8\log 3 - 7\log 2$

$\Leftrightarrow x = \dfrac{\log 5 + 8\log 3 - 7\log 2}{\log 2 + 5\log 3} \Leftrightarrow x \approx 0.896$ ✓

EXERCICE 12

i) $\log x = \log 3 + \log 4 = \log 12 \Leftrightarrow x = 12$ ✓ $\quad x>0$

ii) $\log x = \log 7 - \log 6 + \log 48 \Leftrightarrow$ $\quad x>0$

$\log x = \log \dfrac{7 \cdot 48}{6} \Rightarrow \log x = \log 56 \Leftrightarrow x = 56$ ✓

iii) $\log x = 2 - 3\log 2 + \log 4 - 2\log 5 \Leftrightarrow$ $\quad x>0$

$\Leftrightarrow \log x = 2 + \log\dfrac{4}{8 \cdot 25} = 2 + \log\dfrac{1}{50} \Leftrightarrow$

$\Leftrightarrow \log x = \log 10^2 + \log\dfrac{1}{50} = \log\dfrac{100}{50} = \log 2 \Leftrightarrow x = 2$

iv) $2\log(x-1) = \log(3-x) + \log(3+x)$

$\Leftrightarrow \log(x-1)^2 = \log[(3-x)(3+x)] \Leftrightarrow$

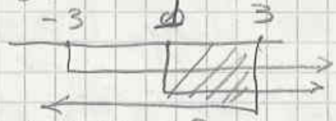
$(x-1)^2 = 9 - x^2 \Leftrightarrow x^2 - 2x + 1 = 9 - x^2 \Leftrightarrow$

$\Leftrightarrow 2x^2 - 2x - 8 = 0 \Leftrightarrow$

$\Leftrightarrow x^2 - x - 4 = 0 \quad \Delta = 1 + 16 = 17$

$x_{1,2} = \dfrac{1 \pm \sqrt{17}}{2} = \begin{cases} x_1 = 2.56 ✓ \\ x_2 = -1.56 ✗ \end{cases}$

$\left.\begin{array}{l} \cdot\, x>1 \\ \cdot\, 3-x>0 \Leftrightarrow x<3 \\ \cdot\, 3+x>0 \Leftrightarrow x>-3 \end{array}\right\}$



$1 < x < 3$

v) $\log(x^2+3x) = \log 5 + \log 4x \Leftrightarrow$

$\Leftrightarrow \log(x^2+3x) = \log 20x \Leftrightarrow$

$\left.\begin{array}{l} \cdot\, x^2+3x>0 \Leftrightarrow x(x+3)>0 \\ \cdot\, x>0 \end{array}\right\} x>0$

$x^2 + 3x = 20x \Leftrightarrow x^2 - 17x = 0$

$\Leftrightarrow x(x-17) = 0 \quad x=0$ ✗ $\quad x = 17$ ✓

vi) $3+\log(5)=\log(x)-2\log(2)$ $\quad x>0$

$\Leftrightarrow \log(10^3\cdot 5)=\log\left(\frac{x}{4}\right) \Leftrightarrow \frac{x}{4}=5000 \Leftrightarrow x=20000$ ✓

vii) $\log_3(x-1)-\log_9(x+1)=0 \Leftrightarrow$ $\quad \left.\begin{array}{l}\cdot\, x>1\\ \cdot\, x>-1\end{array}\right\} x>1$

$\Leftrightarrow \log_3(x-1)-\dfrac{\log_3(x+1)}{\log_3 3^2}=0$

$\Leftrightarrow \log_3(x-1)-\dfrac{\log_3(x+1)}{2}=0 \Leftrightarrow$

$\Leftrightarrow 2\log_3(x-1)=\log_3(x+1) \Leftrightarrow \log_3(x-1)^2=\log_3(x+1)$

$\Leftrightarrow x^2-2x+1=x+1 \Leftrightarrow x^2-3x=0 \Leftrightarrow x(x-3)=0$

$x=0$ ✗ $\quad x=3$ ✓

viii) $1+\log(x)=\log 10^5-3\log x \Leftrightarrow$ $\quad x>0$

$\Leftrightarrow 4\log(x)=5-1=4 \Leftrightarrow \log(x)=1 \Leftrightarrow x=10$ ✓

EXERCICE 13

1. $\log_7(x+5)-\log_{49}x=0$ $\quad \left.\begin{array}{l}x>0\\ x>-5\end{array}\right\} x>0$

$\Leftrightarrow \log_7(x+5)-\dfrac{\log_7 x}{\log_7 7^2}=0 \Leftrightarrow$

$\Leftrightarrow \log_7(x+5)-\dfrac{\log_7 x}{2}=0 \Leftrightarrow 2\log_7(x+5)-\log_7 x=0$

$\Leftrightarrow \log_7\dfrac{(x+5)^2}{x}=0 \Leftrightarrow \dfrac{(x+5)^2}{x}=1 \Leftrightarrow x^2+10x+25=x \Leftrightarrow$

$\Leftrightarrow x^2+9x+25=0 \quad \Delta=81-100<0$ $\quad$ Impossible

2. $\log(x+3)+\log(5-x)=1+\log(-x) \Leftrightarrow$ $\quad \cdot\, x>-3$

$\Leftrightarrow \log(x+3)(5-x)=\log 10(-x) \Leftrightarrow$ $\quad \cdot\, x<5$

$\Leftrightarrow \log(x+3)(5-x)=\log(-10x) \Leftrightarrow$ $\quad \cdot\, -x>0 \Leftrightarrow x<0$

$\Leftrightarrow 5x+15-x^2-3x=-10x \Leftrightarrow$ $\quad -3<x<0$

$\Leftrightarrow x^2-12x-15$

$\Delta=144+60=204 \quad x_{1,2}=\dfrac{12\pm\sqrt{204}}{2}=\begin{cases}x_1=13.14 & ✗\\ x_2=-1.14 & ✓\end{cases}$

3. $5^x+3\cdot 5^{2x}-1=0 \Leftrightarrow 3\cdot 5^{2x}+5^x-1$ $\quad t=5^x$

$3t^2+t-1=0 \quad \Delta=1+12=13 \quad t_{1,2}=\dfrac{-1\pm\sqrt{13}}{6}=\begin{cases}0.43 & ✓\\ -0.77 & ✗\end{cases}$

$t=5^x=0.43 \Rightarrow x=\log_5 0.43 \Leftrightarrow x=\dfrac{\log 0.43}{\log 5}\approx -0.52$

4. $3\cdot 10^x = 7^{3-x} \Leftrightarrow \log(3\cdot 10^x) = \log 7^{3-x} \Leftrightarrow$

$\Leftrightarrow \log 3 + x\log 10 = (3-x)\log 7 \Leftrightarrow$

$\Leftrightarrow \log 3 + x = 3\log 7 - x\log 7 \Leftrightarrow$

$\Leftrightarrow x(1+\log 7) = 3\log 7 - \log 3 \Leftrightarrow$

$\Leftrightarrow x = \dfrac{3\log 7 - \log 3}{1+\log 7} \approx 1.11$

EXERCICE 14

1. $y = \ln x$ $\qquad D_f =]0,+\infty[$

$f(x)=0 \Leftrightarrow \ln x = 0 \Leftrightarrow x = 1$

x	0		1		$+\infty$
f	//	$-$	0	$+$	

2. $f(x) = \log_5 x$ $\qquad D_f =]0,+\infty[$

$\log_5 x = 0 \Leftrightarrow x = 1$

x	0		1		$+\infty$
f	//	$-$	0	$+$	

3. $f(x) = \log_{1/5} x$ $\qquad D_f =]0,+\infty[$

$f(x) = 0 \Leftrightarrow \log_{1/5} x = 0 \Leftrightarrow \left(\frac{1}{5}\right)^0 = x = 1$

x	0		1		$+\infty$
f		$+$	0	$-$	

4. $f(x) = \log_{1/4} x$ $\qquad D_f =]0,+\infty[$

$f(x)=0 \Leftrightarrow \log_{1/4} x = 0 \Leftrightarrow x = 1$

	0		1		$+\infty$
f	//	$+$	0	$-$	

5. $f(x) = \log(x-4)$ $\qquad D_f =]4,+\infty[$

$f(x)=0 \Leftrightarrow (x-4) = 1 \Leftrightarrow x = 5$

x	4		5		$+\infty$
f	//	$-$	0	$+$	

6. $f(x) = \log_{(3\pi-1)}(5-x)$ $\qquad D_f =]-\infty, 5[$

$f(x) = 0 \Leftrightarrow 5-x = 1 \Leftrightarrow x = 4$

	$-\infty$		4		5
f		$+$	0	$-$	//

7. $f(x) = \log_{e^2}(3x+7)$ $\qquad 3x+7>0 \Leftrightarrow x > -7/3$ $\qquad D_f =]-7/3,+\infty[$

$\log_{e^2}(3x+7) = 0 \Leftrightarrow 3x+7 = 1 \Leftrightarrow 3x = -6 \Leftrightarrow x = -2$

x	$-7/3$		-2		$+\infty$
f	//	$-$	0	$+$	

8. $f(x) = \ln x^2$ $\qquad D_f = \mathbb{R}\setminus\{0\}$

$\ln x^2 = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$

	$-\infty$		-1		0		1		$+\infty$
f		$+$	0	$-$	//	$-$	0	$+$	

9. $y = \log(x^2-1)$ $\qquad x^2-1>0 \Leftrightarrow (x-1)(x+1)>0$

$D_f =]-\infty, -1[\cup]1, +\infty[$

	-1		1	
$+$	0	$-$	0	$+$

$f(x) = 0 \Leftrightarrow \log(x^2-1) = 0 \Leftrightarrow x^2-1 = 1 \Leftrightarrow x^2 = 2 \Leftrightarrow x = \pm\sqrt{2}$

	$-\infty$		$-\sqrt{2}$		-1		1		$\sqrt{2}$		$+\infty$
f		$+$	0	$-$		////		$-$	0	$+$	

10. $f(x) = \ln\left(\frac{1}{6}x^2 + \frac{1}{6}x - 1\right)$ $\qquad \frac{1}{6}x^2 + \frac{1}{6}x - 1 > 0 \Leftrightarrow x^2 + x - 6 > 0$

$\Delta = 25 \quad x_{1,2} = \frac{-1 \pm 5}{2} \begin{cases} -3 \\ 2 \end{cases}$

$D_f =]-\infty; -3[\cup]2; +\infty[$

$f(x) = 0 \Leftrightarrow \frac{1}{6}x^2 + \frac{1}{6}x - 1 = 1 \Leftrightarrow x^2 + x - 6 = 6 \Leftrightarrow x^2 + x - 12 = 0$

$\Delta = 1 + 48 = 49 \quad x_{1,2} = \frac{-1 \pm 7}{2} \begin{cases} x_1 = -4 \\ x_2 = 3 \end{cases}$

x	$-\infty$		-4		-3		2		3		$+\infty$
f		$+$	0	$-$		////		$-$	0	$+$	

EXERCICE 15

1) $a(x) = 2e^x + 13 - x^2 \qquad a'(x) = 2e^x - 2x$

2) $b(x) = 5 - 5^x + \frac{1}{5}\ln x \qquad b'(x) = -5^x \ln 5 + \frac{1}{5x}$

3) $c(x) = xe^x \Rightarrow c'(x) = e^x + xe^x = e^x(x+1)$

4) $d(x) = 5^x(x+1) \Rightarrow d'(x) = 5^x \ln 5 (x+1) + 5^x =$
$= 5^x(\ln 5 \cdot (x+1) + 1)$

5) $e(x) = e^x + \ln x \qquad e'(x) = e^x + \frac{1}{x}$

6) $f(x) = \frac{e^x}{\log x} \qquad f'(x) = \frac{e^x \log x - e^x \frac{1}{x \ln 10}}{\log^2 x} =$
$= \frac{e^x(x \log x \cdot \ln x - 1)}{x \log^2 x \ln 10}$

7) $g(x) = e^{5x} \qquad g'(x) = e^{5x} \cdot 5$

8) $h(x) = e^{3x^2} \qquad h'(x) = e^{3x^2} \cdot 6x$

9) $l(x) = (x+\pi)e^{\pi x} \qquad l'(x) = e^{\pi x} + (x+\pi)\pi e^{\pi x} =$
$= e^{\pi x}(1 + (x+\pi)\pi)$

10) $m(x) = 8x \cdot \pi^{x-\pi} \qquad m'(x) = 8\pi^{x-\pi} + 8x\pi^{x-\pi}\ln\pi$

11) $n(x) = \frac{x}{e^x} \qquad n'(x) = \frac{e^x - xe^x}{e^{2x}} = \frac{e^x(1-x)}{e^{2x}} = \frac{1-x}{e^x}$

12) $p(x) = \frac{1-7x}{2^x} \Rightarrow p'(x) = \frac{-7 \cdot 2^x - (1-7x)2^x \ln 2}{2^{2x}} =$
$= \frac{2^x(-7-(1-7x)\ln 2)}{2^{2x}} = \frac{-7-(1-7x)\ln 2}{2^x}$

13) $r(x) = \ln(3-5x) \qquad r'(x) = \frac{-5}{3-5x}$

14) $s(x) = -2\ln(x^2+5) \qquad s'(x) = \frac{-2\cdot 2x}{x^2+5} = \frac{-4x}{x^2+5}$

15) $t(x) = \frac{\ln x}{x} \qquad t'(x) = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1-\ln x}{x^2}$

16) $u(x) = \log(5x+\pi)\cdot e^{\pi x}$

$u'(x) = \frac{1}{(5x+\pi)\ln 10}\, e^{\pi x} + \log(5x+\pi)\,\pi e^{\pi x}$

17) $v(x) = \log_4(3\cdot 2^{x-1}) - e^{2^{x+2}}$

$v'(x) = \frac{3\cdot 2^{x-1}\cdot \ln 2}{3\cdot 2^{x-1}\cdot \ln 4} - e^{2^{x+2}}\cdot 2^{x+2}\cdot \ln 2$

EXERCICE 16.

1) $f(x) = x - \ln x \qquad x_0 = e^2 \qquad f(e^2) = e^2 - 2$

$f'(x) = 1 - \frac{1}{x} \qquad f'(e^2) = 1 - \frac{1}{e^2}$

$e^2 - 2 = \left(1 - \frac{1}{e^2}\right)\cdot e^2 + h \Rightarrow \cancel{e^2} - 2 = \cancel{e^2} - 1 + h \Leftrightarrow$

$\Rightarrow h = -1 \qquad t: y = \left(1 - \frac{1}{e^2}\right)x - 1$

2) $f(x) = x - \ln(4x) \qquad f(e^2) = e^2 - \ln(4e^2) =$

$= e^2 - \ln 4 - 2\,\cancel{\ln e}^{\,1}$

$= e^2 - 2 - \ln 4$

$f'(x) = 1 - \frac{4}{4x} = 1 - \frac{1}{x}$

$f'(e^2) = 1 - \frac{1}{e^2}$

$e^2 - 2 - \ln 4 = \left(1 - \frac{1}{e^2}\right)e^2 + h \Rightarrow$

$\cancel{e^2} - 2 - \ln 4 = \cancel{e^2} - 1 + h \Leftrightarrow h = -1 - \ln 4$

$t: y = \left(1 - \frac{1}{e^2}\right)x - 1 - \ln 4$

EXERCICE 17

$y = e^{-kx^2} \qquad y' = -2kx\cdot e^{-kx^2}$

$y'' = -2k\,e^{-kx^2} + 2kx\cdot 2kx\cdot e^{-kx^2} =$

$= 2k\,e^{-kx^2}\left(-1 + 2kx^2\right)$

$f''(\sqrt{3}) = 0 \Leftrightarrow k = 0 \qquad -1 + 2k\cdot 3 = 0 \Leftrightarrow k = \frac{1}{6}$

EXERCICE 18

1) $\lim\limits_{x \to +\infty} (x+2000)\log x = +\infty$

2) $\lim\limits_{x \to -\infty} (x+10)e^{x-10} = 0$

3) $\lim\limits_{x \to +\infty} x^{10000} e^{x} = +\infty$

4) $\lim\limits_{x \to 0^+} \left(\ln \frac{x}{8}\right) e^{x} = -\infty \cdot 1 = -\infty$

5) $\lim\limits_{x \to 0^+} \left(\frac{\ln(8-x)}{8}\right) \cdot e^{x} = \frac{\ln 8}{8} \cdot e^{0} = \frac{\ln 8}{8}$

6) $\lim\limits_{x \to +\infty} \frac{(2x^7-1)}{1.5^{2x}} = 0$

7) $\lim\limits_{x \to -\infty} \frac{-3x^2}{e^{x}} = -\infty$

8) $\lim\limits_{x \to 0} (3x-2)5^{3x} = -2 \cdot 5^{0} = -2$

9) $\lim\limits_{x \to -\infty} (-3x^9 11^{-x}) = -\infty$

10) $\lim\limits_{x \to 2^+} e^{-\frac{3}{x-2}} = 0$

EXERCICE 19

$h(x) = \frac{12}{4-e^{x}}$

- $4 - e^{x} = 0 \Leftrightarrow e^{x} = 4 \Leftrightarrow x = \ln 4 \qquad D_f = \mathbb{R} \setminus \{\ln 4\}$
- $\lim\limits_{x \to \ln 4} f(x)$ $\begin{cases} \xrightarrow{x<4} +\infty \\ \xrightarrow[x>4]{} -\infty \end{cases}$ AV $x = \ln 4$

EXERCICE 20

- $f_1(x) = \frac{10}{5+e^{x}} \qquad D_f = \mathbb{R}$

- AV $\varnothing$
- AH $\lim\limits_{x \to +\infty} f(x) = 0 \Rightarrow$ AH $y = 0$

  $\lim\limits_{x \to -\infty} f(x) = 2 \Rightarrow$ AH $y = 2$

▶ $f_2(x) = \dfrac{x-3e^x}{e^x-2}$

· $e^x = 2 \Leftrightarrow x = \ln 2$ $\quad D_f = \mathbb{R} \setminus \{\ln 2\}$

· AV $\lim\limits_{x \to \ln 2} f(x) = \dfrac{\text{“}\ln 2 - 6}{0\text{”}}$ $\begin{cases} \xrightarrow{x > \ln 2} -\infty \\ \xrightarrow[x < \ln 2]{} +\infty \end{cases}$ $\quad$ AV $x = \ln 2$

· $\lim\limits_{x \to +\infty} f_2(x) = \lim\limits_{x \to +\infty} \dfrac{e^x\left(\frac{x}{e^x} - 3\right)}{e^x\left(1 - \frac{2}{e^x}\right)} = \dfrac{-3}{1} = -3$ $\quad$ AH $y = -3$

· $\lim\limits_{x \to -\infty} f_2(x) = +\infty.$

EXERCICE 21

▶ $f(x) = e^x$

· $D_f = \mathbb{R}$

· $f(x) = 0$ impossible. $\quad f(x) > 0 \ \forall x \in \mathbb{R}$

· AV $\emptyset$ $\quad$ AH $\lim\limits_{x \to +\infty} f(x) = +\infty$ $\quad \lim\limits_{x \to -\infty} f(x) = 0$ $\quad$ AH $y = 0$

· $f'(x) = e^x$

x	$-\infty$		$+\infty$
f'		$+$	
f		$\nearrow$	

$f''(x) = e^x > 0 \ \forall x \in \mathbb{R}$ $\quad \cup$

▶ $f(x) = e^x - 1$ $\quad D_f = \mathbb{R}$

· $f(x) = 0 \Leftrightarrow e^x = 1 \Leftrightarrow x = 0$ $\quad (0,0)$

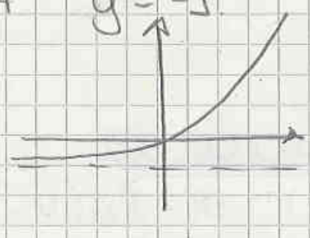
x	$-\infty$		0		$+\infty$
f		$-$	0	$+$	

· AV $\emptyset$ $\quad$ AH: $\lim\limits_{x \to +\infty} f(x) = +\infty$

$\lim\limits_{x \to -\infty} f(x) = -1$ $\quad$ AH $\quad y = -1$

· $f'(x) = e^x > 0 \ \forall x \in \mathbb{R}$ $\quad f \nearrow$

· $f''(x) = e^x > 0 \ \forall x \in \mathbb{R}$ $\quad f \cup$



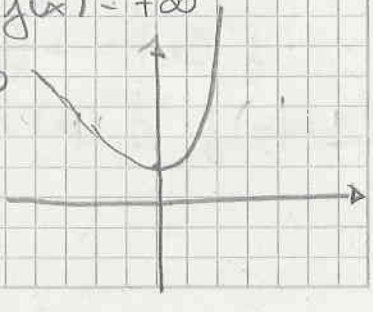
▶ $f(x) = e^x - x$ $\quad D_f = \mathbb{R}$

· $f(x) = 0 \Leftrightarrow e^x = x$ imp $\quad f(x) > 0 \ \forall x \in \mathbb{R}$

· AV $\emptyset$ $\quad$ AH: $\lim\limits_{x \to +\infty} f(x) = +\infty.$ $\quad \lim\limits_{x \to -\infty} f(x) = +\infty$

· $f'(x) = e^x - 1 = 0 \Leftrightarrow x = 0$

x	$-\infty$		0		$+\infty$
f'		$-$	0	$+$	
f		$\searrow$	$f(0) = 1$	$\nearrow$	



· $f''(x) = e^x > 0 \ \forall x \in \mathbb{R}$ $\quad f \cup$

▶ $f(x) = xe^{x}$ $\quad D_f = \mathbb{R}$

· $f(x) = 0 \Leftrightarrow x = 0$ $\quad (0,0)$

	$-\infty$		0		$+\infty$
f		$-$	0	$+$	

· AV $\emptyset$ $\quad$ AH: $\lim\limits_{x\to+\infty} f(x) = +\infty$ $\quad \lim\limits_{x\to-\infty} f(x) = 0$ $\quad$ AH $y = 0$

· $f'(x) = e^{x} + xe^{x} = e^{x}(x+1) = 0 \Leftrightarrow x = -1$

	$-\infty$		-1		$+\infty$
f'		$-$	0	$+$	
f		$\searrow$		$\nearrow$	

$f(-1) = -\frac{1}{e} = -0.37$

$f''(x) = e^{x}(x+1) + e^{x} = e^{x}(x+2) = 0$

$\Leftrightarrow x = -2$

	$-\infty$		-2		$+\infty$
f''		$-$	0	$+$	
		$\frown$	PI	$\smile$	

▶ $f(x) = x^{2}e^{x}$ $\quad D_f = \mathbb{R}$

· $f(x) = 0 \Leftrightarrow x = 0$ $\quad (0,0)$

	$-\infty$		0		$+\infty$
f		$+$	0	$+$	

· AV $\emptyset$ $\quad$ AH: $\lim\limits_{x\to+\infty} f(x) = +\infty$ $\quad \lim\limits_{x\to-\infty} f(x) = 0$ $\quad$ AH $y = 0$

· $f'(x) = 2xe^{x} + x^{2}e^{x} = xe^{x}(2+x) = 0 \Leftrightarrow \begin{matrix} x = 0 \\ x = -2 \end{matrix}$

	$-\infty$		-2		0		$+\infty$
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$		$\searrow$		$\nearrow$	

$f(-2) = \frac{4}{e^{2}} \approx 0.54$

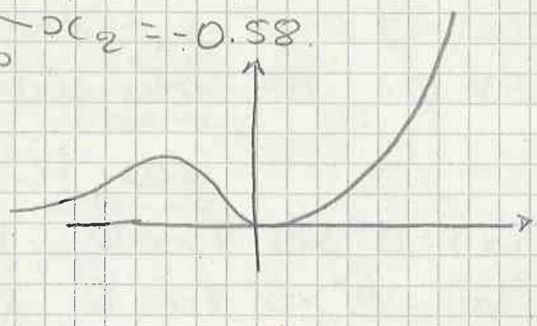
$f(0) = 0$

· $f''(x) = \left(e^{x}(2x+x^{2})\right)' = e^{x}(2x+x^{2}) + e^{x}(2+2x) =$

$= e^{x}(2x + x^{2} + 2 + 2x) = e^{x}(x^{2}+4x+2) = 0$

$\Rightarrow x^{2} + 4x + 2 = 0$ $\quad x_1 = -3.4$, $\quad x_2 = -0.58$

	$-\infty$		-3.4		-0.58		$+\infty$
f''		$+$	0	$-$	0	$+$	
f		$\smile$	PI	$\frown$	PI	$\smile$	



EXERCICE 22

$f(x) = x - \ln(4x)$ $\quad 4x > 0 \Leftrightarrow x > 0$ $\quad D_f =]0, +\infty[$

· $f'(x) = 1 - \frac{1}{x} = 0 \Leftrightarrow x - 1 = 0 \Leftrightarrow x = 1$

	0		1		$+\infty$
f'		$-$	0	$+$	
f		$\searrow$	Min	$\nearrow$	

Min $= f(1) = 1 - \ln 4$

· $f''(x) = +\frac{1}{x^{2}} = 0$ $\quad$ Impossible

$f''(x) > 0 \quad \forall x \in D_f$

EXERCICE 23

$f(x) = \dfrac{\ln x}{x}$ $\qquad D_f =]0,+\infty[$

$f'(x) = \dfrac{\frac{1}{x}\cdot x - \ln x}{x^2} = \dfrac{1-\ln x}{x^2} = 0 \Leftrightarrow \ln x = 1 \Leftrightarrow x = e$

x	0		e		$+\infty$
f'		+	0	−	
f		↗	Max	↘	

$\text{Max} = f(e) = \dfrac{1}{e}$

EXERCICE 24

$f(x) = \ln\left(\dfrac{x^2-1}{x}\right)$

$\dfrac{x^2-1}{x} > 0$

	$-\infty$		-1		0		1		$+\infty$
x^2-1		+	0	−		−	0	+	
x		−		−	0	+		+	
Q		−	0	+	0	−	0	+	

$D_f =]-1,0[\cup]1,+\infty[$

$f'(x) = \dfrac{x}{x^2-1} \cdot \dfrac{2x\cdot x-(x^2-1)}{x^2} = \dfrac{2x^2-x^2+1}{x(x^2-1)} = \dfrac{x^2+1}{x(x^2-1)}$

$f'(x) = 0$ impossible

	-1		0		1		$+\infty$
f'		+				+	
f		↗				↗	

EXERCICE 25

1. $f(x) = x + \ln x$ $\qquad D_f =]0,+\infty[$

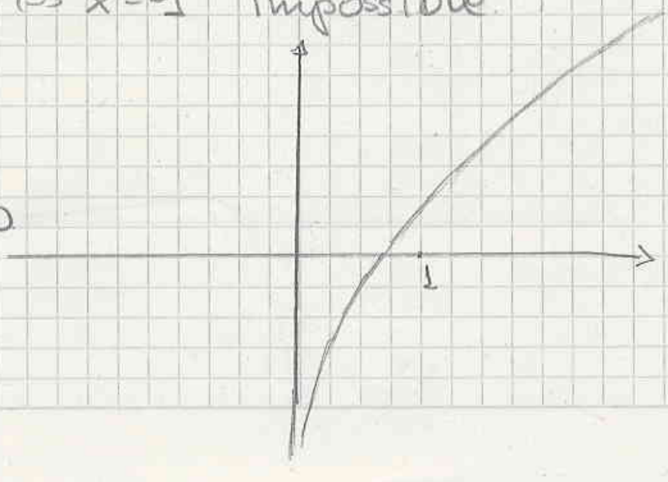
- $f(x) = 0 \Leftrightarrow \ln x = -x \quad \Leftrightarrow x \approx 0.568$

	0		0.568		$+\infty$
f		−	0	+	

- AV: $\lim\limits_{x\to 0} f(x) = -\infty$ $\qquad$ AV: $x = 0$
- AH: $\lim\limits_{x\to +\infty} f(x) = +\infty$ $\qquad \emptyset$
- $f'(x) = 1 + \dfrac{1}{x} = 0 \Leftrightarrow x+1 = 0 \Leftrightarrow x = -1$ impossible

	0		$+\infty$
f'		+	
f		↗	

$f''(x) = -\dfrac{1}{x^2} < 0 \quad \forall x > 0$



2. $f(x) = x - \ln x$ $\qquad D_f =]0, +\infty[$

- $f(x) = 0 \Leftrightarrow x = \ln x$ impossible.

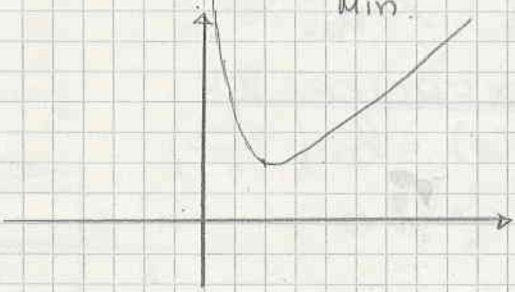
x	0		$+\infty$
f		$+$	

- $\lim\limits_{x \to 0} f(x) = +\infty$ $\quad$ AV : $x = 0$.
- $\lim\limits_{x \to +\infty} f(x) = +\infty$
- $f'(x) = 1 - \frac{1}{x} = 0 \Leftrightarrow x - 1 = 0 \Leftrightarrow x = 1$

$\quad$ Min $= f(1) = 1$.

x	0		1		$+\infty$
f'		$-$	0	$+$	
f		$\searrow$	Min	$\nearrow$	

- $f''(x) = \frac{1}{x^2} > 0$ $\quad \cup$



3. $f(x) = \frac{\ln(x)}{x}$ $\qquad D_f =]0, +\infty[$

- $f(x) = 0 \Leftrightarrow \ln x = 0 \Leftrightarrow x = 1$ $\qquad (1; 0)$

x	0		1		$+\infty$
f	‖	$-$	0	$+$	

- $\lim\limits_{x \to 0} f(x) = -\infty$ $\quad$ AV : $x = 0$.
- $\lim\limits_{x \to +\infty} f(x) = 0$ $\quad$ AH : $y = 0$
- $f'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2} = 0 \Leftrightarrow \ln x = 1 \Leftrightarrow x = e$

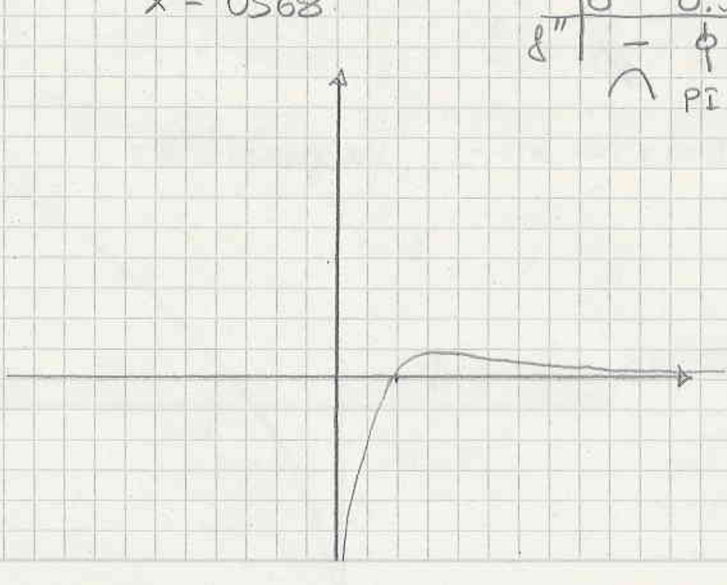
x	0		e		$+\infty$
f'	‖	$-$	0	$+$	
f	‖	$\searrow$	Min	$\nearrow$	

$\quad$ Min $= f(e) = \frac{1}{e}$

- $f''(x) = \frac{-\frac{1}{x} \cdot x^2 - \ln x}{x^2} = \frac{-x - \ln x}{x^2} = 0 \Leftrightarrow \ln x = -x \Leftrightarrow$

$\quad x = 0568$

x	0		0.568		$+\infty$
f''		$-$	0	$+$	
		$\cap$	PI	$\cup$	



4. $f(x) = x\ln x - x \qquad D_f =]0,+\infty[$

$\cdot\ f(x) = 0 \Leftrightarrow x(\ln x - 1) = 0 \Leftrightarrow x = 0$ (imp) $\quad \ln x = 1 \Leftrightarrow x = e \ (e;0)$

	0		e		$+\infty$
f	//	$-$	0	$+$	

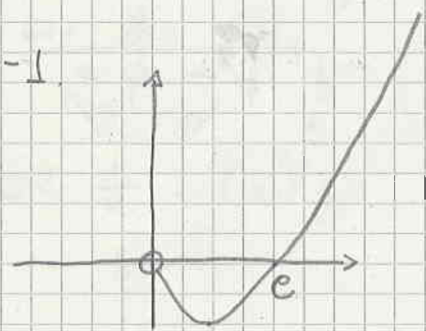
$\cdot\ \lim_{x\to 0} f(x) = 0 \qquad$ Trou $(0;0)$

$\cdot\ \lim_{x\to +\infty} f(x) = +\infty$

$\cdot\ f'(x) = \ln x + x\frac{1}{x} - 1 = \ln x = 0 \Leftrightarrow x = 1$

	0		1		$+\infty$
f'		$-$	0	$+$	
f		$\searrow$	Min	$\nearrow$	

$\text{Min} = f(1) = -1$



$\cdot\ f''(x) = \frac{1}{x} > 0 \quad \forall x \in D_f$

5. $f(x) = (\ln x)^2 \qquad D_f =]0,+\infty[$

$\cdot\ f(x) = 0 \Leftrightarrow \ln x = 0 \Leftrightarrow x = 1 \qquad (1;0)$

	0		1		$+\infty$
f	//	$+$	0	$+$	

$\cdot\ \lim_{x\to 0} f(x) = +\infty \qquad$ AV: $x = 0$

$\cdot\ \lim_{x\to +\infty} f(x) = +\infty$

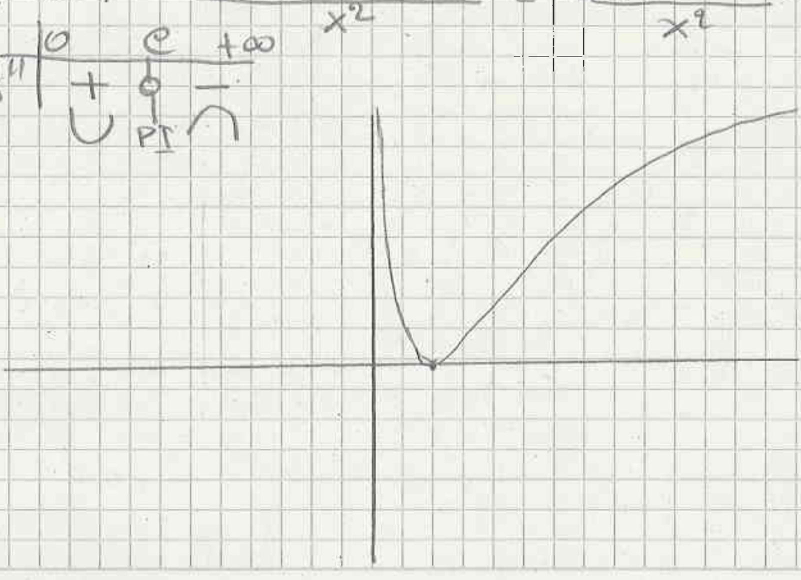
$\cdot\ f'(x) = \frac{2\ln x}{x} = 0 \Leftrightarrow \ln x = 0 \Leftrightarrow x = 1$

	0		1		$+\infty$
f'	//	$-$	0	$+$	
f		$\searrow$		$\nearrow$	

$\text{Min} = f(1) = 0$

$\cdot\ f''(x) = \frac{2\frac{1}{x}\cdot x - 2\ln x}{x^2} = \frac{2(1-\ln x)}{x^2} = 0 \Leftrightarrow \ln x = 1 \Leftrightarrow x = e$

	0		e		$+\infty$
f''		$+$	0	$-$	
		$\cup$	PI	$\cap$	



6. $f(x) = x \ln x$ $\qquad D_f =]0, +\infty[$

· $f(x) = 0 \Leftrightarrow x = 0$ (imp) $\qquad \ln x = 0 \Leftrightarrow x = 1 \quad (1;0)$

·

x	0		1		$+\infty$
f	$\|\|$	$-$	0	$+$	

· $\lim\limits_{x \to 0} f(x) = 0 \qquad$ Trou $(0;0)$

· $\lim\limits_{x \to +\infty} f(x) = +\infty$

· $f'(x) = \ln x + x\frac{1}{x} = \ln x + 1 = 0 \Leftrightarrow \ln x = -1 \Leftrightarrow x = \frac{1}{e}$

x	0		$1/e$		$+\infty$
f'	$\|\|$	$-$	0	$+$	
f		$\searrow$		$\nearrow$	

Min $= f\left(\frac{1}{e}\right) = \frac{-1}{e}$

· $f''(x) = \frac{1}{x} > 0 \quad \forall x \in D_f$

x	0		$+\infty$
f''	$\|\|$	$+$	
f		$\cup$