# LDDR : - Niveau 1 : TE 8 Analyse

## Exercice 1.

$f(x) = \dfrac{x^3-1}{2x^2+x-3}$

$2x^2+x-3=0$

$\Delta = 1+24 = 25 \quad x_{1,2} = \dfrac{-1\pm5}{4} = \begin{cases} 1 \\ -\frac{6}{4} = -\frac{3}{2} \end{cases}$

$f(x) = \dfrac{(x-1)(x^2+x+1)}{2(x-1)(x+\frac{3}{2})} = \dfrac{(x-1)(x^2+x+1)}{(x-1)(2x+3)}$

· $D_f = \mathbb{R} \setminus \{1 ; -\frac{3}{2}\}$

· AV : $\lim\limits_{x\to1} f(x) = \lim\limits_{x\to1} \dfrac{x^2+x+1}{2x+3} = \dfrac{3}{5}$ Trou $(1, \frac{3}{5})$

$\lim\limits_{x\to-3/2} f(x) = \dfrac{"-35/8"}{"0"}$ $\begin{cases} x\to-\frac{3}{2}^+ & +\infty \\ x\to-\frac{3}{2}^- & -\infty \end{cases}$ AV. $x = -\dfrac{3}{2}$

| $2x^2+x-3$ | + | $-3/2$ | − | $1$ | + |
|---|---|---|---|---|---|

· AH/AO.

$$\begin{array}{l|l} x^3-1 & 2x^2+x-3 \\ -x^3-\frac{1}{2}x^2+\frac{3}{2}x & \frac{1}{2}x-\frac{1}{4} \\ \hline -\frac{1}{2}x^2+\frac{3}{2}x-1 & \\ +\frac{1}{2}x^2+\frac{1}{4}x-\frac{3}{4} & \\ \hline \frac{7}{4}x-\frac{7}{4} & \end{array}$$

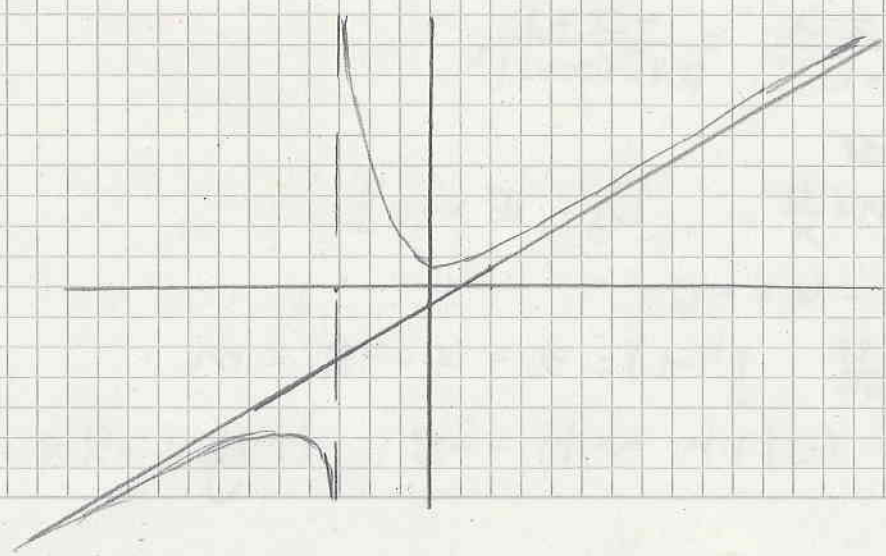
$f(x) = \frac{1}{2}x - \frac{1}{4} + \dfrac{\frac{7}{4}x-\frac{7}{4}}{2x^2+x-3}$

AO : $y = \frac{1}{2}x - \frac{1}{4}$

$x \to \pm\infty \quad f(x) \underset{<}{\overset{>}{}} \frac{1}{2}x - \frac{1}{4}$

· $\cap Ox$ : $f(x) = 0 \Leftrightarrow x^3 = 1 \Leftrightarrow x = 1$ $(1 ; 0)$ impossible

$\cap Oy$ : $f(0) = \dfrac{-1}{-3} = \dfrac{1}{3}$ $(0 ; 1/3)$



## EXERCICE 2

1) $f(x) = 3x^2$ $\quad f(x+h) = 3(x+h)^2 = 3x^2 + 6xh + 3h^2$

$f(x+h) - f(x) = \cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2} = h(6x+3h)$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\cancel{h}(6x+3h)}{\cancel{h}} = 6x = f'(x)$$

2) $f(x) = \frac{1}{x+1}$ $\quad f(x+h) = \frac{1}{x+h+1}$

$$f(x+h) - f(x) = \frac{1}{x+h+1} - \frac{1}{x+1} = \frac{\cancel{x}+\cancel{1}-\cancel{x}-h-\cancel{1}}{(x+1)(x+h+1)} = \frac{-h}{(x+1)(x+h+1)}$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{-\cancel{h}}{(x+1)(x+h+1)}}{\cancel{h}} = -\frac{1}{(x+1)^2} = f'(x)$$

## EXERCICE 3

1) $f(x) = 3x^4 - 7x + 2$ $\quad f'(x) = 12x^3 - 7$

2) $f(x) = \frac{\cos x}{\sin x}$ $\Rightarrow f'(x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

3) $f(x) = \frac{x}{\sqrt[4]{x^3}} = x \cdot x^{-3/4} = x^{1/4} \Rightarrow f'(x) = \frac{1}{4} x^{-3/4} = \frac{1}{4\sqrt[4]{x^3}}$

4) $f(x) = 2x \cdot \sqrt{3x+1}$ $\quad f'(x) = 2\sqrt{3x+1} + \frac{2x \cdot 3}{2\sqrt{3x+1}} =$

$= 2\sqrt{3x+1} + \frac{3x}{\sqrt{3x+1}}$

5) $f(x) = (2 - 5x\cos x)^2$

$f'(x) = 2(2 - 5x\cos x) \cdot (-5\cos x + 5x\sin x)$

6) $f(x) = \frac{\sqrt{x}}{x+1}$ $\quad f'(x) = \frac{\frac{1}{2\sqrt{x}}(x+1) - \sqrt{x} \cdot 1}{(x+1)^2} =$

$= \frac{x+1-2x}{2\sqrt{x}(x+1)^2} = \frac{-x+1}{2\sqrt{x}(x+1)^2}$

## EXERCICE 4

$f(x) = 2x + \frac{4}{x}$ $\quad D_f = \mathbb{R} \setminus \{0\}$

$\cdot f(-1) = -2 - 4 = -6$

$f'(x) = 2 - \frac{4}{x^2}$ $\quad f'(-1) = 2 - 4 = -2 = m$

$-6 = -2 \cdot (-1) + h \Rightarrow h = -8$ $\quad t: y = -2x - 8$