ANALYSE 1. (solutions)

EXERCICE 3

a) $\lim\limits_{x\to 0}\left(\frac{x^3+3x}{x^2-1}\right)=\frac{0}{-1}=0$

b) $\lim\limits_{x\to 2}\left(\frac{(x-5)^2}{x^2-4x+4}\right)=\lim\limits_{x\to 2}\frac{(x-5)^2}{(x-2)^2}=\left[\frac{9}{0^+}\right]=+\infty$

c) $\lim\limits_{x\to -\infty}\left(\frac{-x+x^3+4}{x^2+2x+1}\right)=\lim\limits_{x\to -\infty}\frac{x^3-x+4}{(x+1)^2}=\lim\limits_{x\to -\infty}\frac{x^{\cancel{3}}}{\cancel{x^2}}=-\infty$

d) $\lim\limits_{x\to -1}\left(\frac{x^3-2x^2-x+2}{x^2-x-2}\right)=\lim\limits_{x\to -1}\frac{\cancel{(x+1)}(x^2-3x+2)}{(x-2)\cancel{(x+1)}}=\frac{6}{-3}=-2$

$$\begin{array}{cccc|c} 1 & -2 & -1 & 2 & -1 \\ \downarrow & -1 & 3 & -2 & \\ 1 & -3 & 2 & 0 & \end{array}$$

e) $\lim\limits_{x\to +\infty}(\underset{+\infty\;-\;(+\infty)}{2x-\sqrt{4x^2-3}})=\lim\limits_{x\to +\infty}\frac{4x^2-(4x^2-3)}{2x+\sqrt{4x^2-3}}=\lim\limits_{x\to +\infty}\frac{3}{2x+\sqrt{4x^2-3}}=0$

f) $\lim\limits_{x\to 0}\frac{3x^2-x}{|x|}$ $\quad f(x)=\begin{cases}\frac{3x^2-x}{x}=\frac{\cancel{x}(3x-1)}{\cancel{x}}=3x-1 & x>0\\ \frac{3x^2-x}{-x}=\frac{\cancel{x}(3x-1)}{-\cancel{x}}=-3x+1 & x<0\end{cases}$

Donc $\lim\limits_{x\to 0^-}f(x)=1$
$\lim\limits_{x\to 0^+}f(x)=-1$ et la limite n'existe pas

g) $\lim\limits_{x\to 4}\left(\frac{x^3-8x^2+16x}{x^2-16}\right)=\lim\limits_{x\to 4}\frac{x(x^2-8x+16)}{(x-4)(x+4)}=$

$=\lim\limits_{x\to 4}\frac{x(x-4)^{\cancel{2}}}{\cancel{(x-4)}(x+4)}=0$

h) $\lim\limits_{x\to +\infty}\frac{(x-1)^2}{x^2+x}=\lim\limits_{x\to +\infty}\frac{\cancel{x^2}}{\cancel{x^2}}=1$

i) $\lim\limits_{x\to -\infty}\left(\frac{-x+x^3}{x^3+2x^4+1}\right)=\lim\limits_{x\to -\infty}\frac{\cancel{x^3}}{2x^{\cancel{4}}}=0$

j) $\lim\limits_{x\to 0}\left(\frac{1-\cos x}{3x^2}\right)=\lim\limits_{x\to 0}\frac{1-\cos^2 x}{3x^2(1+\cos x)}=\lim\limits_{x\to 0}\frac{\sin^2 x}{3x^2(1+\cos x)}$

$=\frac{1}{3}\lim\limits_{x\to 0}\underbrace{\left(\frac{\sin x}{x}\right)^2}_{1}\frac{1}{1+\cos x}=\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{6}$

k) $\lim\limits_{x\to 2}\left(\dfrac{\sqrt{3x-2}-2}{4-\sqrt{3x+10}}\right)=\lim\limits_{x\to 2}\dfrac{(\sqrt{3x-2}-2)(\sqrt{3x-2}+2)(4+\sqrt{3x+10})}{(4-\sqrt{3x+60})(4+\sqrt{3x+10})(\sqrt{3x-2}+2)}=$

$=\lim\limits_{x\to 2}\dfrac{(3x-2-4)(4+\sqrt{3x+10})}{(16-(3x+10))(\sqrt{3x-2}+2)}=\lim\limits_{x\to 2}\dfrac{\cancel{3}\cancel{(x-2)}(4+\sqrt{3x+10})}{-\cancel{3}\cancel{(x-2)}(\sqrt{3x-2}+2)}=$

$=-\dfrac{4+4}{2+2}=-\dfrac{8}{4}=-2$

l) $\lim\limits_{x\to -2}\left(\dfrac{x^2-x-6}{\sin(x+2)}\right)=\lim\limits_{x\to 2}\dfrac{(x+2)(x-3)}{\sin(x+2)}=\lim\limits_{x\to 2}\dfrac{(x-3)}{\dfrac{\sin(\cancel{x+2})}{\cancel{x+2}}}=-1$

m) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin^2 x}=\lim\limits_{x\to 0}\dfrac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}}\cdot\dfrac{1}{1+\cos x}=\dfrac{1}{2}$