

LDPR - Niveau 2 TE 6 : Ensembles - Calcul Algébrique

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EXERCICE 1

- 1) " $a^2 < \sqrt{b}$, a au carré" est plus petit que la racine de b
- 2) $A = [-2; \infty[$ $B = [-4; 5[$
 - a) l'intervalle de -4 compris, à 5 pas compris
(fermé) (ouvert)
 - b) $A \cup B = [-4; +\infty[$
 - c) $\bar{A} =]-\infty, 2[$ $\bar{A} \cap B = [-4; 2[$
- 3) $\{x \in \mathbb{R} \mid x^2 > 5\} =]-\infty, -\sqrt{5}[\cup]\sqrt{5}, +\infty[$
- 4) $\{x \in \mathbb{R} \mid -4 < |x| \leq 9\} = [-9; 9]$

EXERCICE 2

- 1) a) $64x^2 - 25 = 0 \Leftrightarrow x^2 = \frac{25}{64} \Leftrightarrow x = \pm \frac{5}{8}$
b) $x^5 - 4x^3 = 0 \Leftrightarrow x^3(x^2 - 4) = 0 \Leftrightarrow x = 0 \text{ ou } x = \pm 2$
- 2) a) $x^2 + 14x - 50 = 0 \Rightarrow (x+7)^2 - 49 - 50 = 0 =$
 $= (x+7)^2 - 99$
b) $2x^2 - 12x + 220 = 2(x^2 - 6x + 110) =$
 $= 2[(x-3)^2 - 9 + 110] = 2[(x-3)^2 + 101]$
- 3) $9x^4 - 6x^3 + x^2 = (3x^2 + x)^2$
- 4) $(x^2+4)(x-2) - 20(x-2) = (x-2)(x^2+4-20) =$
 $= (x-2)(x^2-16) = (x-2)(x-4)(x+4)$

EXERCICE 3

- 1) $\frac{2}{5} - \left(\frac{7}{10}\right)^2 : \frac{35}{4} = \frac{2}{5} - \frac{7^2}{10^2} \cdot \frac{4}{355} = \frac{2}{5} - \frac{1}{125} = \frac{50-1}{125} = \frac{49}{125}$
- 2) $(e^2 f^{-4})^3 : (\sqrt[3]{e^{-6}} f^2)^2 = \frac{e^6}{f^{12}} : (e^{-12/3} f^4) = \frac{e^6}{f^{12}} \cdot \frac{e^4}{f^4} = \frac{e^{10}}{f^{16}}$
- 3) $\sqrt[4]{c^5} \cdot \frac{1}{c^2} = c^{\frac{5}{4}-2} = c^{-\frac{3}{4}}$
- 4) $(25)^{1/2} : 2^{-2} = \frac{(5^2)^{1/2}}{2^2} = \frac{5}{4}$
- 5) $\sqrt{18} - \sqrt{50} = 3\sqrt{2} - 5\sqrt{2} = -2\sqrt{2}$
- 6) $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$
- 7) $\frac{3\sqrt{5}}{2-\sqrt{5}} = \frac{3\sqrt{5}(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})} = \frac{3\sqrt{5}(2+\sqrt{5})}{4-5} = -(6\sqrt{5}+15)$
- 8) $(3+\sqrt{11})(9-2\sqrt{11}) = 27+9\sqrt{11}-6\sqrt{11}-2 \cdot 11 = 5+3\sqrt{11}$

$$9) \frac{2m}{m^2-1} - \frac{2}{m+1} = \frac{2m-2(m+1)}{(m-1)(m+1)} = \frac{2m-2m-2}{(m-1)(m+1)} = \frac{-2}{(m-1)(m+1)}$$

$$10) \frac{\sqrt[3]{x}}{x^2} = \frac{(x^{1/3})^{1/2}}{x^2} = \frac{x^{1/6}}{x^2} = x^{1/6-2} = x^{-11/6} = \frac{1}{\sqrt[6]{x^{11}}}$$

EXERCISE 4

$$1) \frac{2x^2+5x-12}{4(x+4)(x-3)} = \frac{2(x+4)(x-\frac{3}{2})}{4(x+4)(x-3)} = \frac{(x+4)(2x-3)}{4(x+4)(x-3)} = \frac{2x-3}{4(x-3)}$$

$$2) -x^2+2kx-k^2=x \Leftrightarrow -x^2+(2k+1)x-k^2=0$$

$$\Delta=0 \Leftrightarrow (2k+1)^2-4 \cdot k^2=0 \Leftrightarrow$$

$$\Leftrightarrow 4k^2+4k+1-4k^2=0 \Leftrightarrow k=-\frac{1}{4}$$

$$3) ax^2-6x+10=0 \quad \Delta < 0 \Leftrightarrow 36-40a < 0 \Leftrightarrow a > \frac{9}{10}$$

$$4) x^2-mx+9=0 : (-2)^2+m \cdot 2+9=0 \Leftrightarrow$$

$$\Leftrightarrow 4+2m+9=0 \Leftrightarrow m=-\frac{13}{2}$$