

LDDR-Niveau 1-TE 4: Calcul-Algèbre

EXERCICE 1

$$a. 117 + x^3 = \frac{1000}{x^3} \Leftrightarrow$$

$$x \neq 0$$

$$\Leftrightarrow 117x^3 + x^6 - 1000 = 0 \Leftrightarrow x^6 + 117x^3 - 1000 = 0$$

$$x^3 = t, \quad t^2 + 117t - 1000 = 0 \quad \Delta = 133^2$$

$$t_{1,2} = \frac{-117 \pm 133}{2} = \begin{cases} t_1 = 8 \\ t_2 = -125 \end{cases}$$

$$t_1 = 8 = x^3 \Leftrightarrow x = 2$$

$$t_2 = -125 = x^3 \Rightarrow x = -5$$

$$x \neq -2$$

$$b. 1 - \frac{x^2 - 4}{3x + 6} = 0 \Leftrightarrow 1 = \frac{x^2 - 4}{3x + 6} \Leftrightarrow 3x + 6 = x^2 - 4 \Leftrightarrow$$

$$\Leftrightarrow x^2 - 3x - 10 = 0 \quad \Delta = 49 \quad x_{1,2} = \frac{3 \pm 7}{2} \begin{cases} x_1 = 5 \\ x_2 = -2 \text{ impossible} \end{cases}$$

$$c. \frac{x-12}{\sqrt{36-x}} = 2$$

$$36 - x > 0 \Leftrightarrow x < 36$$

$$x - 12 \geq 0 \Leftrightarrow x \geq 12$$

$$\Leftrightarrow x - 12 = 2\sqrt{36-x} \Leftrightarrow (x-12)^2 = 4(36-x) \Leftrightarrow$$

$$\Leftrightarrow x^2 - 24x + 144 = 144 - 4x \Leftrightarrow x^2 - 20x = 0$$

$$\Leftrightarrow x(x-20) = 0 \Rightarrow x = 0 \text{ impossible}$$

$$x = 20 \quad \checkmark$$

$$d. 4x^7 - 9x^9 = 0 \Leftrightarrow$$

$$\Leftrightarrow x^7(4 - 9x^2) = 0 \Leftrightarrow \frac{4}{9} = x^2 \Leftrightarrow x = \pm \frac{2}{3}$$

$$e. 2 - |3x - 1| = x \Leftrightarrow 2 - x = |3x - 1| \Leftrightarrow$$

$$\Leftrightarrow (2-x)^2 = (3x-1)^2 \Leftrightarrow 4 - 4x + x^2 = 9x^2 - 6x + 1 \Leftrightarrow$$

$$\Leftrightarrow 8x^2 - 2x - 3 = 0 \quad \Delta = 100 \quad x_{1,2} = \frac{2 \pm 10}{16} \begin{cases} \frac{3}{4} \\ -\frac{1}{2} \end{cases}$$

$$x = \frac{3}{4}: 2 - |3 \cdot \frac{3}{4} - 1| = \frac{3}{4} \Leftrightarrow | \frac{9}{4} - 1 | = \frac{5}{4} \quad \checkmark$$

$$x = -\frac{1}{2}: 2 - | -\frac{3}{2} - 1 | = -\frac{1}{2} \Leftrightarrow 2 + \frac{1}{2} = | -\frac{5}{2} | \Rightarrow \frac{5}{2} = \frac{5}{2} \quad \checkmark$$

EXERCICE 2

$$a. \begin{array}{r|l} 4x^3 - 2x^2 + 10x + 1 & 2x^2 - 3 \\ -4x^3 & 2x - 1 \\ \hline & +6x \\ & -2x^2 + 16x + 1 \\ & +2x^2 & -3 \\ \hline & 16x - 2 \end{array}$$

$$Q(x) = 2x - 1$$

$$R(x) = 16x - 2$$

b. $p(x) = x^3 + 2x^2 + 2x + 1$ $p(-1) = 0$

$$\begin{array}{r|l} x^3 + 2x^2 + 2x + 1 & x+1 \\ -x^3 - x^2 & \\ \hline x^2 + 2x + 1 & \\ -x^2 - x & \\ \hline x + 1 & \end{array}$$

$P(x) = (x+1)(x^2+x+1)$ $\Delta = 1 - 4 < 0$

c. $P(x) = ax^3 - a^2x - 42$ $P(-2) = 0$

$$\begin{aligned} P(-2) = 0 &\Leftrightarrow -8a + 2a^2 - 42 = 0 \Leftrightarrow \\ &\Leftrightarrow 2a^2 - 8a - 42 = 0 \Leftrightarrow a^2 - 4a - 21 = 0 \\ \Delta &= 100 \quad a_{1,2} = \frac{4 \pm 10}{2} \begin{cases} 7 = a_1 \\ -3 = a_2 \end{cases} \end{aligned}$$

d. $p(x) = -3x^2 + kx - 1$

$$\begin{aligned} \Delta &= 0 \Leftrightarrow k^2 - 4(-3)(-1) = 0 \Leftrightarrow k^2 - 12 = 0 \Leftrightarrow \\ k &= \pm \sqrt{12} \Leftrightarrow k = \pm 2\sqrt{3} \end{aligned}$$

EXERCICE 3

• degré 2 : $P(x) = ax^2 + bx + c$

• $P(x) = (x+5)Q(x)$ ou

$$P(-5) = 0 \Leftrightarrow 25a - 5b + c = 0 \quad \left. \begin{array}{l} 25a - 5b + 50 = 0 \\ 5a - b = -10 \end{array} \right\}$$

• $P(0) = 50 \Leftrightarrow c = 50$

• $P(2) = 35 \Leftrightarrow 4a + 2b + 50 = 35 \Leftrightarrow$

$$\Leftrightarrow \begin{cases} 4a + 2b = -15 \\ 5a - b = -10 \end{cases} \quad \begin{array}{l} 4a + 2b = -15 \\ * 2 \quad 10a - 2b = -20 \\ \hline 14a = -35 \Leftrightarrow a = -\frac{35}{14} \end{array}$$

$$-5 \cdot \frac{35}{14} - b = -10 \Leftrightarrow$$

$$\Leftrightarrow -\frac{175}{14} + 10 = b \Leftrightarrow b = -\frac{35}{14}$$

$$P(x) = -\frac{35}{14}x^2 - \frac{35}{14}x + 50$$

EXERCISE 4

$$\begin{cases} a+2b-c=-8 & (1) \\ 2a+b+c=11 & (2) \\ a+b-2c=-5 & (3) \end{cases}$$

$$(1), (2) \quad a+2b-c=-8$$

$$+ \quad 2a+b+c=11$$

$$\hline 3a+3b=3 \Leftrightarrow a+b=1$$

$$2 \times (2) \quad 4a+2b+2c=22$$

$$(3) + \quad a+b-2c=-5$$

$$\hline 5a+3b=17$$

$$\begin{cases} a+b=1 & \times (-5) \\ 5a+3b=17 \end{cases} \Rightarrow \begin{cases} -5a-5b=-5 \\ 5a+3b=17 \end{cases}$$

$$+ \quad \hline$$

$$-2b=+12 \Leftrightarrow \underline{b=-6}$$

$$a-6=1 \Leftrightarrow \underline{a=7}$$

$$(1): \quad 7-12-c=-8 \Leftrightarrow \underline{c=3}$$