LDDR - Niveau 1 : TE 10 Analyse

EXERCICE 1

1) $\lim\limits_{x\to+\infty} \dfrac{(6x-2)^2}{2x^2-8} = \lim\limits_{x\to+\infty} \dfrac{36x^2}{2x^2} = 18$

2) $\lim\limits_{x\to 2} \dfrac{(6x-2)^2}{2x^2-8} = \dfrac{10^2}{0^+ \text{ ou } 0^-}$ $\begin{cases} x\to 2^+ & +\infty \\ x\to 2^- & -\infty \end{cases}$

x	-2		2	
$2x^2-8$	$+$	0	$-$	0 $+$

3) $\lim\limits_{x\to 0} \dfrac{\sin^2(5x)}{x} = \lim\limits_{x\to 0} \sin 5x \cdot \dfrac{\sin 5x}{5x} \cdot 5 = 5 \cdot 0 = 0$

EXERCICE 2

$f(x) = \dfrac{(x-3)(x+1)}{(x-3)(x+5)} = \dfrac{x^2-2x-3}{x^2+2x-15}$ $\quad D_f = \mathbb{R} \setminus \{3 ; -5\}$

AH : $y = 1$

Trou : $\lim\limits_{x\to 3} f(x) = \dfrac{4}{8} = \dfrac{1}{2}$ $\quad (3 ; \frac{1}{2})$

AV : $\lim\limits_{x\to -5} f(x) = \dfrac{-4}{0^+ \text{ ou } 0^-}$ $\begin{cases} x\to -5^+ & -\infty \\ x\to -5^- & +\infty \end{cases}$ $\quad$ AV : $x = -5$

EXERCICE 3

$f(x) = \dfrac{2x^3+8x^2-10x}{(x+3)(x-1)}$

- $D_f = \mathbb{R} \setminus \{-3 ; 1\}$
- $\cap Ox$: $f(x) = 0 \Leftrightarrow 2x^3+8x^2-10x = 0 \Leftrightarrow 2x(x^2+4x-5) = 0$

  $x = 0$ $\quad (0,0)$ $\quad x^2+4x-5 = 0$ $\quad \Delta = 36$

  $x_{1,2} = \dfrac{-4 \pm 6}{2} = \begin{cases} -5 & (-5;0) \\ 1 & (1;0) \end{cases}$
- $\cap Oy$: $f(0) = 0$ $\quad (0,0)$
- VA : $\lim\limits_{x\to -3} f(x) = \dfrac{48}{0^+ \text{ ou } 0^-}$ $\begin{cases} x\to -3^+ & -\infty \\ x\to -3^- & +\infty \end{cases}$

x		-3		1	
$(x+3)(x-1)$	$+$	0	$-$	0	$+$

AV : $x = -3$

$\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \frac{2x(x-1)(x+5)}{(x+3)(x-1)} = \frac{12}{4} = 3$ Trou $(1;3)$

AH/AO: $2x^3+8x^2-10x \;\big|\; x^2+2x-3$

$+\underline{-2x^3-4x^2+6x} \qquad 2x+4$

$4x^2-4x$

$+\underline{-4x^2-8x+12}$

$-12x+12$

AO: $y = 2x+4$

$f(x) = 2x+4+\frac{-12x+12}{x^2+2x-3}$

$x \to \pm\infty \quad f(x) \underset{>}{<} 2x+4$

EXERCICE 4

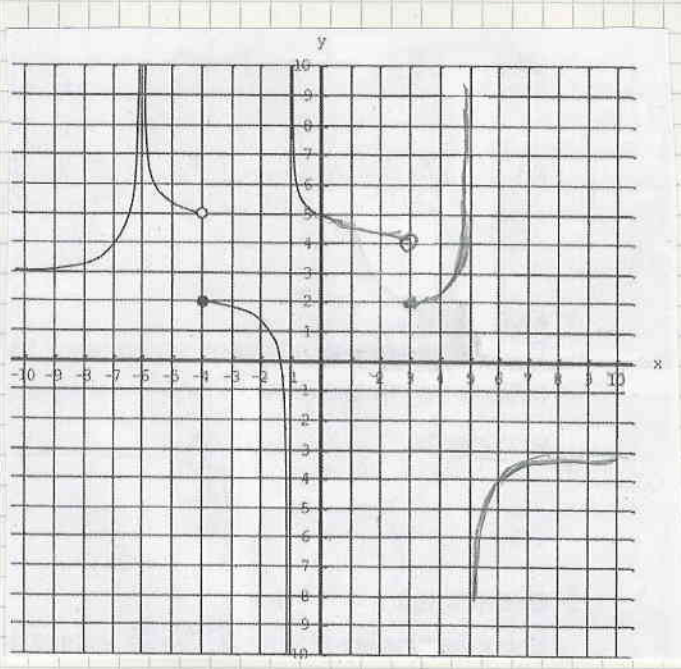
$$f(x) = \begin{cases} \frac{\sin x}{x}, & x<0 \\ 3\cos x + a, & x \geq 0 \end{cases}$$

$\left.\begin{array}{l} \cdot \lim\limits_{x\to 0^-} f(x) = 1 \\ \cdot \lim\limits_{x\to 0^+} f(x) = 3+a \end{array}\right\} \; 1 = 3+a \Rightarrow a = -2$

EXERCICE 5

1) $\lim\limits_{x\to-\infty} f(x) = 3 \quad \lim\limits_{x\to -4^-} f(x) = 5 \quad \lim\limits_{x\to -6} f(x) = +\infty \quad \lim\limits_{x\to 0} f(x) = 5$

$f(-4) = 2$



$D_f = \mathbb{R} \setminus \{-5; -1; 5\}$