

EXERCICE 1

$$(2-3i)^{10}$$

$$\binom{10}{5} 2^{10-5} (-3i)^5 = \frac{10!}{5!5!} \cdot 2^5 \cdot (-3)^5 \cdot i^5 = 252 \cdot 3^5 \cdot (-i) = -1959552i$$

EXERCICE 2

$$1. \frac{1}{1 \cdot 3} + \frac{2^2}{3 \cdot 5} + \dots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}$$

$$\cdot n=1 \quad \frac{1}{1 \cdot 3} = \frac{1 \cdot 1}{2 \cdot (3)} = \frac{1}{3} \quad \checkmark$$

$$\cdot n=k \text{ (Hyp)} \quad \sum_{i=1}^k \frac{i^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{2(2n+1)}$$

$$\cdot n=k+1 \text{ A voir } \sum_{i=1}^{n+1} \frac{i^2}{(2i-1)(2i+1)} = \frac{(n+1)(n+2)}{2(2n+3)}$$

$$\begin{aligned} \sum_{i=1}^{n+1} \frac{i^2}{(2i-1)(2i+1)} &= \frac{n(n+1)}{2(2n+1)} + \frac{(n+1)^2}{(2n+1)(2n+3)} = \\ &= \frac{n(n+1)(2n+3) + 2(n+1)^2}{2(2n+1)(2n+3)} = \frac{(n+1)(2n^2+3n+2n+2)}{2(2n+1)(2n+3)} = \\ &= \frac{(n+1)(2n^2+5n+2)}{2(2n+1)(2n+3)} = \Delta = 25-16=9 \\ &= \frac{2(n+1)(n+2)(n+\frac{1}{2})}{2(2n+1)(2n+3)} = n = \frac{-5 \pm 3}{2} = \begin{matrix} -2 \\ -\frac{1}{2} \end{matrix} \\ &= \frac{(n+1)(n+2)(\cancel{2n+1})}{2(\cancel{2n+1})(2n+3)} = \frac{(n+1)(n+2)}{2(2n+3)} \quad \checkmark \end{aligned}$$

$$2) \quad \sum_{i=1}^{\infty} \frac{i^2}{(2i-1)(2i+1)} = \lim_{n \rightarrow +\infty} \frac{n(n+1)}{2(2n+1)} = \lim_{n \rightarrow +\infty} \frac{n^2+n}{2n+2} = +\infty$$

### EXERCICE 3

a)  $f(x) = \frac{1}{x+3}$

$$f'(x) = -\frac{1}{(x+3)^2}$$

$$f''(x) = \frac{2}{(x+3)^3}$$

$$f'''(x) = -\frac{6}{(x+3)^4}$$

$$f^{(4)}(x) = \frac{24}{(x+3)^5}$$

b)  $f^{(n)}(x) = \frac{(-1)^n \cdot n!}{(x+3)^{n+1}}$

$n=1$ .  $f'(x) = \frac{-1}{(x+3)^2}$  ✓

$n=k$   $f^{(k)}(x) = \frac{(-1)^k k!}{(x+3)^{k+1}}$  "A voir"

$n=k+1$  "A voir"  $f^{(k+1)}(x) = \frac{(-1)^{k+1} (k+1)!}{(x+3)^{k+2}}$

$$\begin{aligned} f^{(k+1)}(x) &= \left( f^{(k)}(x) \right)' = \left( \frac{(-1)^k \cdot k!}{(x+3)^{k+1}} \right)' = \left( (-1)^k \cdot k! (x+3)^{-(k+1)} \right)' \\ &= -(-1)^k \cdot k! (k+1) (x+3)^{-k-1-1} = \frac{(-1)^{k+1} \cdot (k+1)!}{(x+3)^{k+2}} \quad \checkmark \end{aligned}$$

### EXERCICE 4

111 98

a)  $A_9^3 = 9 \cdot 8 \cdot 7 = 504$

b)  $A_{11}^2 = 11 \cdot 10 = 110$

c)  $A_9^3 \cdot A_{11}^2 = 504 \cdot 110 = 55440$

d)  $A_9^1 \cdot A_{11}^2 = 9 \cdot 110 = 990$



EXERCICE 5

Sc 3p

a)  $3^5$  car pour chaque colonne on a 3  
 $= 243$  possibilités

b)  $3^5 \cdot 5! = 29'160$

c)  $C_5^{15} = \binom{15}{5} = 3003$

d)  $A_5^{15} = \frac{15!}{10!} = 360360$