LJP : EXP - LOG : SERIE 1

EXERCICE 1

i) $f(x) = \ln(2x-1)$

· $2x-1>0 \Leftrightarrow x>1/2$ $D_f =]\frac{1}{2} ; +\infty[$

· $f(x)=0 \Leftrightarrow \ln(2x-1)=0 \Leftrightarrow 2x-1=1 \Leftrightarrow 2x=2 \Leftrightarrow x=1$

	1/2		1		+∞
f		−	0	+	

ii) $f(x) = 3\ln(-x)$

· $-x>0 \Leftrightarrow x<0$ $D_f =]-\infty, 0[$

· $f(x) = 3\ln(-x) = 0 \Leftrightarrow -x=1 \Leftrightarrow x=-1$

	−∞		−1		0
f		+	0	−	

iii) $f(x) = \ln(x^2-4x+3)$

$x^2-4x+3>0$

+	0	−	0	+
+	1	−	3	+

· $D_f =]-\infty, 1[\cup]3, +\infty[$

· $f(x)=0 \Leftrightarrow x^2-4x+3=1 \Leftrightarrow x^2-4x+2=0$ $<$ $x_1 = 3.41$ ✓ $x_2 = 0.58$ ✓

−∞		1		3		3,41		+∞
	+		▨		−	0	+	

IV) $f(x) = \dfrac{\ln(x^2-4)}{x-5}$

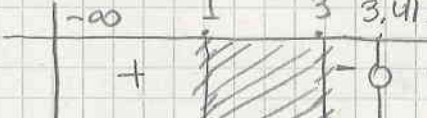
· $x \neq 5$

· $x^2-4>0$ + 0 − 0 + (−2, 2)

$\Big\}$ $D_f =]-\infty, -2[\cup]2, 5[\cup]5, +\infty[$

V) $f(x) = \dfrac{\sqrt{2-x}}{\log x}$

· $x>0$ · $\log x \neq 0 \Leftrightarrow x \neq 1$ · $2-x \geq 0 \Leftrightarrow 2 \geq x$



$D_f =]0, 1[\cup]1, 2]$

VI) $f(x) = \sqrt[3]{\dfrac{x}{2-\log x}}$

· $x>0$ · $2-\log x \neq 0 \Leftrightarrow \log x \neq 2 \Rightarrow 10^{\log x} \neq 10^2 \Leftrightarrow x \neq 100$

$D_f =]0, 100[\cup]100, +\infty[$

VII) $f(x) = \log(x^3 - 6x^2 + 5x)$

- $x^3 - 6x^2 + 5x > 0 \Leftrightarrow x(x^2 - 6x + 5) > 0 \Leftrightarrow x(x-2)(x-3) > 0$

	$-\infty$		0		2		3		$+\infty$
x^2-6x+5		$+$	$\vert$	$+$	0	$-$	0	$+$	
x		$-$	0	$+$	$\vert$	$+$	$\vert$	$+$	
P		$-$	0	$+$	0	$-$	0	$+$	

$D_f =]0, 2[\cup]3, +\infty[$

VIII) $f(x) = 2^{\frac{\sqrt{x+3}}{\log x}}$

- $x + 3 \geq 0 \Leftrightarrow x \geq -3$
- $\log x \neq 0 \Leftrightarrow 10^{\log x} \neq 10^0 \Leftrightarrow x \neq 1$

$D_f = [-3, 1[\cup]1, +\infty[$

IX) $f(x) = \frac{\ln(x^2)}{\sqrt{-x^2+9}}$

- $x^2 > 0 \Leftrightarrow x \neq 0$
- $-x^2 + 9 > 0$ $\quad \begin{array}{ccccc} - & \vert & + & \vert & - \\ \hline & -3 & & 3 & \end{array}$

$D_f =]-3; 0[\cup]0, +3[$

X) $f(x) = 3^{\frac{x}{\sin x}}$

- $\sin x \neq 0 \Leftrightarrow x \neq k\pi \quad k \in \mathbb{Z}$

EXERCICE 2

a) 1) $\log_2 16 = \log_2 2^4 = 4$

2) $\log_7 \left(\frac{1}{49}\right) = \log_7 7^{-2} = -2$

3) $\log_4 1 = 0$

4) $\log_5 5 = 1$

5) $\log_{27}\left(\frac{1}{3}\right) = \frac{\log_3 3^{-1}}{\log_3 3^3} = \frac{-1}{3}$

6) $\log_{16} 8 = \frac{\log_2 8}{\log_2 16} = \frac{3}{4}$

7) $\log_2(2\sqrt{2}) = \log_2(2 \cdot 2^{1/2}) = \log_2 2^{3/2} = \frac{3}{2}$

8) $\log_{\sqrt{2}}(8\sqrt{2}) = \frac{\log_2 8\sqrt{2}}{\log_2 \sqrt{2}} = \frac{\log_2 2^3 2^{1/2}}{\log_2 2^{1/2}} = \frac{\log_2 2^{7/2}}{\log_2 2^{1/2}} = \frac{7/2}{1/2} = 7$

b) 1) $\log_y(49) = 2 \Leftrightarrow y^2 = 49 \Leftrightarrow y = 7$ $\quad y > 0 \quad y \neq 1$

2) $\log_4 y = -3 \Leftrightarrow y = 4^{-3}$ $\quad y > 0$

3) $\log_{1/2}(y) = 8 \Leftrightarrow y = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$ $\quad y > 0$

4) $\log_y(1296) = 4 \Leftrightarrow y^4 = 1296 \Rightarrow y = 6$ $\quad y > 0 \quad y \neq 1$

c) 1) $\log(pqr) = \log p + \log q + \log r$

2) $\log(pq^2r^3) = \log p + 2\log q + 3\log r$

3) $\log(100pr^5) = \log 10^2 + \log p + 5\log r = 2 + \log p + 5\log r$

4) $\log\left(\sqrt{\frac{p}{q^2r}}\right) = \frac{1}{2}\log\frac{p}{q^2r} = \frac{1}{2}\left(\log p - 2\log q - \log r\right)$

5) $\log\left(\frac{qr^2p}{10}\right) = \log q + 2\log r + \log p - 1$

6) $\log\left(\sqrt{\frac{10p^{10}r}{q}}\right) = \frac{1}{2}\log\frac{10p^{10}r}{q} =$

$= \frac{1}{2}\left(\log 10 + 10\log p + \log r - \log q\right) =$

$= \frac{1}{2}\left(1 + 10\log p + \log r - \log q\right)$

d) 1) $2\log 5 + \log 4 = \log 5^2 + \log 4 = \log(5^2 \cdot 4) = \log 100 = 2$

2) $2\log 2 + \log 150 - \log(6000) = \log\left(\frac{2^2 \cdot 150}{6000}\right) =$

$= \log\left(\frac{6\cancel{00}}{60\cancel{00}}\right) = \log\left(\frac{1}{10}\right) = \log 10^{-1} = -1$

3) $\log 24 - \frac{1}{2}\log 9 + \log 125 = \log\left(\frac{24 \cdot 125}{9^{1/2}}\right) =$

$= \log\left(\frac{3000}{3}\right) = \log 1000 = \log 10^3 = 3$

4) $3\log 2 + 3\log 5 - \log 10^6 = \log\frac{2^3 \cdot 5^3}{10^6} = \log\frac{1000}{10^6} =$

$= \log\frac{1}{10^3} = \log 10^{-3} = -3$

5) $\frac{1}{2}\log 16 + \frac{1}{0}\log 8 = \log(16^{1/2} \cdot 8^{1/3}) = \log(4 \cdot 2) = \log 8$

6) $\log(64) - 2\log(4) + 5\log 2 - \log(2^7) =$

$= \log\frac{64 \cdot 2^5}{4^2 \cdot 2^7} = \log 4 \cdot 2^{-2} = \log 1 = 0$