

Exercice 1

a) $y = (x^3 - 7) \cdot \sin^2(x)$, $y' = 3x^2 \cdot \sin^2(x) + (x^3 - 7) \cdot 2\sin(x)\cos(x)$

b) $y = \frac{(1 + \sin(x))^2}{\cos(x)}$, $y' = \frac{2\cos^2(x) \cdot (1 + \sin(x)) + (1 + \sin(x))^2 \cdot \sin(x)}{\cos^2(x)}$

c) $y = \tan(3 + \sqrt{x})$, $y' = \frac{1}{\cos^2(3 + \sqrt{x})} \cdot \frac{1}{2\sqrt{x}} = (1 + \tan^2(3 + \sqrt{x})) \cdot \frac{1}{2\sqrt{x}}$

Exercice 2

$$f(x) = 2x + \frac{4}{(2x+5)^2} = 2x + 4 \cdot (2x+5)^{-2}$$

$$f'(x) = 2 - 16 \cdot (2x+5)^{-3} = 2 - \frac{16}{(2x+5)^3}$$

$$f'(x) = 0 \Leftrightarrow (2x+5)^3 = 8 \Leftrightarrow 2x+5 = 2 \Leftrightarrow x = -\frac{3}{2} \text{ d'où } H\left(-\frac{3}{2}; -2\right)$$

Exercice 3

$$f(x) = \frac{ax^2 + b}{3x+5}, x = -2, t: x + y - 1 = 0 \Leftrightarrow t: y = -x + 1, \text{ la pente vaut } -1$$

a) $y' = \frac{6ax^2 + 10ax - 3ax^2 - 3b}{(3x+5)^2} = \frac{3ax^2 + 10ax - 3b}{(3x+5)^2}$

b) $x = -2$ dans t donne $y = 3$, d'où $T(-2; 3)$

$$f(-2) = 3 \Rightarrow 4a + b = -3 \text{ et } f'(-2) = -1 \Rightarrow -8a - 3b = -1$$

On a donc un système de 2 équations à 2 inconnues : $\begin{cases} 4a + b = -3 \\ -8a - 3b = -1 \end{cases}$

dont la solution est $a = -\frac{5}{2}$ et $b = 7$.

Exercice 4

$$f(x) = \frac{2x+6}{x+2}, A(1; 0), f'(x) = \frac{-2}{(x+2)^2}$$

$$\frac{y}{x-1} = y' \Leftrightarrow \frac{2x+6}{x+2} \cdot \frac{1}{x-1} = \frac{-2}{(x+2)^2} \Leftrightarrow \frac{2x+6}{x-1} = \frac{-2}{x+2}$$

$$2x^2 + 10x + 12 = -2x + 2 \Leftrightarrow 2x^2 + 12x + 10 = 0 \Leftrightarrow x^2 + 6x + 5 = 0$$

$$\Leftrightarrow (x+1) \cdot (x+5) = 0 \text{ et les points cherchés sont } P_1(-1; 4) \text{ et } P_2\left(-5; \frac{4}{3}\right)$$