ANALYSE

EXERCICE 1

$f(x) = \frac{1}{x^2}$

a) $f'(x) = -\frac{1}{x^4} \cdot 2x = -\frac{2}{x^3}$

b) $f^s(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$

$f(x+h) = \frac{1}{(x+h)^2}$ $\quad f(x-h) = \frac{1}{(x-h)^2}$

$f(x+h) - f(x-h) = \frac{1}{(x+h)^2} - \frac{1}{(x+h)^2} = \frac{(x-h)^2 - (x+h)^2}{(x+h)^2(x-h)^2} =$

$= \frac{x^2 - 2xh + h^2 - x^2 - 2xh - h^2}{(x+h)^2(x-h)^2} = \frac{-4xh}{(x+h)^2(x-h)^2}$

$\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h} = \lim_{h \to 0} \frac{-4x\cancel{h}}{(x+h)^2(x-h)^2 \cdot 2\cancel{h}} = \frac{-4x}{2x^2 \cdot x^2} =$

$= -\frac{2}{x^3} = f'(x)$

c) $f'(2) = -\frac{2}{8} = -\frac{1}{4} = m$ $\quad f(2) = \frac{1}{4}$

$y = mx + h \quad \rightarrow \frac{1}{4} = -\frac{1}{4} \cdot 2 + h \Rightarrow h = \frac{1}{4} + \frac{2}{4} \Rightarrow h = \frac{3}{4}$

$t: \ y = -\frac{1}{4}x + \frac{3}{4}$

EXERCICE 2

$f(x) = -x + \sqrt{x^2+1}$ a) $x^2 + 1 \geq 0 \quad \forall x \in \mathbb{R} \Rightarrow D_f = \mathbb{R}$

b) AV $\emptyset$

AH/AO: $+\infty$ $\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{-x + \sqrt{x^2+1}}{x} =$

$= \lim_{x \to +\infty} \frac{x^2 + 1 - x^2}{x(\sqrt{x^2+1} + x)} = 0 = m$

$h = \lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{(\sqrt{x^2+1} - x)(\sqrt{x^2+1} + x)}{(\sqrt{x^2+1} + x)} = \lim_{x \to +\infty} \frac{\cancel{x^2} + 1 - \cancel{x^2}}{\sqrt{x^2+1} + x} = 0$

$\Rightarrow \underline{y = 0 \quad (+\infty) \text{ AH}}$

$-\infty$ $\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{\cancel{x}\left(-1 - \sqrt{1 + \frac{1}{x^2}}\right)}{\cancel{x}} = -2 = m$

$h = \lim_{x \to -\infty} (f(x) + 2x) = \lim_{x \to -\infty} (x + \sqrt{x^2+1}) = \lim_{x \to -\infty} \frac{\cancel{x^2} - \cancel{x^2} - 1}{x - \sqrt{x^2+1}} = 0$

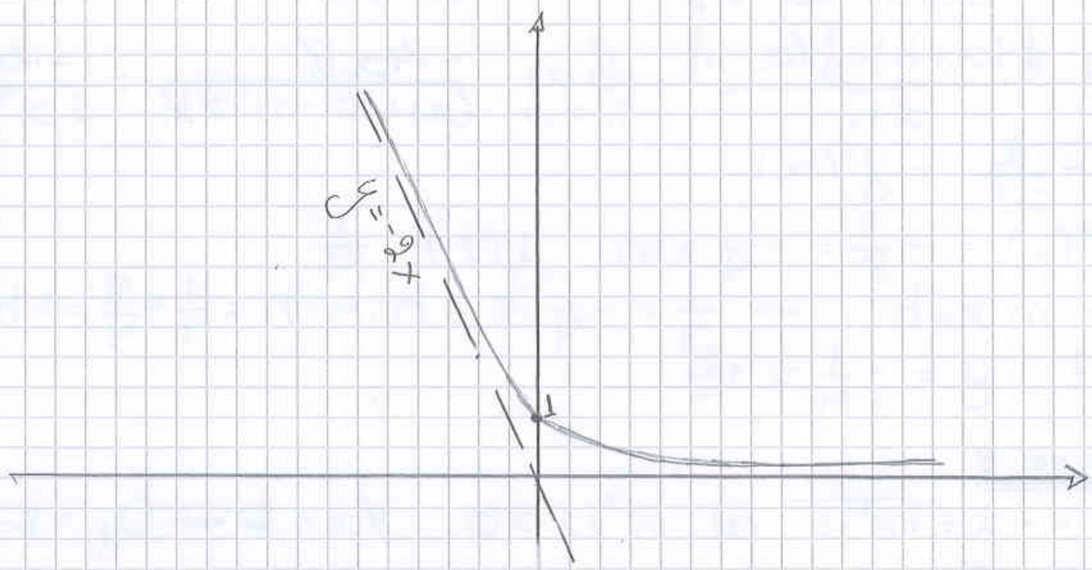
$\Rightarrow \underline{y = -2x \quad \text{AO en } x \to -\infty}$

c) $f'(x) = -1 + \dfrac{2x}{2\sqrt{x^2+1}} = -1 + \dfrac{x}{\sqrt{x^2+1}}$

$f'(x) = 0 \Leftrightarrow \dfrac{x}{\sqrt{x^2+1}} = 1 \Leftrightarrow x = \sqrt{x^2+1} \overset{x \geq 0}{\Leftrightarrow} x^2 = x^2 + 1 \Leftrightarrow$
$1 = 0$ Impossible

$\Rightarrow$ Il n'y a pas de PTH

d) Ox : $y = 0 \Leftrightarrow -x + \sqrt{x^2+1} = 0 \Leftrightarrow \sqrt{x^2+1} = x \overset{x \geq 0}{\Leftrightarrow}$
$x^2 + 1 = x^2 \Rightarrow 1 = 0$ impossible
$\Rightarrow$ le graphe de f ne coupe pas l'axe Ox
Oy : $x = 0$ $f(0) = 1$ $I_y(0;1)$



EXERCICE 3

a) $\lim\limits_{x \to +\infty} \dfrac{(x-1)(x-4)}{(2x-4)^2} = \lim\limits_{x \to +\infty} \dfrac{x^2}{4x^2} = \dfrac{1}{4}$

b) $\lim\limits_{x \to 0} \dfrac{1-\cos^2 x}{x \sin 2x} = \lim\limits_{x \to 0} \dfrac{\sin^2 x}{x \cdot 4 \sin^2 x \cos^2 x} = \lim\limits_{x \to 0} \dfrac{1}{4x\cos^2 x}$

$= \begin{cases} +\infty & x > 0 \\ -\infty & x < 0 \end{cases}$ $\Rightarrow$ la limite n'existe pas

c) $\lim\limits_{x \to 0} \dfrac{x^2}{\sqrt{x^2+4} - 2} = \lim\limits_{x \to 0} \dfrac{x^2(\sqrt{x^2+4}+2)}{x^2 + \cancel{4} - \cancel{4}} = 4$

EXERCICE 4

$$f(x) = \frac{ax^2+bx+c}{x+d}$$

- le graphe passe par $A(2;-2)$.

$f(2) = -2 \Leftrightarrow \frac{4a+2b+c}{2+d} = -2 \Leftrightarrow 4a+2b+c = -4-2d \Leftrightarrow$

$\Leftrightarrow 4a+2b+c+2d = -4 \quad (1)$

- AV $x=-3$: $-3+d=0 \Leftrightarrow \underline{d=3}$
- AO : $y=-2x+1$

$$\begin{array}{l|l} ax^2+bx+c & x+3 \\ \cline{2-2} -ax^2-3ax & ax+b-3a \\ \hline (b-3a)x+c & \\ -(b-3a)x-3b+9a & \\ \hline c-3b+9a & \end{array}$$

AO : $\left.\begin{array}{l} y = ax+b-3a \\ y = -2x+1 \end{array}\right\} \Rightarrow$

$\Rightarrow \underline{-2=a}$

$b-3a=1 \Rightarrow b=1-6$

$\Rightarrow \underline{b=-5}$

On remplace en (1) : $-8-10+c+6=-4 \Leftrightarrow$

$\Leftrightarrow \underline{c=8}$

$$f(x) = \frac{-2x^2-5x+8}{x+3}$$

EXERCICE 5

$$f(x) = \begin{cases} x^2+a & x \leq 9 \\ \frac{\sqrt{x}-3}{x-9} & x > 9 \end{cases}$$

- $\lim_{x\to 9^-} f(x) = f(9) = 81+a$
- $\lim_{x\to 9^+} f(x) = \lim_{x\to 9^+} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9^+} \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} = \frac{1}{6}$

Alors $81+a = \frac{1}{6} \Rightarrow a = \frac{1}{6} - 81 = -\underline{\frac{485}{6} = a}$