1 DDR - Niveau 1 : Limites - Continuité

EXERCICE 1.

1. $f(x) = \begin{cases} -1 & x < 1 \\ 2 & x = 1 \\ 3 & x > 1 \end{cases}$

a) $\lim\limits_{\substack{x\to 1\\<}} f(x) = -1$ b) $\lim\limits_{\substack{x\to 1\\>}} f(x) = 3$

c) $f(1) = 2$ d) $\lim\limits_{x\to 1} f(x)$ $\emptyset$

EXERCICE 2

$f(x) = \dfrac{1}{x+1}$ $D_f = \mathbb{R} \setminus \{-1\}$

a) $\lim\limits_{\substack{x\to -1\\<}} f(x) = -\infty$ b) $\lim\limits_{\substack{x\to -1\\>}} f(x) = +\infty$ c) $f(-1)$ $\emptyset$

d) $\lim\limits_{x\to -1} f(x)$ $\emptyset$ car $\lim\limits_{\substack{x\to -1\\>}} f(x) \neq \lim\limits_{\substack{x\to -1\\<}} f(x)$

e) $\lim\limits_{\substack{x\to 1\\<}} f(x) = \dfrac{1}{2}$ f) $\lim\limits_{\substack{x\to 1\\>}} f(x) = \dfrac{1}{2}$ g) $f(1) = \dfrac{1}{2}$ h) $\lim\limits_{x\to 1} f(x) = \dfrac{1}{2}$

EXERCICE 3

$f(x) = \dfrac{\sin(x)}{x}$

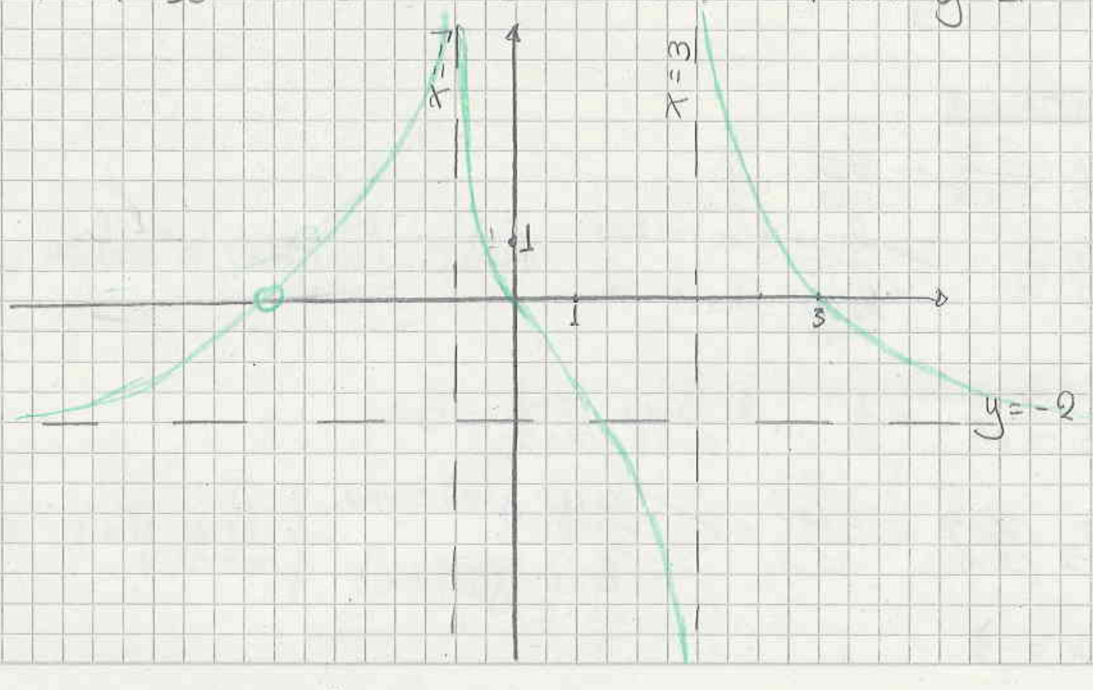
a) $\lim\limits_{\substack{x\to 0\\<}} f(x) = 1$ b) $\lim\limits_{\substack{x\to 0\\>}} f(x) = 1$ c) $f(0) = 1$ d) $\lim\limits_{x\to 0} f(x) = 1$

EXERCICE 4

a) $\lim\limits_{x\to -\infty} f(x) = 1$ b) $\lim\limits_{x\to +\infty} f(x) = 1$ c) $\lim\limits_{x\to -6} f(x) = 1$ d) $\lim\limits_{x\to 0} f(x) = +\infty$

e) $D_f = \mathbb{R} \setminus \{-3.75 ; 0 ; 0.7\}$

f) AV : $x = -3.75$ $x = 0$ $x = 0.7$ AH : $y = 1$



EXERCICE 6

a) $\lim\limits_{y\to 0} \frac{1}{y-1} = -1$

b) $\lim\limits_{x\to 0} \frac{3x^2-2x}{x+1} = 0$

c) $\lim\limits_{x\to 3} \frac{x^2-2x-3}{x-3} = \text{"}\frac{0}{0}\text{"} \xRightarrow{\text{fact}} \lim\limits_{x\to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} = 4$

$\Delta = 4+12 = 16 \quad x_{1,2} = \frac{2\pm 4}{2} = \left\langle \begin{matrix} 3 \\ -1 \end{matrix} \right.$ Trou $(3;4)$

d) $\lim\limits_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \text{"}\frac{0}{0}\text{"} = \lim\limits_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}} = 4$ Trou $(4;4)$

e) $\lim\limits_{x\to 0} \frac{x^2}{\sqrt{x^2+1}-1} = \text{"}\frac{0}{0}\text{"} = \lim\limits_{x\to 0} \frac{x^2(\sqrt{x^2+1}+1)}{x^2+1-1} =$

$= \lim\limits_{x\to 0} \frac{\cancel{x^2}(\sqrt{x^2+1}+1)}{\cancel{x^2}} = 2$ Trou $(0;2)$

f) $\lim\limits_{x\to 3} \frac{x^2-5x+6}{x^2-4x+3} = \text{"}\frac{0}{0}\text{"} = \lim\limits_{x\to 3} \frac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} = \frac{3-2}{3-1} = \frac{1}{2}$ Trou $(3;\frac{1}{2})$

g) $\lim\limits_{x\to 2} \frac{x^2+x+1}{x^2+x-6} = \lim\limits_{x\to 2} \frac{x^2+x+1}{(x+3)(x-2)} = \text{"}\frac{7}{0}\text{"}$

$\Delta = 1+24 = 25 \quad x_{1,2} = \frac{-1\pm 5}{2} = \left\langle \begin{matrix} -3 \\ 2 \end{matrix} \right.$

x	$-\infty$		-3		2		$+\infty$
x^2+x-2		+	0	−	0	+	

$\lim\limits_{x\to 2^-} f(x) = -\infty \quad \lim\limits_{x\to 2^+} f(x) = +\infty$

AV: $x = 2$

h) $\lim\limits_{x\to 0} \frac{x^2-2x}{x} = \text{"}\frac{0}{0}\text{"} = \lim\limits_{x\to 0} \frac{\cancel{x}(x-2)}{\cancel{x}} = -2$ Trou $(0;-2)$

i) $\lim\limits_{\substack{x\to 0 \\ >}} \log(x) = -\infty$

EXERCICE 7

a) $\lim\limits_{x\to 5} \frac{5x}{5+x} = \frac{25}{10} = \frac{5}{2}$

b) $\lim\limits_{x\to 0} \frac{1}{x} = \left\langle \begin{matrix} \lim\limits_{x\to 0^+} f(x) = +\infty \\ \lim\limits_{x\to 0^-} f(x) = -\infty \end{matrix} \right.$ Alors $\lim\limits_{x\to 0} f(x)$ n'existe pas

c) $\lim\limits_{x\to 3} \frac{x-2}{x+2} = \frac{1}{5}$ d) $\lim\limits_{x\to 2} \frac{x-2}{x+2} = 0$

e) $\lim\limits_{x\to -2} \frac{x-2}{x+2} = \text{"}\frac{-4}{0}\text{"} \quad \left\langle \begin{matrix} \lim\limits_{\substack{x\to -2 \\ >}} f(x) = -\infty \\ \lim\limits_{\substack{x\to -2 \\ <}} f(x) = +\infty \end{matrix} \right\} \lim\limits_{x\to -2} f(x) \quad \emptyset$

AV: $x = -2$

f) $\lim\limits_{x\to 2} \sqrt{4x+1} = 3$ g) $\lim\limits_{x\to \frac{\pi}{2}} \sin(2x) = \sin\pi = 0$

h) $\lim\limits_{x\to 3} \frac{(x-3)^{\cancel{2}}}{\cancel{x-3}} = "\frac{0}{0}" = \lim\limits_{x\to 3}(x-3) = 0$ Trou $(3;0)$

i) $\lim\limits_{x\to 2} \frac{1}{4-2x} = "\frac{1}{0}" < \begin{matrix} \lim\limits_{\substack{x\to 2\\ >}} f(x) = -\infty \\ \lim\limits_{\substack{x\to 2\\ <}} f(x) = +\infty \end{matrix} \Big\}$ $\lim\limits_{x\to 2} f(x)$ n'existe pas

x	$-\infty$		2		$+\infty$
$4-2x$		$+$	0	$-$	

AV : $x = 2$

j) $\lim\limits_{x\to 1} \frac{(3x-1)^2}{(2x+1)^3} = \frac{(3-1)^2}{(2+1)^3} = \frac{4}{27}$

k) $\lim\limits_{x\to 4} \sqrt{25-x^2} = \sqrt{25-16} = \sqrt{9} = 3$

l) $\lim\limits_{x\to -2} \frac{x^2+4}{x^3+8} = "\frac{8}{0}" = < \begin{matrix} \lim\limits_{\substack{x\to -2\\ >}} f(x) = +\infty \\ \lim\limits_{\substack{x\to -2\\ <}} f(x) = -\infty \end{matrix}$ AV $x = -2$

m) $\lim\limits_{\substack{x\to 0\\ >}} \log(2x) = -\infty$ AV: $x = 0$ n) $\lim\limits_{x\to 0} 5^x = 5^0 = 1$

o) $\lim\limits_{x\to 1} \frac{2x-2}{x-1} = "\frac{0}{0}" = \lim\limits_{x\to 1} \frac{2\cancel{(x-1)}}{\cancel{x-1}} = 2$ Trou $(1;2)$

p) $\lim\limits_{x\to 1} \frac{x^2-3x+2}{x-1} = "\frac{0}{0}" = \lim\limits_{x\to 1} \frac{(x-2)\cancel{(x-1)}}{\cancel{x-1}} = -1$ Trou $(1;-1)$

q) $\lim\limits_{x\to 4} \frac{x-4}{x^2-x-12} = "\frac{0}{0}" = \lim\limits_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(x+3)} = \frac{1}{7}$ Trou $(4;\frac{1}{7})$

r) $\lim\limits_{x\to -3} \frac{x^2+x-6}{-3x^2+6x+45} = -\frac{1}{3} \lim\limits_{x\to -3} \frac{x^2+x-6}{x^2-2x-15} = "\frac{0}{0}"$

$= -\frac{1}{3} \lim\limits_{x\to -3} \frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x-5)} = -\frac{1}{3} \cdot \frac{-5}{-8} = -\frac{5}{24}$ Trou $(-3;-\frac{5}{24})$

EXERCICE 8

a) $f(x) = \frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1 & x > 0 \\ \frac{-x}{x} = -1 & x < 0 \end{cases}$ $D_f = \mathbb{R}^*$

$\lim\limits_{\substack{x\to 0\\ >}} f(x) = 1$ $\lim\limits_{\substack{x\to 0\\ <}} f(x) = -1$ $\lim\limits_{x\to 0} f(x)$ n'existe pas

b) $f(x) = \begin{cases} \frac{x^2-2x}{x} = \frac{x(x-2)}{x} = x-2 & x > 0 \\ \frac{x^2-2x}{-x} = -\frac{x(x-2)}{x} = -x+2 & x < 0 \end{cases}$ $D_f = \mathbb{R}^*$

$\lim\limits_{\substack{x\to 0\\ >}} f(x) = -2$ $\lim\limits_{\substack{x\to 0\\ <}} f(x) = 2$ $\lim\limits_{x\to 0} f(x)$ n'existe pas

c) $f(x) = \frac{|x|+x^2}{x} = \begin{cases} \frac{x+x^2}{x} = \frac{\cancel{x}(x+1)}{\cancel{x}} = x+1 & x>0 \\ \frac{-x+x^2}{x} = \frac{\cancel{x}(-1+x)}{\cancel{x}} = x-1 & x<0 \end{cases}$ $D_f = \mathbb{R}^*$

$\lim\limits_{x\to 0^+} f(x) = 1$ $\lim\limits_{x\to 0^-} f(x) = -1$ $\lim\limits_{x\to 0} f(x)$ $\emptyset$

d) $\lim\limits_{x\to 4} \frac{x-4}{x^2-7x+12} = \text{"}\frac{0}{0}\text{"} = \lim\limits_{x\to 4} \frac{\cancel{x-4}}{(x-3)\cancel{(x-4)}} = \frac{1}{1} = 1$

Trou $(4;1)$

e) $\lim\limits_{x\to 2} \frac{x+1}{x^2-4} = \text{"}\frac{3}{0}\text{"}$ $\begin{cases} \lim\limits_{x\to 2^+} f(x) = +\infty \\ \lim\limits_{x\to 2^-} f(x) = -\infty \end{cases}$ A V $x=2$

$$\begin{array}{c|ccccccc} x & -\infty & & -2 & & 2 & & +\infty \\ \hline x^2-4 & & + & 0 & - & 0 & + & \end{array}$$

f) $f(x) = \frac{|x-2|}{x^2-3x+2} = \begin{cases} \frac{x-2}{x^2-3x+2} & x \geqslant 2 \\ \frac{-x+2}{x^2-3x+2} & x<2 \end{cases}$

$D_f = \mathbb{R} \setminus \{2;1\}$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} \frac{\cancel{x-2}}{\cancel{(x-2)}(x-1)} = 1$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(x-1)} = -1$

$\lim\limits_{x\to 2} f(x)$ n'existe pas

EXERCICE 9

a) $\lim\limits_{x\to 0} \frac{\tan(x)}{x} = 1$

b) $\lim\limits_{x\to 0} \frac{\cos x}{x} = \begin{cases} \lim\limits_{x\to 0^+} f(x) = \text{"}\frac{1}{0}\text{"} = +\infty \\ \lim\limits_{x\to 0^-} f(x) = \text{"}\frac{1}{0}\text{"} = -\infty \end{cases}$ A V $x=0$

c) $\lim\limits_{x\to 0} \frac{1-\cos x}{\sin^2 x} = \lim\limits_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{\sin^2 x(1+\cos x)} = \lim\limits_{x\to 0} \frac{1-\cos^2 x}{\sin^2 x(1+\cos x)} =$

$= \lim\limits_{x\to 0} \frac{\sin^2 x}{\sin^2 x(1+\cos x)} = \frac{1}{2}$ Trou $(0;\frac{1}{2})$

d) $\lim\limits_{x\to 0} \frac{1-\cos^2 x}{x\cdot\tan x} = \text{"}\frac{0}{0}\text{"} = \lim\limits_{x\to 0} \frac{\sin^{\cancel{2}} x}{x\,\frac{\cancel{\sin x}}{\cos x}} = \lim\limits_{x\to 0} \frac{\sin x\cdot\cos x}{x}$

$= \lim\limits_{x\to 0} \overset{\to 1}{\frac{\sin x}{x}}\cdot\cos x = 1\cdot 1 = 1$ Trou $(0;1)$

e) $\lim\limits_{x\to 0} \frac{\sin x}{2x^2-3x} = (\text{"}\frac{0}{0}\text{"}) = \lim\limits_{x\to 0} \frac{\overset{\to 1}{\sin x}}{\cancel{x}(2x-3)} = -\frac{1}{3}$ Trou $(0;-\frac{1}{3})$

f) $\lim\limits_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos^2 x} = \lim\limits_{x\to\frac{\pi}{2}} \frac{(1-\sin x)(1+\sin x)}{\cos^2 x(1+\sin x)} = \lim\limits_{x\to\frac{\pi}{2}} \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}(1+\sin x)} = \frac{1}{2}$

Trou $(\frac{\pi}{2};\frac{1}{2})$

EXERCICE 10

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = \lim_{x \to 0} \frac{(\cos x - 1)(\cos x + 1)}{x(\cos x + 1)} = \lim_{x \to 0} \frac{\cos^2 x - 1}{x(\cos x + 1)} =$$

$$= - \lim_{x \to 0} \frac{\sin^2 x}{x} \cdot \frac{1}{\cos x + 1} = - \lim_{x \to 0} \underbrace{\frac{\sin x}{x}}_{1} \frac{\sin x}{\cos x + 1} = \frac{-\sin 0}{\cos 0 + 1} = 0$$

EXERCICE 11

$D_f = \mathbb{R} \setminus \{-2 ; 3 ; 5\}$

La fonction est continue sur les intervalles $]-\infty, -2[$; $]-2 ; 1[$; $]1 ; 3[$; $]3 ; 5[$ et $]5, +\infty[$ mais elle n'est pas continue sur $x = 1$.

$\lim\limits_{x \to 5} f(x) = 1.5$ $\quad \lim\limits_{\substack{x \to -2 \\ >}} f(x) = +\infty$ $\quad \lim\limits_{x \to 1} f(x)$ n'existe pas

$\lim\limits_{\substack{x \to 1 \\ <}} f(x) = 1$ $\quad \lim\limits_{\substack{x \to 1 \\ >}} f(x) = -1.5$ $\quad \lim\limits_{x \to +\infty} f(x) = 2$

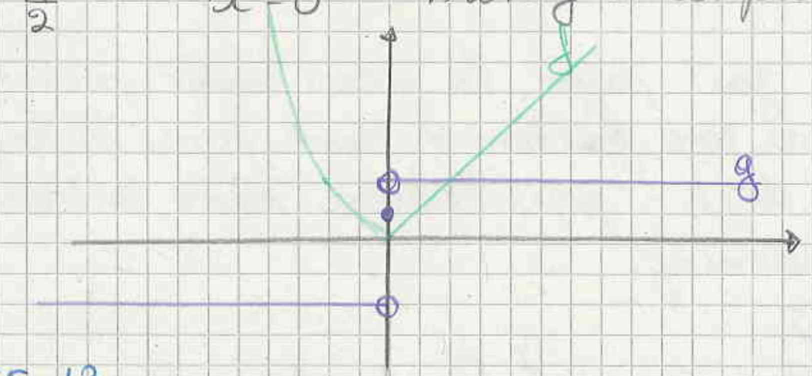
EXERCICE 12

$f(x) = \begin{cases} x^2 & x \leq 0 \\ x & x > 0 \end{cases}$ $\quad \lim\limits_{x \to 0^-} f(x) = 0 = \lim\limits_{x \to 0^+} f(x) = f(0)$

Alors f est continue en $x = 0$

$g(x) = \begin{cases} \frac{x}{x} = 1 & x > 0 \\ -\frac{x}{x} = -1 & x < 0 \\ \frac{1}{2} & x = 0 \end{cases}$ $\quad \lim\limits_{x \to 0^+} f(x) = 1 \neq \lim\limits_{x \to 0^-} f(x) = -1 \neq f(0)$

Alors f n'est pas continue en 0

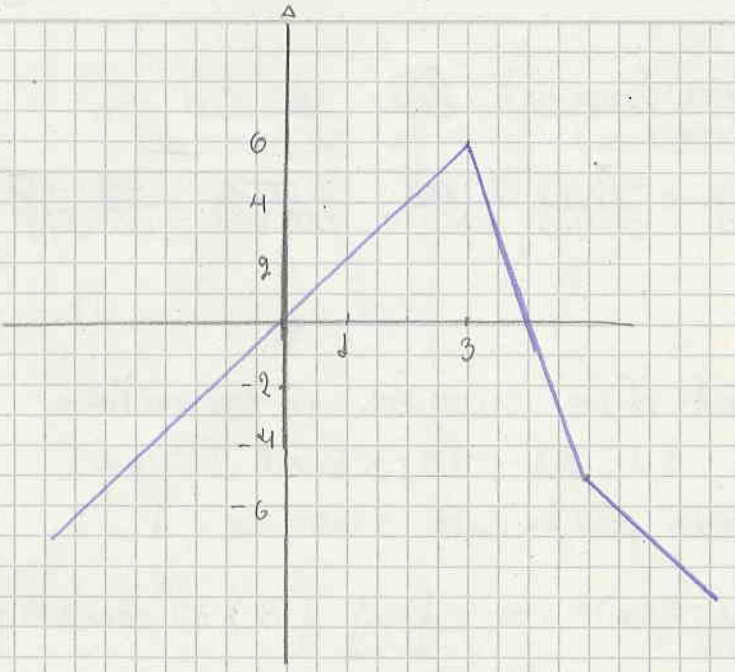


EXERCICE 13

$f(x) = \begin{cases} 2x & x \leq 3 \\ ax + b & 3 < x \leq 5 \\ -x & x > 5 \end{cases}$ $\quad$ continue: $\lim\limits_{x \to x_0^-} f(x) = \lim\limits_{x \to x_0^+} f(x) = f(x_0)$

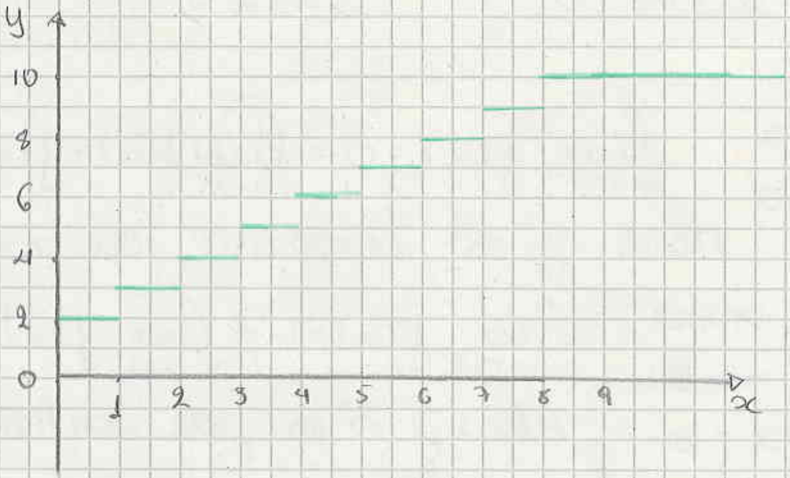
$\left.\begin{array}{l} \cdot\ x = 3 : \lim\limits_{x \to 3^-} f(x) = 6 = \lim\limits_{x \to 3^+} f(x) = 3a + b \Leftrightarrow 3a + b = 6 \\ \cdot\ x = 5 \ \lim\limits_{x \to 5^-} f(x) = 5a + b = \lim\limits_{x \to 5^+} f(x) = -5 \Leftrightarrow 5a + b = -5 \end{array}\right\} (-)$

$-2a = 11 \Leftrightarrow a = -\frac{11}{2}$ et $b = \frac{45}{2}$



EXERCICE 14

1.



2. Le parking fait payer la même somme pour une heure complète ou une fraction d'heure. Ainsi le tarif fait un "saut" de 1.- pour chaque heure entière passée dans le parking

EXERCICE 15

a) $f(x) = \frac{x^2-1}{x+1}$ $\quad a = -1 \notin D_f \quad \Rightarrow$ discontinue

b) $f(x) = \begin{cases} \frac{x^2-1}{x+1} & x \neq -1 \\ 6 & x = -1 \end{cases}$

$$\lim_{x \to -1} f(x) = \lim_{x \to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = -2 \neq f(-1) \Rightarrow \text{discontinue}$$

c) $f(x) = \begin{cases} \frac{x^2-2x-8}{x-4} & x \neq 4 \\ 6 & x = 4 \end{cases}$

$$\lim_{x \to 4} f(x) = \lim_{x \to 4} \frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}} = 6 = f(4) \Rightarrow f \text{ continue}$$

d) $f(x) = \begin{cases} 1-x & x \le 2 \\ x^2-2x & x > 2 \end{cases}$

$\lim_{\substack{x \to 2 \\ <}} f(x) = -1 \neq \lim_{\substack{x \to 2 \\ >}} f(x) = 0 \Rightarrow f$ discontinue

EXERCICE 16

a) $\lim_{x \to +\infty} \frac{2x}{x+2} = \lim_{x \to +\infty} \frac{2\cancel{x}}{\cancel{x}} = 2$ AH : $y = 2$

b) $\lim_{x \to +\infty} \frac{1}{x} = 0$ AH : $y = 0$

c) $\lim_{x \to +\infty} \frac{x^2-2x}{x} = \lim_{x \to +\infty} \frac{x^{\cancel{2}}}{\cancel{x}} = \lim_{x \to +\infty} x = +\infty$

d) $\lim_{x \to -\infty} \frac{2x^2+x-1}{x^3+1} = \lim_{x \to -\infty} \frac{2x^2}{x^3} = 0$ AH : $y = 0$

e) $\lim_{x \to -\infty} \frac{3x^2-2x+2}{x+1} = \lim_{x \to -\infty} \frac{3x^2}{\cancel{x}} = -\infty$

f) $\lim_{x \to +\infty} \frac{2x^2+2x-15}{3x^2+8x+15} = \lim_{x \to +\infty} \frac{2\cancel{x^2}}{3\cancel{x^2}} = \frac{2}{3}$ AH : $y = \frac{2}{3}$

g) $\lim_{x \to +\infty} \frac{2x^2+2x-15}{x^2+6x+9} = \lim_{x \to +\infty} \frac{2\cancel{x^2}}{\cancel{x^2}} = 2$ AH : $y = 2$

h) $\lim_{x \to -\infty} \frac{-3x^2+2}{4x+4} = \lim_{x \to -\infty} \frac{-3x^{\cancel{2}}}{4\cancel{x}} = +\infty$

i) $\lim_{x \to -\infty} \frac{x^2-2x+1}{x^2-2x+1} = \lim_{x \to -\infty} \frac{x^2}{x^2} = 1$ AH : $y = 1$

EXERCICE 17

▸ $f(x) = \frac{2x-1}{x^2-x-2}$ $\quad x^2-x-2 = 0 \quad \Delta = 1+8 = 9 \quad x_{1,2} = \frac{1 \pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$

$D_f = \mathbb{R} \setminus \{-1 ; 2\}$

· $x = -1$ $\quad \lim_{x \to -1} f(x) = "\frac{-3}{0}" \quad \lim_{x \to -1^+} f(x) = -\infty \quad \lim_{x \to -1^-} f(x) = +\infty$

x	$-\infty$		-1		2		$+\infty$
x^2-x-2		+	0	−	0	+	

AV $x = -1$

· $x = 2$ $\quad \lim_{x \to 2^+} f(x) = +\infty \quad \lim_{x \to 2^-} f(x) = -\infty$ $\quad$ AV : $x = 2$

▸ $g(x) = \frac{4-x^2}{x^2-x-2} = \frac{(2-x)(2+x)}{(x-2)(x+1)} = \frac{-\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)}$

$\lim_{x \to 2} g(x) = -\frac{4}{3}$ $\quad$ Trou $(2 ; -\frac{4}{3})$

▶ $h(x) = \frac{3x^4+12x^3+9x^2}{4x^3+8x^2-12x} = \frac{3x^2(x^2+4x+3)}{4\cancel{x}(x^2+2x-3)} = \frac{3\cancel{(x+3)}(x+1)}{4\cancel{(x+3)}(x-1)} =$

$= \frac{3(x+1)}{4(x-1)}$

$D_h = \mathbb{R} \setminus \{0; -3; 1\}$

$x = 0 \quad \lim_{x \to 0} h(x) = \frac{3}{-4}$ Trou $(0; -\frac{3}{4})$

$x = -3 \quad \lim_{x \to -3} h(x) = \frac{-6}{-16} = \frac{3}{8}$ Trou $(-3; \frac{3}{8})$

$$x = 1 \quad \left.\begin{array}{l} \lim_{x \to 1^+} h(x) = \frac{6}{0^+} = +\infty \\ \lim_{x \to 1^-} h(x) = \frac{6}{0^-} = -\infty \end{array}\right\} \text{AV} : x = 1$$

EXERCICE 18

$f_1(x) = \frac{1}{2(x+1)}$ AV : $x = -1$ AH : $y = 0$

$f_2(x) = \frac{x^2}{2(x^2-1)}$ AV : $x = -1$ $x = 1$ AH : $y = \frac{1}{2}$

$f_3(x) = \frac{x^2}{2(x+1)}$ AV : $x = -1$ AO : $y = \frac{1}{2}x - \frac{1}{2}$

EXERCICE 19

a) $\lim_{x \to \infty} (2x^3 - 7x) = +\infty$ $\emptyset$

b) $\lim_{x \to \infty} \frac{1}{x^2-1} = 0$ AH : $y = 0$

c) $\lim_{x \to \infty} \frac{3x-2}{5x+6} = \frac{3}{5}$ AH : $y = \frac{3}{5}$

d) $\lim_{x \to -\infty} 2^x = 0$ AH : $y = 0$

e) $\lim_{x \to \infty} \frac{4x^2-5x+6}{2x^2-7x} = 2$ AH : $y = 2$

f) $\lim_{x \to -\infty} \frac{x^3+2x^2+1}{x^4-3x^3+x} = \lim_{x \to -\infty} \frac{x^3}{x^4} = 0$ $y = 0$

g) $\lim_{x \to \infty} \frac{1}{\log(x)} = 0$ AH : $y = 0$

h) $\lim_{x \to \infty} \frac{(2x^2-1)^3}{(x^3-5x)^2} = \lim_{x \to \infty} \frac{8x^6}{x^6} = 8$ AH : $y = 8$

EXERCICE 20

a) $f(x) = \dfrac{x^3+3}{x^2+x+1}$ $\left.\begin{array}{l}\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \dfrac{x^3}{x^2} = \lim\limits_{x\to+\infty} x = +\infty \\ \lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} x = -\infty\end{array}\right\}$ 4em graphe

b) $g(x) = \dfrac{3x^2}{x^2+x+1}$ $\lim\limits_{x\to\pm\infty} f(x) = 3$ AH: $y = 3$; 3em graphe

c) $h(x) = \dfrac{x}{x^2+x+1}$ $\lim\limits_{x\to\pm\infty} f(x) = 0$ AH: $y = 0$; 1em graphe

d) $l(x) = 3^{-x}$ $\lim\limits_{x\to+\infty} l(x) = 0$ $\lim\limits_{x\to-\infty} e^{-x} = +\infty$; 2em graphe

EXERCICE 21

a) $f(x) = \dfrac{5-x-4x^2}{2x+1}$

$$\begin{array}{r|l} -4x^2-x+5 & 2x+1 \\ \hline +4x^2+2x & -2x+\frac{1}{2} \\ \hline x+5 & \\ -x-\frac{1}{2} & \\ \hline \frac{9}{2} & \end{array}$$

AO: $y = -2x + \frac{1}{2}$

$x \to +\infty$ $f(x) \xrightarrow{>} -2x+\frac{1}{2}$

$x \to -\infty$ $f(x) \xrightarrow[<]{} -2x+\frac{1}{2}$

b) $f(x) = \dfrac{3x^2-5x+2}{4x^2-2x+3}$ AO: $y = \frac{3}{4}$

$x \to +\infty$ $\to$ $f(x) \xrightarrow{<} \frac{3}{4}$

$x \to -\infty$ $\to$ $f(x) \xrightarrow{>} \frac{3}{4}$

c) $f(x) = \dfrac{3x^3+2x+1}{10x+2}$ Pas d'asymptote

d) $f(x) = \dfrac{2x^3-x}{5x^2+10x-1}$

$$\begin{array}{r|l} 2x^3 \quad -x & 5x^2+10x-1 \\ \hline -2x^3-4x^2+\frac{2}{5}x & \frac{2}{5}x-\frac{4}{5} \\ \hline \cancel{-4x^2}-\frac{3}{5}x & \\ \cancel{4x^2}+8x-\frac{4}{5} & \\ \hline \frac{37}{5}x-\frac{4}{5} & \end{array}$$

AO: $y = \frac{2}{5}x - 4/5$

$x \to +\infty$ $f(x) \xrightarrow{>} \frac{2}{5}x - \frac{4}{5}$

$x \to -\infty$ $f(x) \xrightarrow[<]{} \frac{2}{5}x - \frac{4}{5}$

EXERCICE 22

a) $f(x) = \dfrac{x^2+x-4}{x^2-1}$ $D_f = \mathbb{R} \setminus \{1, -1\}$

AV : $\lim\limits_{x\to-1} f(x) = \text{“}\dfrac{-4}{0}\text{”} = \left\langle \begin{array}{l}\lim\limits_{x\to-1^-} f(x) = -\infty \\ \lim\limits_{x\to-1^+} f(x) = +\infty\end{array}\right.$ AV $x = -1$

$$\begin{array}{c|ccccccc} x & -\infty & & -1 & & 1 & & +\infty \\ \hline x^2-1 & & + & 0 & - & 0 & + & \end{array}$$

$\lim\limits_{x\to 1} f(x) = "\frac{-2}{0}" \left< \begin{array}{l} \lim\limits_{x\to 1^-} f(x) = +\infty \\ \lim\limits_{x\to 1^+} f(x) = -\infty \end{array}\right.$ AV : $x = 1$

AH/AO : $\lim\limits_{x\to\pm\infty} f(x) = 1$ AH : $y = 1$

$x \to \pm\infty \to f(x) \xrightarrow[<]{>} 1$

b) $f(x) = \frac{4x^2+5}{x-2}$

$$\begin{array}{r|l} 4x^2 \quad +5 & x-2 \\ \hline -4x^2+8x & 4x+8 \\ \hline 8x+5 & \\ -8x+16 & \\ \hline 21 & \end{array}$$

AO: $y = 4x+8$

$x \to \pm\infty \Rightarrow f(x) \xrightarrow[<]{>} 4x+8$

AV: $D_f = \mathbb{R} \setminus \{2\}$

$\lim\limits_{x\to 2} f(x) = "\frac{21}{0}" = \left< \begin{array}{l} \lim\limits_{x\to 2^+} f(x) = +\infty \\ \lim\limits_{x\to 2^-} f(x) = -\infty \end{array}\right.$ AV; $x = 2$

c) $f(x) = \frac{8x^4-3x^2+1}{4x^3+10}$

$$\begin{array}{r|l} 8x^4 \quad -3x^2 \quad +1 & 4x^3+10 \\ \hline -8x^4 \qquad -20x & 2x \\ \hline -3x^2-20x+1 & \end{array}$$

AO: $y = 2x$

$x \to \pm\infty \quad f(x) \xrightarrow[>]{<} 2x$

AV: D_f : $4x^3+10 = 0 \Leftrightarrow 4x^3 = -10 \Leftrightarrow x^3 = -\frac{5}{2} \Rightarrow x = -\sqrt[3]{\frac{5}{2}}$

$\left.\begin{array}{l} \lim\limits_{x\to -\sqrt[3]{\frac{5}{2}}^-} f(x) = -\infty \\ \lim\limits_{x\to -\sqrt[3]{\frac{5}{2}}^+} f(x) = +\infty \end{array}\right\}$ AV: $x = -\sqrt[3]{\frac{5}{2}}$

EXERCICE 23

a) $f(x) = \frac{3}{x-1}$ $D_f = \mathbb{R} \setminus \{1\}$

AV: $\lim\limits_{x\to 1} f(x) = "\frac{3}{0}" = \left< \begin{array}{l} \lim\limits_{x\to 1^+} f(x) = +\infty \\ \lim\limits_{x\to 1^-} f(x) = -\infty \end{array}\right.$ AV ; $x = 1$

AO/AH: $\lim\limits_{x\to\pm\infty} f(x) = 0$ AH : $y = 0$

$x \to +\infty \quad f(x) \xrightarrow{>} 0 \qquad x \to -\infty \quad f(x) \xrightarrow{<} 0$

b) $f(x) = \frac{x^2-4x}{x^2-4x+3}$ $\quad x^2-4x+3=0 \Rightarrow x=3 \quad x=1$

$D_f = \mathbb{R}\setminus\{1;3\}$

AV: $\lim_{x\to3} f(x) = \text{"}\frac{-3}{0}\text{"} = \left< \begin{array}{l} \lim_{x\to3^-} f(x) = +\infty \\ \lim_{x\to3^+} f(x) = -\infty \end{array} \right.$ $\quad$ AV $x=3$

x	$-\infty$		1		3		$+\infty$
x^2-4x+3		+	0	−	0	+	

$\lim_{x\to1} f(x) = \text{"}\frac{-3}{0}\text{"} = \left< \begin{array}{l} \lim_{x\to1^-} f(x) = -\infty \\ \lim_{x\to1^+} f(x) = +\infty \end{array} \right.$ $\quad$ AV $x=1$

AH: $\lim_{x\to\pm\infty} f(x) = 1$ $\quad$ AH: $y=1$

$x\to+\infty \quad f \xrightarrow{\leq} 1 \qquad x\to-\infty \quad f(x) \xrightarrow{\leq} 1$

c) $f(x) = \frac{x^2}{x^2-4}$ $\quad x^2-4=0 \Leftrightarrow x=\pm2 \quad D_f = \mathbb{R}\setminus\{-2;2\}$

AV: $\lim_{x\to-2} f(x) = \text{"}\frac{4}{0}\text{"} \left< \begin{array}{l} \lim_{x\to-2^+} f(x) = -\infty \\ \lim_{x\to-2} + f(x) = +\infty \end{array} \right.$ $\quad$ AV; $x=-2$

x	$-\infty$		−2		2		$+\infty$
x^2-4		+	0	−	0	+	

$\lim_{x\to2} f(x) = \text{"}\frac{4}{0}\text{"} \left< \begin{array}{l} \lim_{x\to2^+} f(x) = +\infty \\ \lim_{x\to2^-} f(x) = -\infty \end{array} \right.$ $\quad$ AV: $x=2$

AH: $\lim_{x\to\pm\infty} f(x) = 1$ $\quad$ AH: $y=1$

$x\to+\infty \quad f(x) \xrightarrow{\geq} 1 \qquad x\to-\infty \quad f(x) \xrightarrow{\geq} 1$

d) $f(x) = \frac{2x-1}{x^2-x-2}$ $\quad x^2-x-2=0 \quad \Delta = 1+8=9 \quad x_{1,2} = \frac{1\pm3}{2} \left< \begin{array}{l} 2 \\ -1 \end{array} \right.$

$D_f = \mathbb{R}\setminus\{2;-1\}$

AV: $\lim_{x\to-1} f(x) = \text{"}\frac{-3}{0}\text{"} \left< \begin{array}{l} \lim_{x\to-1^-} f(x) = -\infty \\ \lim_{x\to-1^+} f(x) = +\infty \end{array} \right.$ $\quad$ AV $x=-1$

x	$-\infty$		−1		2		$+\infty$
x^2-x-2		+	0	−	0	+	

$\lim_{x\to2} f(x) = \text{"}\frac{3}{0}\text{"} = \left< \begin{array}{l} \lim_{x\to2^+} f(x) = +\infty \\ \lim_{x\to2^-} f(x) = -\infty \end{array} \right.$ $\quad$ AV: $x=2$

AH: $\lim_{x\to\pm\infty} f(x) = 0$ $\quad$ AH: $y=0$

$x\to+\infty \quad f(x) \xrightarrow{\geq} 0 \qquad x\to-\infty \quad f(x) \xrightarrow{\leq} 0$

e) $f(x) = \dfrac{x^2-2x+3}{x^2-3x+2}$ $x^2-3x+2=0$ $\Delta = 9-8=1$ $x_{1,2} = \dfrac{3\pm1}{2} = \begin{cases} 2 \\ 1 \end{cases}$

$D_f = \mathbb{R} \setminus \{1;2\}$

AV: $\lim\limits_{x\to1} f(x) = \text{"}\dfrac{2}{0}\text{"} \begin{cases} \lim\limits_{x\to1^+} f(x) = -\infty \\ \lim\limits_{x\to1^-} f(x) = +\infty \end{cases}$ AV: $x=1$

x	$-\infty$		1		2		$+\infty$
x^2-3x+2		$+$	0	$-$	0	$+$	

$\lim\limits_{x\to2} f(x) = \text{"}\dfrac{3}{0}\text{"} = \begin{cases} \lim\limits_{x\to2^+} f(x) = +\infty \\ \lim\limits_{x\to2^-} f(x) = -\infty \end{cases}$ AV: $x = 2$

AH: $\lim\limits_{x\to\pm\infty} f(x) = 1$ AH: $y=1$

$x\to+\infty$ $f(x) \xrightarrow{>} 1$ $x\to-\infty$ $f(x) \xrightarrow{<} 1$

f) $f(x) = \dfrac{x^3+2}{x^2+1}$ $D_f = \mathbb{R}$

AV: $\emptyset$

AO: $\begin{array}{l|l} x^3+1 & x^2+1 \\ \cline{2-2} \underline{-x^3-x} & x \\ -x+1 & \end{array}$ AO: $y=x$

$x\to+\infty$ $f(x) \xrightarrow{<} x$ $x\to-\infty$ $f(x) \xrightarrow{>} x$

g) $f(x) = 2x+1+\dfrac{3}{x}$ $D_f = \mathbb{R}\setminus\{0\}$

AV: $\lim\limits_{x\to0} f(x) = \text{"}\dfrac{3}{0}\text{"} \begin{cases} \lim\limits_{x\to0^+} f(x) = +\infty \\ \lim\limits_{x\to0^-} f(x) = -\infty \end{cases}$ AV: $x=0$

AO: $y = 2x+1$ $x\to+\infty$ $f(x) \xrightarrow{>} 2x+1$

$x\to-\infty$ $f(x) \xrightarrow{<} 2x+1$

h) $f(x) = \dfrac{x^3-3x^2+2}{x^2-3}$ $D_f = \mathbb{R}\setminus\{\pm\sqrt{3}\}$

AV: $\lim\limits_{x\to\sqrt{3}} f(x) = \text{"}\dfrac{-1.8}{0}\text{"} \begin{cases} \lim\limits_{x\to\sqrt{3}^+} f(x) = -\infty \\ \lim\limits_{x\to\sqrt{3}^-} f(x) = +\infty \end{cases}$ AV $x=\sqrt{3}$

x	$-\infty$		$-\sqrt{3}$		$\sqrt{3}$		$+\infty$
x^2-3		$+$	0	$-$	0	$+$	

$\lim\limits_{x\to-\sqrt{3}} f(x) = \text{"}\dfrac{-12.2}{0}\text{"} = \begin{cases} \lim\limits_{x\to-\sqrt{3}^+} f(x) = +\infty \\ \lim\limits_{x\to-\sqrt{3}^-} f(x) = -\infty \end{cases}$ AV: $x=-\sqrt{3}$

AO: $\begin{array}{l|l} x^3-3x^2+2 & x^2-3 \\ \cline{2-2} \underline{-x^3+3x} & x-3 \\ -3x^2+3x+2 & \\ \underline{+3x^2-9} & \\ 3x-7 & \end{array}$ AO: $y=x-3$

$x\to+\infty$ $f(x) \xrightarrow{>} x-3$

$x\to-\infty$ $f(x) \xrightarrow{<} x-3$

i) $f(x) = \frac{x^2-2x}{x-1}$ $\qquad D_f = \mathbb{R} \setminus \{1\}$

AV: $\lim\limits_{x \to 1} f(x) = \text{"}\frac{-1}{0}\text{"} < \begin{cases} \lim\limits_{x \to 1^+} f(x) = -\infty \\ \lim\limits_{x \to 1^-} f(x) = +\infty \end{cases}$ $\qquad$ AV : $x = 1$

AO: $\begin{array}{r|l} x^2 - 2x & x-1 \\ \cancel{-x^2} + x & x-1 \\ \hline -x & \\ +x-1 & \\ \hline -1 & \end{array}$

AO : $y = x - 1$

$x \to +\infty \quad f(x) \xrightarrow{<} x-1$

$x \to -\infty \quad f(x) \xrightarrow{>} x-1$

j) $f(x) = \tan(x) = \frac{\sin x}{\cos x}$ $\qquad \cos x = 0 \Leftrightarrow x = k\pi + \frac{\pi}{2} \quad k \in \mathbb{Z}$

$\lim\limits_{x \overset{<}{\underset{>}{\to}} \frac{\pi}{2}} f(x) = \pm\infty$ $\qquad$ AV: $x = \frac{\pi}{2}$

k) $f(x) = \frac{3x^2+x-4}{(x+1)^2}$ $\qquad D_f = \mathbb{R} \setminus \{-1\}$

AV: $\lim\limits_{x \to -1} f(x) = \text{"}\frac{-2}{0}\text{"} = +\infty$ $\qquad$ AV : $x = -1$

AH : $\lim\limits_{x \to \pm\infty} \frac{3x^2+x-4}{x^2+2x+1} = 3$ $\qquad$ AH $y = 3$

$x \to +\infty \quad f(x) \xrightarrow{<} 3 \qquad x \to -\infty \quad f(x) \xrightarrow{>} 3$

l) $f(x) = 5x - 3 + \frac{1}{\sqrt{x^2-1}}$ $\qquad x^2 - 1 > 0 \quad \begin{array}{ccccc} + & \phi & - & \phi & + \\ & -1 & & 1 & \end{array}$

$D_f =]-\infty; -1[\cup]1, +\infty[$

AV: $\lim\limits_{x \to -1} f(x) = +\infty$ $\qquad$ AV $x = -1$

$\lim\limits_{x \to 1} f(x) = +\infty$ $\qquad$ AV $x = 1$

AO : $y = 5x - 3$ $\qquad x \to +\infty \quad y \xrightarrow{>} 5x-3$

$x \to -\infty \quad y \xrightarrow{>} 5x-3$

EXERCICE 24

AO $y = 3x - 4$ $\qquad$ AV : $x = -2$

$f(x) = 3x - 4 + \frac{1}{x+2} = \frac{3x^2+2x-7}{x+2}$

EXERCICE 25

$f(x) = \frac{ax^2+bx+c}{x+d}$ $\qquad (2;2) \qquad$ AV : $x = -3$ $\qquad$ AO: $y = -2x+1$

- AV : $x = -3 \Rightarrow d = 3$
- $(2;2) \in$ graphe de f : $2 = \frac{4a+2b+c}{5} \Leftrightarrow 10 = 4a + 2b + c$
- AO: $y = -2x+1$

$\begin{array}{r|l} ax^2 + bx + c & x+3 \\ -ax^2 - 3ax & ax + b - 3a = -2x + 1 \\ \hline (b-3a)x + c & \\ -(b-3a)x - 3b + 9a & \\ \hline c - 3b + 9a & \end{array}$

$\underline{a = -2} \quad b - 3a = 1 \Rightarrow \underline{b = -5}$

$10 = -8 - 10 + c \Leftrightarrow \underline{c = 28}$

EXERCICE 26

▸ $f(x) = \dfrac{x^2-3x}{x-2}$ $D_f = \mathbb{R} \setminus \{2\}$

· $\cap Ox$: $f(x) = 0 \Leftrightarrow x^2 - 3x = 0 \Leftrightarrow x(x-3) = 0$ $x = 0$ $I_1(0;0)$
$x = 3$ $I_2(3;0)$

· $\cap Oy$ $x = 0$ $I_y(0;0)$

x	$-\infty$	0		2		3	$+\infty$
x^2-3x	+	0	−		−	0	+
$x-2$	−		−	0	+		+
f	−	0	+	‖	−	0	+

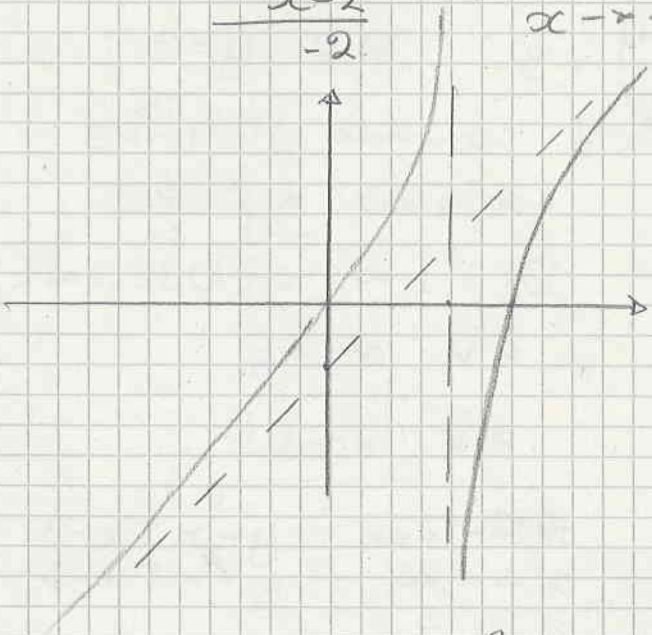
AV : $\lim\limits_{x \to 2} f(x) = \text{"}\dfrac{-2}{0}\text{"}$ $\left<\begin{array}{l}\lim\limits_{x\to 2^+} f(x) = +\infty \\ \lim\limits_{x\to 2^-} f(x) = -\infty\end{array}\right.$ $x = 2$

AO :
$$\begin{array}{r|l} x^2-3x & x-2 \\ \hline -x^2+2x & x-1 \\ -x & \\ x-2 & \\ -2 & \end{array}$$

AO : $y = x - 1$

$x \to +\infty$ $f(x) \xrightarrow{<} x-1$

$x \to -\infty$ $f(x) \xrightarrow{>} x-1$



▸ $g(x) = \frac{1}{2}x + 1 + \frac{3}{x} = \dfrac{x^2+2x+6}{2x}$ $D_g = \mathbb{R} \setminus \{0\}$

· $\cap Ox$: $g(x) = 0 \Leftrightarrow x^2 + 2x + 6 = 0$ $\Delta = 4 - 24 < 0$ $\emptyset$

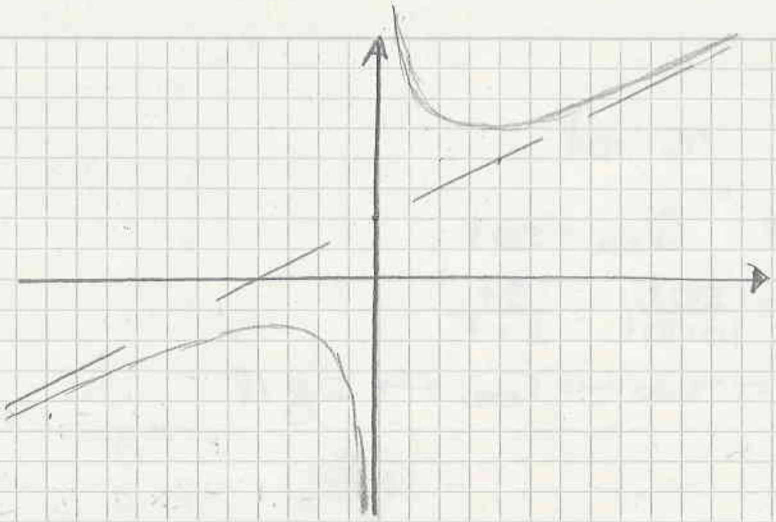
$\cap Oy$: $x = 0$ Impossible

x	$-\infty$	0	$+\infty$
f	−	‖	+

· AV : $\lim\limits_{x \to 0} f(x) = \text{"}\dfrac{6}{0}\text{"} = \left<\begin{array}{l}\lim\limits_{x\to 0^+} f(x) = +\infty \\ \lim\limits_{x\to 0} f(x) = -\infty\end{array}\right.$

AO : $y = \frac{1}{2}x + 1$ $x \to +\infty$ $f(x) \xrightarrow{>} \frac{1}{2}x + 1$

$x \to -\infty$ $f(x) \xrightarrow[<]{} \frac{1}{2}x + 1$



- $h(x) = \frac{3x^2+x-4}{(x+1)^2}$ $D_h = \mathbb{R} \setminus \{-1\}$

. $\cap O_x$: $3x^2+x-4=0$ $\Delta = 49$ $x_{1,2} = \frac{-1 \pm 7}{6} = \begin{cases} -\frac{4}{3} \\ 1 \end{cases}$

$(1;0)$ $(-\frac{4}{3};0)$

. $\cap O_y$: $x=0$ $f(0) = -4$ $(0;-4)$

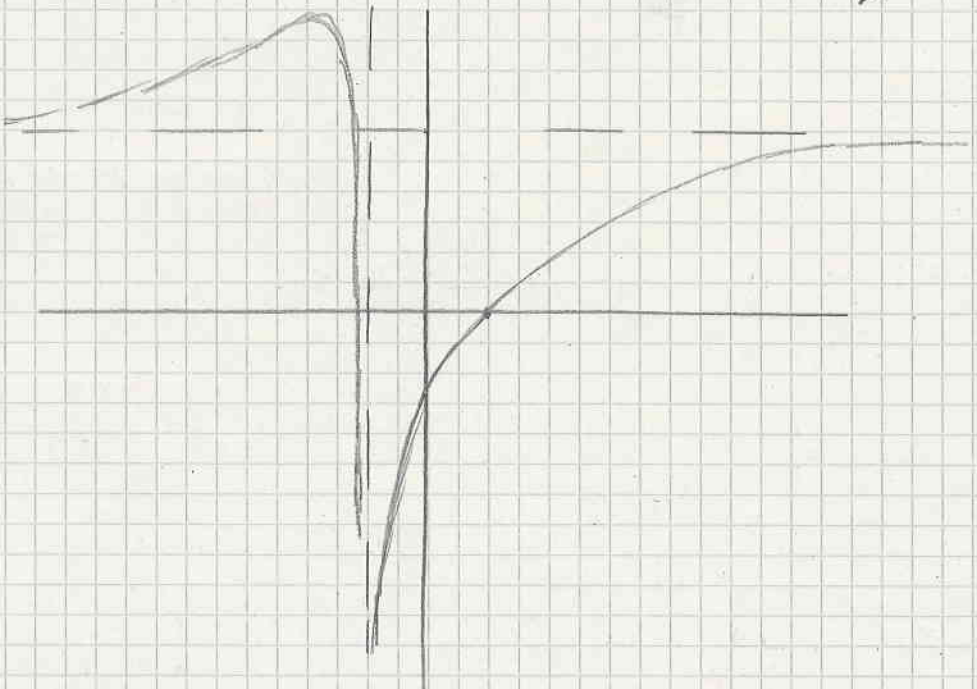
. TS:

x	$-\infty$		$-4/3$		-1		1		$+\infty$
$3x^2+x-4$		+	0	−		−	0	+	
$(x+1)^2$		+		+	0	+		+	
f		+	0	−	▨	−	0	+	

. AV $\lim_{x \to -1} f(x) = "\frac{-2}{0}" = -\infty$ AV $x=-1$.

AH : $y = 3$ $x \to +\infty$ $f(x) \overset{<}{\to} 3$

$x \to -\infty$ $f(x) \underset{>}{\to} 3$



EXERCICE 27

10l/h 5g/l $V_0 = 10l$

a) $V_t = 10 + 10t$ $Q_{sel} = 50t$

b) $C_{sel} = \frac{Q_s}{V_t} = \frac{50t}{10+10t} = \frac{5t}{1+t}$

Donc $t \to +\infty \Rightarrow C_{sel} \longrightarrow 5 g/l$