LJP: Fonctions exponentielles et logarithmes

Exercice 1

$$a^{m-2} \cdot a^{3-m} = a^{\cancel{m}-2+3-\cancel{m}} = a$$

$$\frac{(16x^4y^2)^n}{(4x^2y)^{2n-1}} = \frac{4^{2n}x^{4n}y^{2n}}{4^{2n-1}x^{4n-2}y^{2n-1}} = 4^{2n-2n+1}x^{4n-4n+2}y^{2n-2n-1} =$$

$$= \frac{4 \cdot x^2}{y}$$

$$\sqrt[3]{a \cdot \sqrt{a^4}} = \sqrt[3]{a\, a^{4/2}} = \sqrt[3]{a^3} = a$$

$$\sqrt[3]{a\sqrt[3]{a^4\sqrt[3]{a^6}}} = \sqrt[3]{a\sqrt[3]{a^4a^2}} = \sqrt[3]{a\sqrt[3]{a^6}} = \sqrt[3]{a\,a^2} = \sqrt[3]{a^3} = a$$

$$a^{2n-9} \cdot a^{8-n} = a^{2n-9+8-n} = a^{n-1}$$

$$\frac{(27a^6b^3)^{n+1}}{(3a^2b)^{3n+3}} = \frac{\cancel{3^{3n+3}}\,\cancel{a^{6n+6}}\,\cancel{b^{3n+3}}}{\cancel{3^{3n+3}}\,\cancel{a^{6n+6}}\,\cancel{b^{3n+3}}} = 1$$

$$\sqrt{a}\,\sqrt[5]{a^3} \cdot (\sqrt[10]{a})^4 = a^{1/2} \cdot a^{3/5}\,a^{4/10} = a^{\frac{1}{2}+\frac{3}{5}+\frac{2}{5}} =$$

$$= a^{\frac{5+6+4}{10}} = a^{15/10} = a^{3/2} = \sqrt{a^3}$$

Exercice 2

$$\log_2(64) = \log_2 2^6 = 6$$

$$\log_2(1024) = \log_2 2^{10} = 10$$

$$\log_5(125) = \log_5 5^3 = 3$$

$$\log 10 = 1$$

$$\log_4(256) = \log_4 4^4 = 4$$

$$\log_3(243) = \log_3 3^5 = 5$$

$$\log_2\left(\frac{1}{2}\right) = \log_2 2^{-1} = -1$$

$$\log_4(4^{-3}) = -3$$

$$\log_5(0.008) = \log_5 5^{-3} = -3$$

Exercice 3

1) $\log(0.1a^3bc^2) = \log 10^{-1} + \log a^3 + \log b + \log c^2 =$

$= -1 + 3\log a + \log b + 2\log c$

2) $\log\left(\frac{a^2\sqrt{b}}{c}\right) = \log a^2 + \log b^{1/2} - \log c = 2\log a + \frac{1}{2}\log b - \log c$

3) $\log \sqrt[3]{a^5b^3c} = \frac{1}{3}\log(a^5b^3c) = \frac{1}{3}(5\log a + 3\log b + \log c)$

4) $\log\left(10a^2b^3\sqrt{c^2}\right)^2 = 2(\log 10 + 2\log a + 3\log b + \log c)$

5) $\log\sqrt[3]{a^4b\sqrt{c}} = \frac{1}{3}\left(4\log a + \log b + \frac{1}{2}\log c\right)$

6) $\log\left[\left(\frac{a}{b^2}\right)^3\sqrt[3]{\frac{a^2}{b}}\right] = 3\log\left(\frac{a}{b^2}\right) + \frac{1}{3}\log\frac{a^2}{b} =$

$= 3(\log a - 2\log b) + \frac{1}{3}(2\log a - \log b) =$

$= 3\log a - 6\log b + \frac{2}{3}\log a - \frac{1}{3}\log b =$

$= \frac{11}{3}\log a - \frac{19}{3}\log b$

EXERCICE 4

1) $\log_6(18) + \log_6(2) = \log_6(36) = \log_6 6^2 = 2$

2) $\log_2(24) - \log_2(3) = \log_2\frac{24}{3} = \log_2 8 = \log_2 2^3 = 3$

3) $\log 2 + \log 5 = \log 10 = 1$

4) $2\log 20 - \log 4 = \log\frac{20^2}{4} = \log\frac{400}{4} = \log 100 = 2$

5) $\log 90 + \log 4 - 2\log 6 = \log\frac{90\cdot 4}{6^2} = \log\frac{360}{36} = \log 10 = 1$

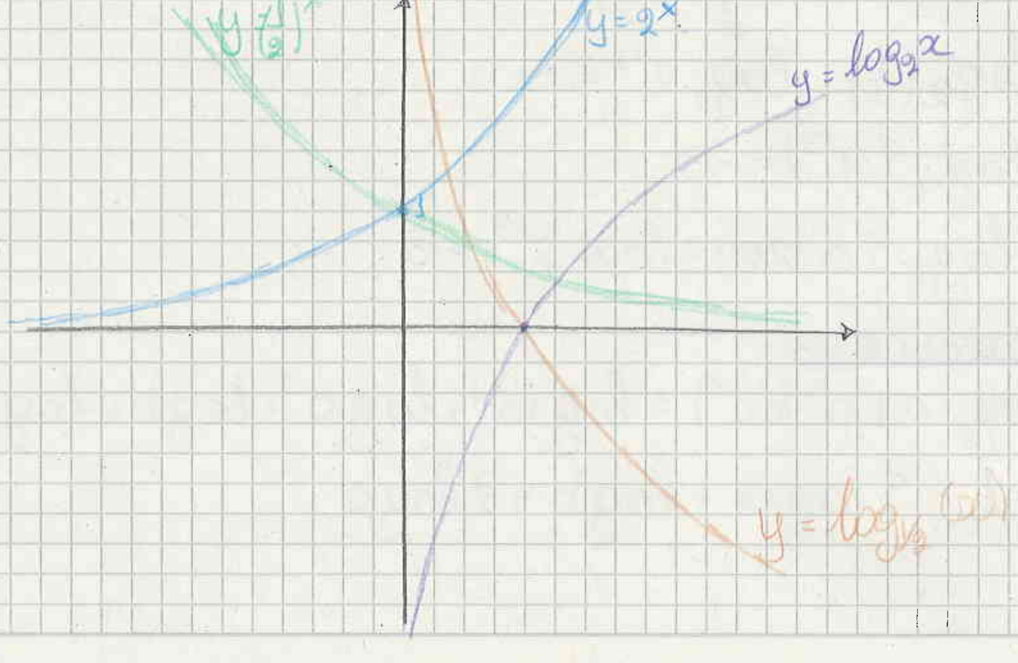
6) $-2\log(5) - 2\log(2) = \log 5^{-2} + \log 2^{-2} =$

$= \log(5\cdot 2)^{-2} = -2\log 10 = -2$

EXERCICE 5

a) $y = 2^x$ $D_f = \mathbb{R}$ b) $y = \log_2 x$ $D_f =]0,+\infty[$

c) $y = \left(\frac{1}{2}\right)^x$ $D_f = \mathbb{R}$ d) $y = \log_{1/2}(x)$ $D_f =]0,+\infty[$



EXERCICE 6

1) $\log(x) = 9 \Leftrightarrow x = 10^9$

$x > 0$

2) $\log(x) = 1 + \log(7) \Leftrightarrow x = 10^{1+\log 7} = 10 \cdot 10^{\log 7} = 10 \cdot 7 = 70$

$x > 0$

3) $\log(x+1) = 3 \Leftrightarrow (x+1) = 10^3 \Rightarrow x = 1000 - 1 = 999$

$x > -1$

4) $\log_x(2) = 3 \Leftrightarrow x^3 = 2 \Rightarrow x = \sqrt[3]{2}$

$x > 0 \quad x \neq 1$

5) $27^{x+2} = 3^{5x+8} \Leftrightarrow 3^{3x+6} = 3^{5x+8} \Leftrightarrow 3x + 6 = 5x + 8 \Leftrightarrow 2x = -2 \Leftrightarrow$

$x = -1$

6) $8^{4x+1} = 1 \Leftrightarrow 8^{4x+1} = 8^0 \Leftrightarrow 4x + 1 = 0 \Leftrightarrow x = -1/4$

7) $5^{3x+1} = \frac{1}{25} = 5^{-2} \Leftrightarrow 3x + 1 = -2 \Leftrightarrow 3x = -3 \Leftrightarrow x = -1$

8) $2^{3x+4} = 45 \Leftrightarrow 3x + 4 = \log_2 45 \Leftrightarrow x = \frac{\log_2 45 - 4}{3}$

9) $\log_2(81) = x$

10) $\log_3(x-2) + \log_3(x-4) = 2 \Leftrightarrow \log(x^2 - 6x + 8) = \log 10^2 \Leftrightarrow \begin{cases} x_1 = 13 \checkmark \\ x_2 = -7 \times \end{cases}$

$x - 2 > 0 \Leftrightarrow x > 2 \quad x - 4 > 0 \Leftrightarrow x > 4 \Rightarrow \underline{x > 4}$

11) $\log(2x) + 3 = 2\log(x) \Leftrightarrow \log(2x) - \log x^2 = -3 \Leftrightarrow$

$\underline{x > 0} \quad \Leftrightarrow \log \frac{2x}{x^2} = 3 \Leftrightarrow \frac{2}{x} = 10^3 \Leftrightarrow x = \frac{2}{1000} = \frac{1}{500}$

12) $7^{x+1} = 5 \cdot 3^x \Leftrightarrow$

$\Leftrightarrow (x+1)\log 7 = \log 5 + x \log 3 \Leftrightarrow$

$\Leftrightarrow x \log 7 + \log 7 = \log 5 + x \log 3 \Leftrightarrow$

$\Leftrightarrow x(\log 7 - \log 3) = \log 5 - \log 7 \Leftrightarrow$

$\Leftrightarrow x = \frac{\log 5 - \log 7}{\log 7 - \log 3}$

13) $\log(x^2 + x) = 0 \Leftrightarrow$

$x^2 + x > 0 \Leftrightarrow x(x+1) > 0$

	-1		0	
$+$	φ	$-$	φ	$+$

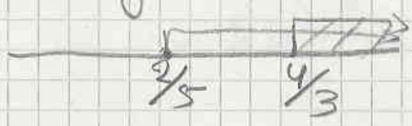
$\underline{x < -1}$ ou $\underline{x > 0}$

$\log(x^2 + x) = 0 \Leftrightarrow x^2 + x = 1 \Leftrightarrow x^2 + x - 1 = 0$

$\Delta = 1 + 4 = 5 \quad x_{1,2} = \frac{-1 \pm \sqrt{5}}{2} \begin{cases} x_1 \approx -1.62 \checkmark \\ x_2 \approx 0.6 \checkmark \end{cases}$

14) $\log(3x-4) + \log(10x-4) = 2\log(5x-2) \quad (1)$

$\left.\begin{aligned} &3x - 4 > 0 \Leftrightarrow x > 4/3 \\ &10x - 4 > 0 \Leftrightarrow x > 2/5 \\ &5x - 2 > 0 \Leftrightarrow x > 2/5 \end{aligned}\right\}$



$\underline{x > 4/3}$

(1) $\log \frac{(3x-4)(10x-4)}{(5x-2)^2} = 0 \Leftrightarrow \frac{(3x-4)(10x-4)}{(5x-2)^2} = 1 \Leftrightarrow$

$\Leftrightarrow 30x^2 - 40x - 12x + 16 = 25x^2 - 20x + 4 \Leftrightarrow$

$\Leftrightarrow 5x^2 - 32x + 12 = 0 \qquad \Delta = 784$

$x_{1,2} = \frac{32 \pm 28}{10} \begin{cases} x_1 = 6 \ ✓ \\ x_2 = 0.4 \ × \end{cases}$

15) $\ln(x+1) - \ln(2-x) + \ln 2 = \ln 7 - \ln(4-x)$ (1)

$\left.\begin{array}{l} x > -1 \\ 2 - x > 0 \Leftrightarrow x < 2 \\ 4 - x > 0 \Leftrightarrow x < 4 \end{array}\right\}$

$-1 < x < 2$

(1) $\ln\left(\frac{(x+1) \cdot 2}{2-x}\right) = \ln \frac{7}{4-x} \Leftrightarrow$

$\Leftrightarrow \frac{2x+2}{2-x} = \frac{7}{4-x} \Leftrightarrow (2x+2)(4-x) = 7(2-x) \Leftrightarrow$

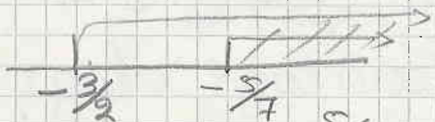
$\Leftrightarrow 8x + 8 - 2x^2 - 2x = 14 - 7x \Leftrightarrow$

$\Leftrightarrow -2x^2 + 6x + 7x - 6 = 0 \Leftrightarrow 2x^2 - 13x + 6 = 0$

$\Delta = 121 \qquad x_{1,2} = \frac{13 \pm 11}{4} = \begin{cases} x_1 = 6 \ × \\ x_2 = \frac{1}{2} \ ✓ \end{cases}$

16) $\log(\sqrt{7x+5}) + \log(\sqrt{2x+3}) = 1 + \log(4.5)$ (1)

$\left.\begin{array}{l} 7x + 5 > 0 \Leftrightarrow x > -5/7 \\ 2x + 3 > 0 \Leftrightarrow x > -3/2 \end{array}\right\}$



$x > -5/7$

(1) $\Leftrightarrow \log(\sqrt{7x+5} \cdot \sqrt{2x+3}) = \log(45) \Leftrightarrow$

$\Leftrightarrow \sqrt{14x^2 + 10x + 21x + 15} = 45 \Leftrightarrow$

$\Leftrightarrow 14x^2 + 31x + 15 = 2025 \Leftrightarrow$

$\Leftrightarrow 14x^2 + 31x - 2010 = 0$

$\Delta = 113521 \qquad x_{1,2} = \frac{-31 \pm \sqrt{113521}}{28} \begin{cases} x_1 \approx 10.9 \ ✓ \\ x_2 \approx -13.14 \ × \end{cases}$

17) $e^{2x} + e^x - 6 = 0 \qquad t = e^x > 0$

$t^2 + t - 6 = 0 \qquad \Delta = 1 + 24 = 25 \qquad t_{1,2} = \frac{-1 \pm 5}{2} \begin{cases} t_1 = -3 \\ t_2 = 2 \end{cases}$

$e^x = -3$ impossible

$e^x = 2 \Leftrightarrow x = \ln 2$ ✓

18) $3e^{4x} - 5e^{2x} = 0 \qquad t = e^{2x}$

$3t^2 - 5t = 0 \Leftrightarrow t(3t-5) = 0 \Leftrightarrow t = 0 \quad t = 5/3$.

$e^{2x} = 0$ impossible

$e^{2x} = \frac{5}{3} \Leftrightarrow 2x = \ln\left(\frac{5}{3}\right) \Leftrightarrow x = \frac{1}{2}\ln\left(\frac{5}{3}\right)$

19) $2e^{6x} + 5e^{3x} - 3 = 0 \qquad t = e^{3x}$

$2t^2 + 5t - 3 = 0 \qquad \Delta = 25 + 24 = 49 \quad t_{1,2} = \frac{-5 \pm 7}{4}$

$e^{3x} = -3$ impossible $\qquad t_1 = -3 \quad t_2 = \frac{1}{2}$.

$e^{3x} = \frac{1}{2} \Leftrightarrow 3x = \ln 2^{-1} \Leftrightarrow$

$\Leftrightarrow x = -\frac{1}{3}\ln 2$.

EXERCICE 7

a) $y = e^{2x} \Rightarrow y' = 2e^{2x}$

b) $y = xe^x \Rightarrow y' = e^x + xe^x = e^x(x+1)$

c) $y = e^{1/x} \Rightarrow y' = -\frac{1}{x^2}e^{1/x} \qquad \left(\text{car } \left(\frac{1}{x}\right)' = -\frac{1}{x^2}\right)$

d) $y = x^2e^x \Rightarrow y' = 2xe^x + x^2e^x = (x^2+2x)e^x$

e) $y = e^{\sin x} \Rightarrow y' = e^{\sin x}\cdot\cos x$

f) $y = xe^{x^2} \Rightarrow y' = e^{x^2} + 2x^2e^{x^2} = e^{x^2}(1+2x^2)$

g) $y = \ln(2x) \Rightarrow y' = \frac{1}{2x}\cdot 2 \Rightarrow y' = \frac{1}{x}$

h) $y = x\ln(x) \Rightarrow y' = \ln x + \frac{x}{x} \Rightarrow y' = \ln x + 1$

i) $y = \ln\left(\frac{1}{x}\right) = \ln x^{-1} = -\ln x$

$y' = -\frac{1}{x}$

j) $y = x^2\ln x \Rightarrow y' = 2x\ln x + x^2\frac{1}{x} \Rightarrow$

$y' = 2x\ln x + x = x(2\ln x + 1)$

k) $y = \ln(x^2) \Rightarrow y' = \frac{1}{x^2}\cdot 2x \Rightarrow y' = \frac{2}{x}$

l) $y = \ln(e^x) = x \Rightarrow y' = 1$

m) $y = e^x\ln(x) \Rightarrow y' = e^x\ln x + e^x\frac{1}{x} = e^x\left(\ln x + \frac{1}{x}\right)$

n) $y = \ln(6x^3 - 3x^2 - 7) \Rightarrow y' = \frac{18x^2 - 6x}{6x^3 - 3x^2 - 7}$

EXERCICE 8

- $\ln(x^2) \quad x \neq 0$
- $\ln(x^2+1) \quad x \in \mathbb{R}$
- $\ln(x-3) \quad x > 3$
- $\frac{\ln(x)}{x} \quad x > 0$
- $\frac{1}{\ln x} \quad x > 0 \quad \ln x \neq 0 \Rightarrow x \neq 1$

$x \in]0,1[\cup]1,+\infty[$

EXERCICE 9

a) $\lim\limits_{x\to+\infty} \ln(x^3) = +\infty$

b) $\lim\limits_{x\to 0} \dfrac{\ln(x)}{x} = -\infty$

c) $\lim\limits_{x\to+\infty} \dfrac{\ln(x)}{x^2} = 0$

d) $\lim\limits_{x\to+\infty} \ln\left(\dfrac{x+1}{x-1}\right) = \lim\limits_{t\to 1} \ln 1 = 0$

e) $\lim\limits_{x\to-\infty} x \cdot e^{x} = 0$

f) $\lim\limits_{x\to+\infty} \dfrac{e^{x}}{x} = +\infty$

g) $\lim\limits_{x\to+\infty} (e^{-x})^2 = 0$

h) $\lim\limits_{x\to+\infty} (x^2 e^{-x}) = 0$

i) $\lim\limits_{x\to 0} (x^2+1)\ln(x) = -\infty$

j) $\lim\limits_{x\to-\infty} (x^2+2)e^{x} = 0$

k) $\lim\limits_{x\to-\infty} \overset{\to 0}{\dfrac{2x-1}{x^3+2}} \overset{+\infty}{\ln(x^2+1)} = 0$

l) $\lim\limits_{x\to+\infty} x e^{1/x} = +\infty$

m) $\lim\limits_{x\to-\infty} \dfrac{x^2}{e^{x^3-3x+4}} = +\infty$

n) $\lim\limits_{x\to 0} \underset{1}{e^{-x^2}}(x^2-12x+3) = 3$

EXERCICE 10

1) $f(x) = (x+1)e^{x}$ $\qquad D_f = \mathbb{R}$

$\cap Ox$ · $f(x) = 0 \Leftrightarrow (x+1) = 0 \Leftrightarrow x = -1$ $\qquad (-1;0)\ \cap Ox$

$\cap Oy$ · $y = 0 \quad f(0) = 1$ $\qquad (0;1)$

· TS

x	$-\infty$		-1		$+\infty$
f		$-$	0	$+$	

· AH: $\lim\limits_{x \to +\infty} f(x) = +\infty$ $\qquad \emptyset$

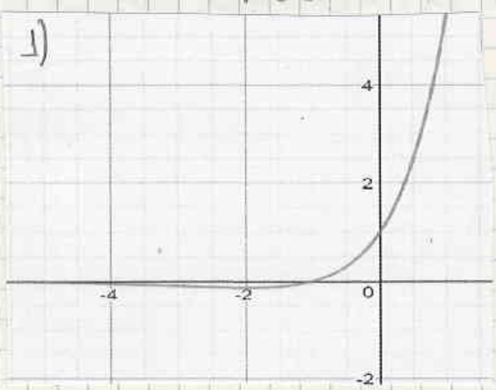
$\lim\limits_{x \to -\infty} f(x) = 0$ $\qquad$ AH: $y = 0$

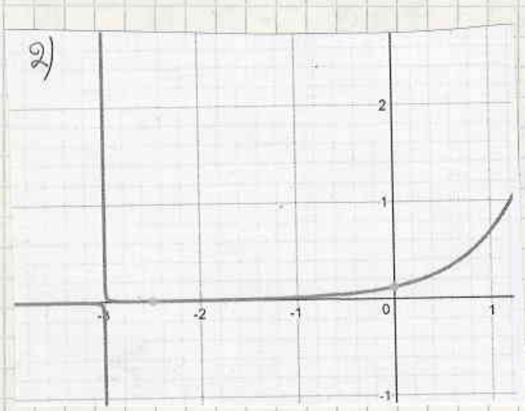
· AV $\emptyset$

· $f'(x) = e^{x} + (x+1)e^{x} = e^{x}(x+2) = 0 \Leftrightarrow x = -2$

x	$-\infty$		-2		$+\infty$
f'		$-$	0	$+$	
f		$\nearrow$	Max	$\searrow$	

Max $= f(-2) = -e^{-2} \approx -0{,}135$





2) $f(x) = \dfrac{e^{2x-1}}{x+3}$ $\qquad D_f = \mathbb{R} \setminus \{-3\}$

· $\cap Ox$: $f(x) = 0$ impossible

· $\cap Oy$: $y = 0 \quad f(0) = \dfrac{e^{-1}}{3} \approx 0.123$

· TS

x	$-\infty$		-3		$+\infty$
f		$-$	//	$+$	

· AV: $\lim\limits_{x \to -3} f(x) = \text{"}\dfrac{e^{-7}}{0}\text{"}$ $\begin{cases} +\infty & x > -3 \\ -\infty & x < -3 \end{cases}$ $\qquad$ AV: $x = -3$

AH: $\lim\limits_{x \to +\infty} f(x) = +\infty$ $\qquad \emptyset$

$\lim\limits_{x \to -\infty} f(x) = 0$ $\qquad$ AH: $y = 0$

- $f'(x) = \dfrac{2e^{2x-1}\cdot(x+3) - e^{2x-1}}{(x+3)^2} = \dfrac{e^{2x-1}(2x+6-1)}{(x+3)^2} = \dfrac{e^{2x-1}(2x+5)}{(x+3)^2} = 0 \Leftrightarrow$

$\Leftrightarrow x = -3/2$

x	$-\infty$		-3		$-3/2$		$+\infty$
f'		$-$	$\|\|$	$-$	0	$+$	
f		$\searrow$		$\searrow$	Min	$\nearrow$	

$\text{Min} = f(-3/2) \simeq 0.0122$

3) $f(x) = x^2 \cdot e^{-x}$ $\qquad D_f = \mathbb{R}$

- $\cap Ox$: $f(x) = 0 \Leftrightarrow x = 0$ $\qquad (0;0)$

$\cap Oy$: $x = 0$ $f(0) = 0$.

x	$-\infty$		0		$+\infty$
f		$+$	0	$+$	

- AH: $\lim\limits_{x\to+\infty} f(x) = 0$ $\qquad$ AH: $y = 0$

$\lim\limits_{x\to-\infty} f(x) = +\infty$ $\qquad \emptyset$

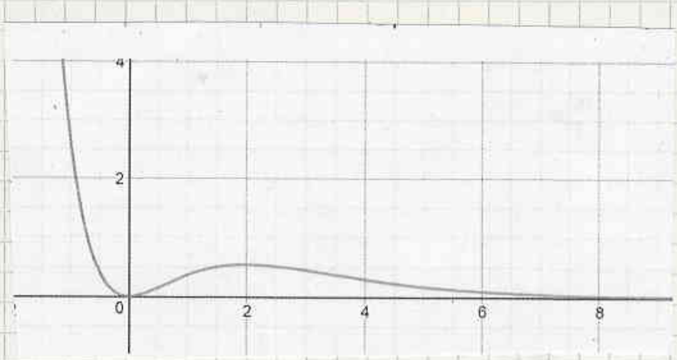
AV $\emptyset$

- $f'(x) = 2xe^{-x} - x^2e^{-x} = e^{-x}\cdot x(2-x) = 0 \Leftrightarrow x = 0 \quad x = 2$

x	$-\infty$		0		2		$+\infty$
f'		$-$	0	$+$	0	$-$	
f		$\searrow$	Min	$\nearrow$	Max	$\searrow$	

Min $f(0) = 0$

Max $f(2) = 4e^{-2} = 0.54$



4) $f(x) = \ln(x^2)$ $\qquad x^2 > 0 \Leftrightarrow x \neq 0$ $\qquad D_f = \mathbb{R}\setminus\{0\}$

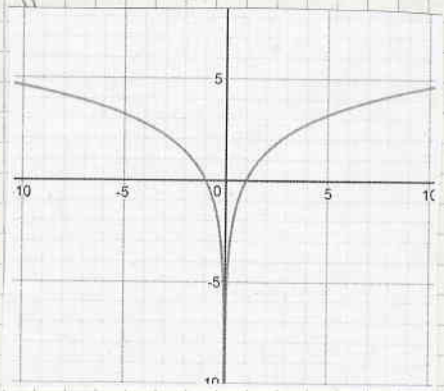
- $\cap Ox$: $f(x) = 0 \Leftrightarrow \ln(x^2) = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$ $\quad (1;0)\ (-1;0)$
- $\cap Oy$: $x = 0$ impossible.

x	$-\infty$		-1		0		1		$+\infty$
f		$+$	0	$-$	$\|\|$	$-$	0	$+$	

- AH $\lim\limits_{x\to\pm\infty} f(x) = +\infty$ $\qquad \emptyset$
- AV: $\lim\limits_{x\to 0} f(x) = -\infty$ $\qquad$ AV: $x = 0$

- $f'(x) = \frac{1}{x^2}\, 2x = \frac{2}{x} = 0$ impossible

x	$-\infty$		0		$+\infty$
f'		$+$	$\|$	$+$	
f		$\nearrow$	$\|$	$\nearrow$	



5) $f(x) = \frac{20\cdot e^{-2x}}{(x-3)^2}$ $\qquad D_f = \mathbb{R}\setminus\{3\}$

- $\cap Ox$: $f(x) = 0 \Leftrightarrow 20e^{-x} = 0$ impossible
- $\cap Oy$ $f(0) = \frac{20}{9} \simeq 2.22$ $\qquad I(0; 2.22)$
-

x	$-\infty$		3		$+\infty$
f		$+$	$\|$	$+$	

- AV: $\lim\limits_{x\to 3} f(x) = +\infty$ $\qquad$ AV: $x = 3$

AH: $\lim\limits_{x\to +\infty} f(x) = 0$ $\qquad$ AH: $y = 0$

$\lim\limits_{x\to -\infty} f(x) = +\infty$ $\qquad \emptyset$

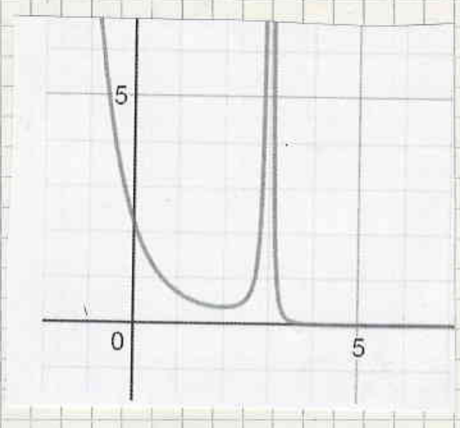
- $f'(x) = \frac{-40e^{-2x}(x-3)^2 - 20\cdot e^{-x}\cdot 2(x-3)}{(x-3)^4} =$

$= \frac{-40e^{-2x}(x-3)\left[x-3+1\right]}{(x-3)^4} = \frac{-40e^{-2x}\cancel{(x-3)}(x-2)}{(x-3)^{\cancel{4}3}} =$

$= \frac{-40e^{-2x}(x-2)}{(x-3)^3} = 0 \Leftrightarrow x = 2$

x	$-\infty$		2		3		$+\infty$
f'		$-$	0	$+$	$\|$	$-$	
f		$\searrow$	Min	$\nearrow$	$\|$	$\searrow$	

$\text{Min} = f(2) = \frac{20e^{-4}}{1} \simeq 0.37$



6) $f(x) = 2e^{x} - e^{2x}$ $\qquad D_f = \mathbb{R}$

· $\cap Ox$: $f(x) = 0 \Leftrightarrow e^{x}(2-e^{x}) = 0 \Leftrightarrow e^{x} = 2 \Leftrightarrow x = \ln 2 \approx 0.7$ $\quad (0.7, 0)$

· $\cap Oy$: $x = 0$ $\; f(0) = 2 - 1 = 1$ $\quad (0;1)$

· TS

x	$-\infty$		$\ln 2$		$+\infty$
f		$+$	0	$-$	

· AV $\emptyset$

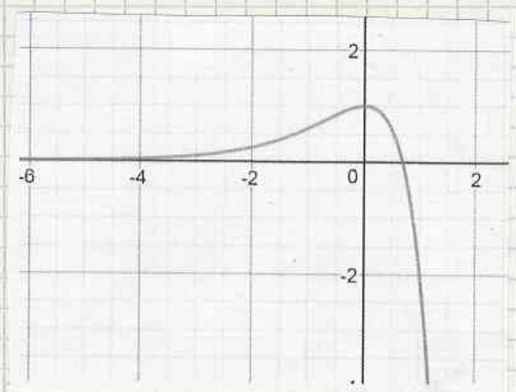
· AH : $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} e^{x}(2-e^{x}) = +\infty(-\infty) = -\infty$ $\quad \emptyset$

$\lim\limits_{x\to-\infty} f(x) = 0$ $\quad$ AH : $y = 0$

· $f'(x) = 2 - 2e^{2x} = 0 \Leftrightarrow 1 = e^{2x} \Leftrightarrow 2x = 0 \Leftrightarrow x = 0$

x	$-\infty$		0		$+\infty$
f'		$+$	0	$-$	
f		$\nearrow$	Max	$\searrow$	

Max $= f(0) = 1$



7) $f(x) = \ln(x^2 - x - 2)$ $\qquad x^2 - x - 2 > 0 \Leftrightarrow (x+2)(x-1) > 0$

	-2		1	
$+$	0	$-$	0	$+$

$D_f =]-\infty; -2[\cup]1, +\infty[$

· $\cap Ox$: $f(x) = 0 \Leftrightarrow x^2 - x - 2 = 1 \Leftrightarrow x^2 - x - 3 = 0$ $\quad x_1 = 2.3$, $x_2 = -6$

· $\cap Oy$: $x = 0$ impossible

	$-\infty$		-6		-2		1		2.3		$+\infty$
f		$+$	0	$-$		//////		$-$	0	$+$	

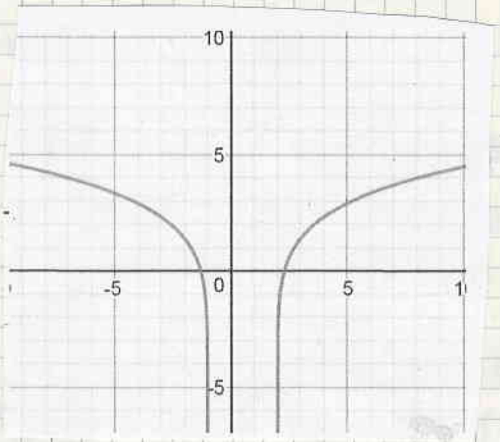
· AV : $\lim\limits_{x\to-2} f(x) = -\infty$ $\qquad$ AV $x = -2$, $x = 1$

$\lim\limits_{x\to 1} f(x) = -\infty$

· AH : $\lim\limits_{x\to\pm\infty} f(x) = +\infty$ $\qquad \emptyset$

$\cdot f'(x) = \frac{1}{x^2-x-2} \cdot (2x-1) = \frac{2x-1}{x^2-x-2} = 0 \Leftrightarrow x = \frac{1}{2}$ impossible

x	$-\infty$	-2	1	$+\infty$
f'	$-$	//////	$+$	
f	$\searrow$		$\nearrow$	



EXERCICE 11

a) $f(x) = (2x^2+ax)e^{-x}$

$f'(x) = (4x+a)e^{-x} - (2x^2+ax)e^{-x} = e^{-x}(4x+a-2x^2-ax) =$

$= e^{-x}(-2x^2+(4-a)x+a)$

$\Delta = (4-a)^2 - 4\cdot(-2)a = 16 - 8a + a^2 + 8a = a^2+16 > 0 \; \forall a \in \mathbb{R}$

$\Rightarrow$ l'équation $f(x) = 0$ a 2 solutions $\forall a \in \mathbb{R}$

$\Rightarrow f$ admet 2 p t h

b) $f'(0) = 3 \Leftrightarrow 1 \cdot (a) = 3 \Leftrightarrow a = 3$

EXERCICE 12

$f(x) = 4e^{-2x} \quad g(x) = e^{x+3}$

a) $f(x) = g(x) \Leftrightarrow 4e^{-2x} = e^{x+3} \Leftrightarrow \ln(4e^{-2x}) = \ln(e^{x+3})$

$\Leftrightarrow \ln 4 + \ln e^{-2x} = \ln(e^{x+3}) \Leftrightarrow \ln 4 - 2x = x+3 \Leftrightarrow$

$\Leftrightarrow \ln 4 = 3x+3 \Leftrightarrow 3x = \ln 4 - 3 \Leftrightarrow x = \frac{\ln 4 - 3}{3} \approx -0{,}538$

$y = e^{\frac{\ln 4 - 3}{3}+3} = e^{\frac{\ln 4 - 3 + 9}{3}} = e^{\frac{\ln 4 + 6}{3}} = e^{\frac{\ln 4}{3}} \cdot e^2 = \sqrt[3]{4} \cdot e^2 \approx 11{,}73$

$I(-0{,}538 ; 11{,}73)$

b) $f'(x) = -8e^{-2x} \qquad f'(-1+\frac{2}{3}\ln 2) = -8e^{-2(\frac{2\ln 2}{3}-1)} \approx -23{,}46$

$t : 11{,}73 = -23{,}46 \cdot (-0{,}538) + h \Leftrightarrow h \approx -0{,}89$

$y = -23{,}43x - 0{,}89$

EXERCICE 13

$f(x) = 10^5 \cdot a^t \quad f(45) = 5\cdot 10^4 = 10^5 \cdot a^{45} \Leftrightarrow a^{45} = \frac{1}{2} \Rightarrow a = \sqrt[45]{\frac{1}{2}}$

$\Rightarrow a \approx 0{,}985$

EXERCICE 14

V(t) $V(t) = a e^{bt}$ $a = 72342$ $b = 0.034$

a) $V(8) = 72342 \cdot e^{0.034 \cdot 8} \simeq 94955 \ m^3$

b) $V(t) = 10^5 = 72342 \cdot e^{0.034t} \Leftrightarrow e^{0.034t} = 1.382 \Rightarrow$

$0.034t = \ln 1.382 \Rightarrow t = 9.52$ ans

c) $t = -15$ $V(-15) = 72342 \cdot e^{0.034 \cdot (-15)} \simeq 43441 \ m^3$

EXERCICE 15

$P(t) = P_0 (1 - \frac{5}{100})^t = P_0 (0,95)^t$

$P(t_0) = \frac{P_0}{2} = P_0 (0.95)^t \Leftrightarrow 0.95^t = \frac{1}{2} \Leftrightarrow t \ln 0.95 = \ln \frac{1}{2} \Rightarrow$

$\Rightarrow t = \frac{-\ln 2}{\ln(0.95)} \Rightarrow t \approx 13.5$ ans

EXERCICE 16

$t = 0$ 100

$t = 1$ $2 \cdot 100$ $P(t) = 100 \cdot 2^t$

$t = 2$ $2 \cdot 2 \cdot 100$

⋮

$t = x$ $2^x \cdot 100$

1) $t = 2.5$ h $P(t) = 100 \cdot 2^{2.5} = 565.7$

2) $P(t) = 10^5 = 10^2 \cdot 2^t \Leftrightarrow 10^3 = 2^t \Leftrightarrow t \log 2 = 3 \Rightarrow t \simeq 10$ h

EXERCICE 17

$A = 10 \cdot 0.8^n$

a) $A(1) = 10 \cdot 0.8 = 8$ mg

la perte est $10 - 8 = 2$ mg $\rightsquigarrow \frac{2}{10} \cdot 100 = 20\%$

b) $A(n) = 2 \Rightarrow 10 \cdot 0.8^n = 2 \Rightarrow 0.8^n = \frac{1}{5} \Rightarrow n \log 0.8 = -\log 5$

$\Rightarrow n \simeq 7.213$ h $= 7$ h 12 m 45 sec

c) $P = P_0 \ a^t$

$P(2.5) = \frac{P_0}{2} = P_0 \ a^{2.5} \Leftrightarrow \frac{1}{2} = a^{2.5} \Rightarrow 2.5 \log a = -\log 2 \Rightarrow$

$\log a = -0.12 \Rightarrow a = 10^{-0.12} \Rightarrow a = 0.76$

$P(1) = P_0 \cdot 0.76^1$

Donc après 1h il reste $0.76 P_0$ ⇒ il élimine $0.24 P_0$ ou 24%

EXERCICE 18

a) $P(t) = P_0 \cdot 2^t$

$P(t) = 10 \cdot P_0 = P_0 \cdot 2^t \Leftrightarrow 2^t = 10 \Leftrightarrow t \log 2 = 1 \Rightarrow t \approx 3{,}32h$

b) $P(t) = P_0 \cdot a^t$

$P(80) = 4 P_0 = P_0 a^{80} \Leftrightarrow a^{80} = 4 \Rightarrow a = \sqrt[80]{4} \approx 1.02$

EXERCICE 19

$P(t) = P_0 \, a^t$

$P(15) = P_0 \cdot a^{15} = \frac{30}{100} P_0 \Leftrightarrow a^{15} = \frac{3}{10} \Rightarrow a \approx 0{,}923$

$P(t) = P_0 \cdot 0{,}923^t$

$P(t_0) = \frac{P_0}{2} = P_0 \cdot 0.923^t \Rightarrow t \ln 0.923 = -\ln 2 \Rightarrow$

$\Rightarrow t \approx 8{,}651$

EXERCICE 20

- $P(t) = P_1 \left(1 + \frac{3.2}{100}\right)^t = P_1 \, (1.032)^t$
- $P(0) = 55 \cdot 10^3 = P_1$

$\Big\}\ P_1(t) = 55 \cdot 10^3 (1{,}032)^t$

- $P(t) = P_2 \cdot \left(1 - \frac{2.3}{100}\right)^t = P_2 \, (0{,}977)^t$

$P(0) = P_2 = 90000$

$\Big\}\ P_2(t) = 9 \cdot 10^4 \cdot 0{,}977^t$

En 1er Janvier 2001 pour le forêt A:

$P(3) = 55 \cdot 10^3 \cdot 1{,}032^3 = 60450{,}76$

Donc $\left.\begin{array}{l} P_A(t) = 60450{,}76 \cdot 1{,}032^t \\ P_B(t) = 9 \cdot 10^4 \cdot 0.977^t \end{array}\right\} P_A(t) = P_B(t) \Rightarrow$

$\Rightarrow 0{,}672 = 0{,}947^t \Rightarrow t = 7{,}258$ ans $= 7$ ans 3 mois

Alors le volume sera identique en 2008, 1 avril

EXERCICE 21

$N(t) = \frac{1000}{1 + 999 \cdot 10^{-0{,}17t}}$

1) $N(20) = \frac{1000}{1 + 999 \cdot 10^{-0{,}17 \cdot 20}} = 715$

2) $N(t) = 600 \Leftrightarrow \frac{10\cancel{00}}{1 + 999 \cdot 10^{-0{,}17t}} = 6\cancel{00} \Leftrightarrow$

$\Leftrightarrow \frac{10}{6} - 1 = 999 \cdot 10^{-0{,}17t} \Leftrightarrow 10^{-0{,}17t} = 6.673 \cdot 10^{-4} \Leftrightarrow$

$\Leftrightarrow -0{,}17t = -3.176 \Rightarrow t \approx 18{,}68$ jours

EXERCICE 22

$P_0 = 10^{-12}$ $\quad N = 10\log\left(\frac{P}{P_0}\right)$

1) $P = 10^{-6} \Rightarrow N = 10\log\left(\frac{10^{-6}}{10^{-12}}\right) = 10\log 10^6 = 60$

2) $N_2 = 90 \Rightarrow 10\log\left(\frac{P_2}{10^{-12}}\right) \Rightarrow 9 = \log(P_2 \cdot 10^{12}) \Rightarrow$

$\Rightarrow 10^9 = P_2 \cdot 10^{12} \Rightarrow P_2 = 10^{-3}$

$P_1 = 10^{-6} \longrightarrow P_2 = 10^{-3}$ facteur 10^3

3) $P = 1$ $\quad N = 10\log\left(\frac{1}{10^{-12}}\right) = 10\log 10^{12} = 120 > 90$
$\Rightarrow$ oui.