ANALYSE 2 — TE 22.10.2015

EXERCICE 1

$f(x) = \sqrt{4x+1}$

$f(x+\Delta x) = \sqrt{4(x+\Delta x)+1} = \sqrt{4x+4\Delta x+1}$

$f(x+\Delta x) - f(x) = \sqrt{4x+4\Delta x+1} - \sqrt{4x+1} = \dfrac{\cancel{4x}+4\Delta x+\cancel{1}-\cancel{4x}-\cancel{1}}{\sqrt{4x+4\Delta x+1}+\sqrt{4x+1}} =$

$= \dfrac{4\Delta x}{\sqrt{4x+4\Delta x+1}+\sqrt{4x+1}}$

$\lim\limits_{\Delta x\to 0} \dfrac{f(x+\Delta x)-f(x)}{\Delta x} = \lim\limits_{\Delta x\to 0} \dfrac{4\cancel{\Delta x}}{\left(\sqrt{4x+4\Delta x+1}+\sqrt{4x+1}\right)\cancel{\Delta x}} = \dfrac{4}{2\sqrt{4x+1}}$

$\Rightarrow f'(x) = \dfrac{2}{\sqrt{4x+1}}$

EXERCICE 2

$$f(x) = \begin{cases} \dfrac{3-\sqrt{3-2x}}{x+3} & x < -3 \\ ax & -3 \leq x \leq 2 \\ \dfrac{x^3-8}{x^2-4} + b & x > 2 \end{cases}$$

• $\underline{x=-3}$ $\lim\limits_{x\to -3^-} f(x) = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to -3^-} \dfrac{9-(3-2x)}{(x+3)(3+\sqrt{3-2x})} =$

$= \lim\limits_{x\to -3^-} \dfrac{6+2x}{(x+3)(3+\sqrt{3-2x})} = \lim\limits_{x\to -3^-} \dfrac{2(\cancel{x+3})}{(\cancel{x+3})(3+\sqrt{3-2x})} = \dfrac{2}{6} = \dfrac{1}{3}$

$\lim\limits_{x\to -3^+} f(x) = -3a$

Il faut que $-3a = \dfrac{1}{3} \Leftrightarrow \boxed{a = -\dfrac{1}{9}}$

• $\underline{x=2}$ $\lim\limits_{x\to 2^-} f(x) = 2a = -\dfrac{2}{9}$

$\lim\limits_{x\to 2^+} f(x) = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 2^+} \left[\dfrac{(\cancel{x-2})(x^2+2x+4)}{(\cancel{x-2})(x+2)} + b\right] =$

$= \dfrac{12}{4} + b \to \lim\limits_{x\to 2^+} f(x) = 3+b$

Il faut que : $-\dfrac{2}{9} = 3+b \Leftrightarrow b = -\dfrac{2}{9} - \dfrac{27}{9} \Leftrightarrow$

$\boxed{b = -\dfrac{29}{9}}$

EXERCICE 3

$f(x) = \sin^2 x - \cos^2 x$ $\qquad$ $A\left(\frac{\pi}{6} ; y\right)$

$f'(x) = 2\sin x\cos x + 2\cos x \sin x = 4\cos x \sin x$

$f'\left(\frac{\pi}{6}\right) = 4\cos\frac{\pi}{6}\sin\frac{\pi}{6} = \cancel{4}\cdot\frac{\sqrt{3}}{\cancel{2}}\cdot\frac{1}{\cancel{2}} = \sqrt{3}$

vecteur directeur: $\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$

tangente : $f\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$

$y = mx + b$

$-\frac{1}{2} = \sqrt{3}\cdot\frac{\pi}{6} + b \Leftrightarrow b = -\frac{1}{2} - \frac{\sqrt{3}\pi}{6}$

$t : \underline{y = \sqrt{3}x - \frac{1}{2} - \frac{\sqrt{3}\pi}{6}}$

EXERCICE 4

a) $[(2x^4+3)\cdot(3x^2-x)]' = (6x^6+9x^2-2x^5-3x)' =$
$= 36x^5 - 10x^4 + 18x - 3$

ou $[(2x^4+3)(3x^2-x)]' =$

$8x^3\cdot(3x^2-x) + (2x^4+3)(6x-1) =$
$= 24x^5 - 8x^4 + 12x^5 + 18x - 2x^4 - 3 =$
$= 36x^5 - 10x^4 + 18x - 3.$

b) $\left(\frac{x\sqrt{x^3}}{\sqrt[3]{x}\cdot x^2}\right)' = \left(\frac{x\cdot x^{3/2}}{x^{1/3}\cdot x^2}\right)' = \left(\frac{x^{5/2}}{x^{7/3}}\right)' = \left(x^{\frac{5}{2}-\frac{7}{3}}\right)' =$

$\left(x^{1/6}\right)' = \frac{1}{3}x^{-5/6} = \frac{1}{3\sqrt[6]{x^5}}$

c) $\left(\frac{\tan x}{\sin x\cos x}\right)' = \left(\frac{\cancel{\sin x}}{\cancel{\sin x}\cdot\cos^2 x}\right)' = \left(\frac{1}{\cos^2 x}\right)' = -\frac{2\cancel{\cos x}\sin x}{\cos^{\cancel{4}} x}$

$= \frac{-2\sin x}{\cos^3 x}$

d) $\left(\frac{5}{4x^2-3x}\right)' = \frac{-5(8x-3)}{(4x^2-3x)^2} = \frac{-5(8x-3)}{(4x^2-3x)^2}$