without « formulaire »

Exercise 1 / 6 pts

We consider the polynomial $p(z) = z^3 + 5z^2 + 4z + a$, with real coefficients ($a \in \mathbb{R}$).
Given that $z_1 = 2i$ is a root of p, determine the value of a and find the other roots of p.

Exercise 2 / 6 pts

The « inversion » function is $f(z) = \frac{1}{\bar{z}}$

It is such that $f(x+iy) = \frac{x}{x^2+y^2} + i \cdot \frac{y}{x^2+y^2}$ and $f\big(rcis(\varphi)\big) = \frac{1}{r}cis(\varphi)$.

a) Precisely describe the image of the line $y = 4x$.

b) Give the cartesian equation and the geometrical description of the image under f of the line $d: y = -4$

Exercise 3 / 7 pts

We consider the function $f(z) = i\,\overline{z^2}$.

a) Give the image of $z_1 = 2 - i$.

b) The image of $z = x + iy$ is $f(z) = u + iv$. Express u and v as functions of x and y.

c) Determine the image of the line $y = -3$ under f. Give the name of the curve.

Exercise 4 / 7 pts

We consider the function $f(z) = \frac{z}{z-1}$.

a) Determine its domain.

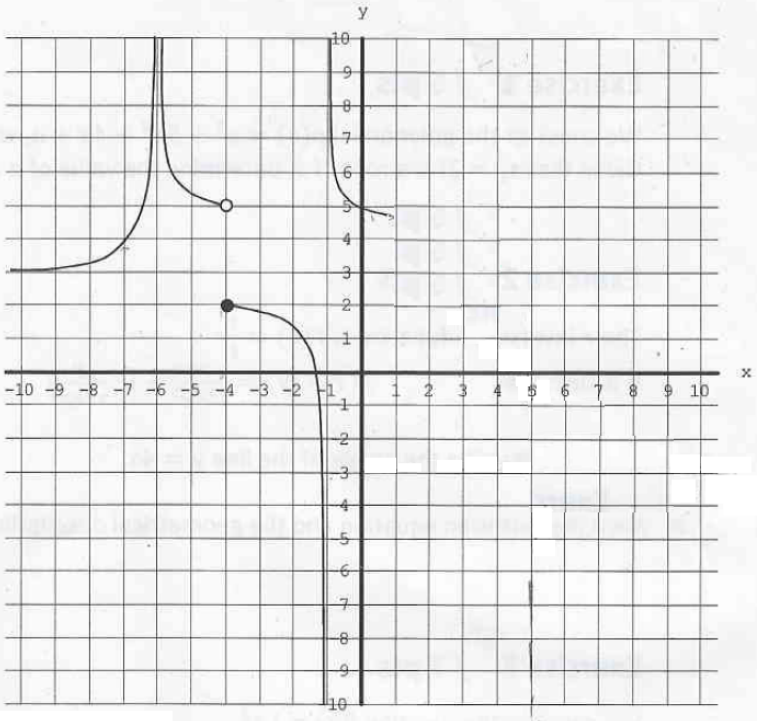
The image of $z = x + iy$ is $f(z) = u + iv = \frac{x^2-x+y^2}{(x-1)^2+y^2} + i\frac{-y}{(x-1)^2+y^2}$

b) Describe the set $A = \{z \in \mathbb{C} | Re(f(z)) = 1\}$, by giving its equation and its geometrical description.

c) Describe the set $B = \left\{z \in \mathbb{C} \middle| 2 \cdot \underbrace{Re(f(z))}_{u} = \underbrace{Im(f(z))}_{v}\right\}$, by giving its equation and its geometrical description.

Exercise 5 / 8 pts

The graph of the function f is partially given.



a) By observing the graph, determine

$\lim_{x\to-\infty} f(x) =$

$\lim_{x\to-4^-} f(x) =$

$\lim_{x\to-6} f(x) =$

$\lim_{x\to0} f(x) =$

$f(-4) =$

b) Continue the graph such that

$f(3) = 2$

$\lim_{x\to5} f(x) = n.d.$

$\lim_{x\to+\infty} f(x) = -3$

c) Determine the domain of that completed function

Exercise 6 / 6 pts

Determine the following limits.
The result should not be obtained with the calculator but justified.

a) $\lim_{x\to-\infty} 7x^3 - 4x^2 =$

b) $\lim_{x\to+\infty} \frac{(6x-2)^2}{2x^2-8} =$

c) $\lim_{x\to2} \frac{(6x-2)^2}{2x^2-8} =$

Exercise 7 / 10 pts

a) Give the domain of the following functions and identify, with justification, the holes, vertical asymptotes and horizontal asymptotes : $f(x) = \frac{3(x-2)(x+1)}{(x-2)(x-5)}$ and $g(x) = \frac{3x-6}{(x-2)^2}$

b) Give a function whose graph is even and that has the period $T = 3\pi$. What's the range of that function ?

c) Give a function whose domain is $D =]-2;+\infty[\setminus \{3\}$