1.7 Exercises

1.1 : Differentiate the following functions :

1) $f(x) = 8x^3 - 9x^2 + 11x - 5$
2) $f(x) = \sin(x) \cdot x^2$
3) $f(x) = \dfrac{1}{4x^3 - 2}$
4) $f(x) = \dfrac{5 - 7x}{9x + 3}$
5) $f(x) = \sqrt{x^2 + 2x - 5}$
6) $f(x) = \tan(5\sqrt{x})$

✐

1.2 : Determine the equations of the tangents to the given curves, at the given points :

1) $f(x) = x^3 - 2x + 7$ at the point whose abscissa is -2
2) $f(x) = \dfrac{4 - x}{x^2}$ at the point(s) whose ordinate is 3
3) $f(x) = \cos(3x)$ at the point with abscissa $\frac{\pi}{6}$

✐

1.3 : Given the piecewise function $f(x) = \begin{cases} x^2 - 6x + k & , x \leq 3 \\ \dfrac{x^2 - 4x + 3}{x - 3} & , x > 3 \end{cases}$

For which value of k is that function continuous on $\mathbb{R}$?

✐

1.4 : Given the piecewise function $f(x) = \begin{cases} \dfrac{2x + 8}{x^2 - 16} & , x < -4 \\ x^3 + ax^2 - 5x + b & , x \geq -4 \end{cases}$

For which values of a and b is the function differentiable on $\mathbb{R}$?

✐

1.5 : Determine the largest vertical distance between the graphs of the functions $f(x) = \frac{x^3}{8}$ and $g(x) = \sqrt{x}$.

1) on the interval $[0; 4]$
2) on the interval $[0; 2]$

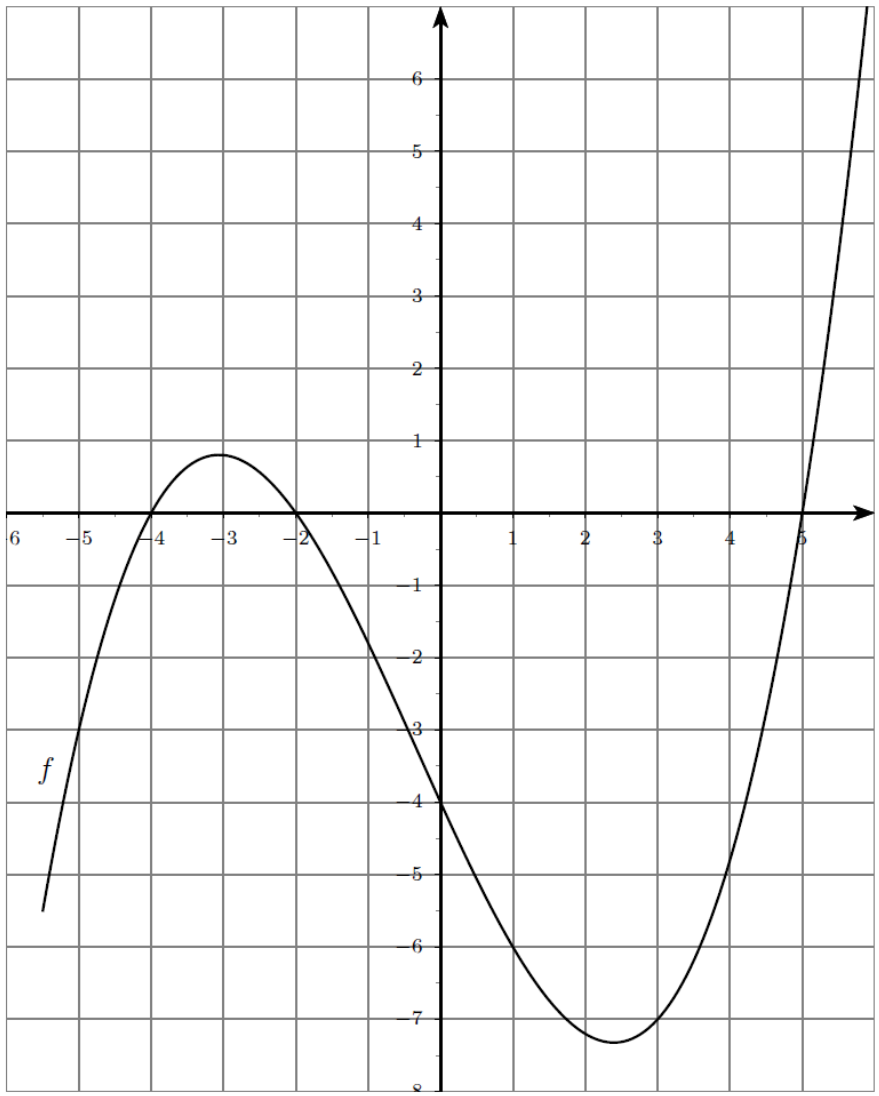
✐

1.6 : Determine the minimal distance from the curve $f(x) = \frac{2}{1+x^2}$ to the origin.

✐

1. 7 : Here is the graph of a function f.

1) Sketch the curve $y = f'(x)$ using the given graph.
2) Determine the equation of the curve f.
3) On which interval(s) is it convex ?
4) What is the « maximal » decreasing slope ?



1.8 : Answer the following :

1) Determine the equation of the tangent to the following curves at the point whose abscissa is given :

   a) $y = \ln(-x)$ at $x = -\dfrac{1}{3}$ b) $y = \ln(2x)$ at $x = \dfrac{1}{2}$

2) Find the stationary points of the following functions and indicate their type. Sketch the curves.

   a) $y = x - \ln(x)$ b) $y = \dfrac{1}{2}x^2 - \ln(2x)$ c) $y = x^2 - \ln(x^2)$

3) Determine the equation of the normal to the curve $y = \ln(2x - 3)$ at $x = 2$.

✎

1.9 : Answer the following :

1) Differentiate the functions $f(x) = \mathrm{e}^{3x}$, $g(x) = \mathrm{e}^{-x}$, $h(x) = \mathrm{e}^{3-2x}$ and $i(x) = \mathrm{e}^{\sin(2x+3)}$

2) Determine the equation of the tangent to the following curves at the given abscissa :

   a) $y = x - \mathrm{e}^{2x}$ at $x = 0$ b) $y = \mathrm{e}^{6-2x}$ at $x = 3$

3) Find all the stationary points of $y = 7x^2 - \mathrm{e}^{x^2}$ and determine their type.

✎

1.10 : Determine the acute angle between the curves f and g at their intersection point :

1) $f(x) = \mathrm{e}^{x+2}$ and $g(x) = \mathrm{e}^{-x}$

2) $f(x) = \mathrm{e}^{2x}$ and $g(x) = 2\mathrm{e}^{3x}$

✎

1.11 : Differentiate $f(x) = 3^x \cdot x^3$, $g(x) = \frac{\mathrm{e}^x - 1}{\mathrm{e}^x + 1}$, $h(x) = 2^{\sqrt{x^2+1}}$ and $i(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}$

✎

1.12 : Solve :

1) $3 \cdot 2^x > 5^x$ 2) $\dfrac{\mathrm{e}^x + \mathrm{e}^{-x}}{2} = 2$ 3) $\mathrm{e}^{2x} = 6 - \mathrm{e}^x$

✎

1.13 : Analyze, with the second derivative, the functions :

1) $f(x) = \dfrac{\ln^2(x)}{x}$

2) $f(x) = (x-1)^2 \cdot \mathrm{e}^x$

3) $f(x) = (2x^2 - 4) \cdot \mathrm{e}^{-x}$

4) $f(x) = \dfrac{1}{x-3} \cdot \mathrm{e}^{-x}$

1.14 : Find the value of c prescribed in Rolle's Theorem for $f(x) = x^3 - 12x$ on the interval $[0; b]$.

1.15 : Does Rolle's Theorem apply to the functions $f(x) = \frac{x^2-4x}{x-2}$ and $g(x) = \frac{x^2-4x}{x+2}$?

1.16 : Check the mean value theorem with $f(x) = 2x^2 - 7x + 10$, $a = 2$ and $b = 5$.

1.17 : If $f'(x) = 0$ at each point of the interval $]a; b[$, prove that f is constant in this interval.

1.18 : We consider the function $f(x) = x^3 - 6x$. Determine Δy, dy and $\Delta y - dy$. Then, find numerical values for them with $x = 1$ and $\Delta x = 0,01$.

1.19 : Use differentials to approximate $\sqrt[3]{124}$ and $\sin(60°1')$.

1.20 : Use **l'Hospital's rule**, whenever possible :

1) $\displaystyle\lim_{x\to 0} \frac{\sin(2x)}{x} =$

2) $\displaystyle\lim_{x\to 0} \frac{\mathrm{e}^x}{x^2} =$

3) $\displaystyle\lim_{x\to +\infty} \frac{x + \sin x}{x} =$

4) $\displaystyle\lim_{x\to 0} \frac{3x}{x^2+1} =$

5) $\displaystyle\lim_{x\to 0} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} =$

6) $\displaystyle\lim_{x\to 1} \frac{x^2+2x-3}{x-1} =$

7) $\displaystyle\lim_{x\to 1} \frac{x^5 - 2x^4 + x^3 + 2x^2 - 4x + 2}{x^3 - 3x + 2} =$

8) $\displaystyle\lim_{x\to 0} \frac{x}{x + \sin x} =$

9) $\displaystyle\lim_{x\to e} \frac{\ln(x) - 1}{x - \mathrm{e}} =$

10) $\displaystyle\lim_{x \overset{>}{\to} 0} \frac{\ln\left(x^2+1\right)}{x} =$