LPDR - Niveau 2 - TE 10 : Polynômes - Equations

EXERCICE 1

a) $\frac{x}{2} + \frac{6}{x} = \frac{x}{3} + 2 \overset{\cdot 6x}{\Leftrightarrow} 3x^2 + 36 = 2x^2 + 12x \Leftrightarrow$ $\boxed{x \neq 0}$

$\Leftrightarrow x^2 - 12x + 36 = 0 \quad \overset{\Delta = 0}{\Leftrightarrow} (x-6)^2 = 0 \Leftrightarrow x = 6$ ✓

b) $\frac{2t+m}{t+m} = m + 1 \overset{\cdot(t+m)}{\Leftrightarrow} 2t + m = (m+1)(t+m) \Leftrightarrow$

$\Leftrightarrow 2t + \cancel{m} = tm + m^2 + t + \cancel{m} \Leftrightarrow$

$\Leftrightarrow 2t - tm - t = m^2 \Leftrightarrow (2 - m - 1)t = m^2 \Leftrightarrow (1-m)t = m^2$

· Si $m = 1$ on a: $0 = 1$ imp.

· Si $m \neq 1$ $t = \frac{m^2}{1-m}$

c) $y^5 - 6y^3 = 27y \Leftrightarrow y^5 - 6y^3 - 27y = 0 \Leftrightarrow$

$\Leftrightarrow y(y^4 - 6y^2 - 27) = 0$ $\quad y = 0$

$y^4 - 6y^2 - 27 = 0$

We put: $y^2 = u \geq 0$ $\quad u^2 - 6u - 27 = 0$

$\Delta = 36 + 108 \Leftrightarrow \Delta = 144 = 12^2$

$u_{1,2} = \frac{6 \pm 12}{2} = \begin{cases} u_1 = 9 = y^2 \Leftrightarrow y = \pm 3 \\ u_2 = -3 < 0 \times \end{cases}$

We have 3 roots: $y = 3$, $y = -3$, $y = 0$

d) $\sqrt{3x-2} - \sqrt{x} = 2 \Leftrightarrow$

$\Leftrightarrow \sqrt{3x-2} = 2 + \sqrt{x} \overset{(\ )^2}{\Leftrightarrow}$

$3x - 2 = 4 + 4\sqrt{x} + x \Leftrightarrow$

$2x - 6 = 4\sqrt{x} \Leftrightarrow x - 3 = 2\sqrt{x}$

$\overset{(\ )^2}{\Leftrightarrow} x^2 - 6x + 9 = 4x \Leftrightarrow$

$x^2 - 10x + 9 = 0$

$\Delta = 100 - 36 = 64 = 8^2$

$x_{1,2} = \frac{10 \pm 8}{2} = \begin{cases} 9 & ✓ \\ 1 & \times \end{cases}$ $\quad \boxed{x = 9}$

$\left.\begin{array}{l} 3x - 2 \geq 0 \Leftrightarrow x \geq \frac{2}{3} \\ x \geq 0 \end{array}\right\} \Rightarrow$

$\Rightarrow \left.\begin{array}{l} x \geq 2/3 \\ x \geq 3 \end{array}\right\} \Rightarrow$

$x \geq 3$

Exercice 2

$$4x^2-6x-1=0 \overset{:4}{\Leftrightarrow} x^2-\frac{3}{2}x=\frac{1}{4} \Leftrightarrow$$

$$\Leftrightarrow x^2-\frac{3}{2}x+\left(\frac{3}{4}\right)^2=\left(\frac{3}{4}\right)^2+\frac{1}{4} \Leftrightarrow$$

$$\Leftrightarrow \left(x-\frac{3}{4}\right)^2=\frac{9}{16}+\frac{4}{16}=\frac{13}{16} \Leftrightarrow$$

$$\left(x-\frac{3}{4}\right)^2=\left(\frac{\sqrt{13}}{4}\right)^2 \Leftrightarrow x-\frac{3}{4}=\frac{\sqrt{13}}{4} \Rightarrow \underline{x=\frac{3-\sqrt{13}}{4}}$$

$$x-\frac{3}{4}=-\frac{\sqrt{13}}{4} \Rightarrow \underline{x=\frac{3-\sqrt{13}}{4}}$$

Exercice 3

We should have $\Delta=0$.

$$\Delta=(6-2m)^2-4m\cdot 4m=0 \Leftrightarrow$$

$$\Leftrightarrow 36-24m+4m^2-16m^2=0 \Leftrightarrow$$

$$\Leftrightarrow 12m^2+24m-36=0 \Leftrightarrow m^2+2m-3=0 \Leftrightarrow$$

$$\Delta_1=4+12=16 \quad m_{1,2}=\frac{-2\pm 4}{2}= \begin{cases} \underline{-3=m_1} \\ \underline{1=m_2} \end{cases}$$

Exercice 4

$$P(x)=4x^3-8x^2-11x-3$$

r.p. $\pm 1, \pm 3$ $\quad P(1)=-18\neq 0 \quad P(-1)=-4\neq 0$

$P(3)=108-72-33-3=0 \quad \underline{x=3}$

$P(x)$ is divisible by $x-3$: Horner

4	−8	−11	−3	3
↓	12	12	3	
4	4	1	0	

$$P(x)=(x-3)(4x^2+4x+1)=(x-3)(2x+1)^2$$

So, the roots are

$$x=3 \quad x=-\tfrac{1}{2}\ (m=2)$$