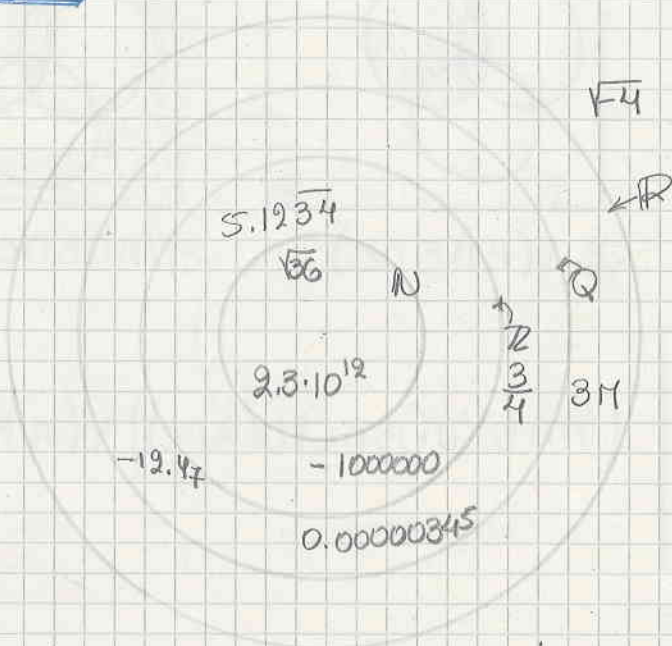


# LPDR-Niveau 1: Ensembles - Calcul Algébrique

## EXERCICE 1



## EXERCICE 2

$$A = \{x \in \mathbb{N} : x < 5\} = \{0; 1; 2; 3; 4\}$$

$$B = \{x \in \mathbb{Z} : x^2 \leq 3\} = \{-1; 0; 1\}$$

$$C = \{x^2 : x \in \mathbb{N} \text{ et } 2 < x < 6\} = \{9; 16; 25\}$$

$$D = \{3x : x \in \mathbb{Z}\} = \{\dots; -9; -6; -3; 0; 3; 6; 9; \dots\}$$

$$E = \left\{ \frac{1}{x} : x \in \mathbb{N}^* \right\} = \left\{ 1; \frac{1}{2}; \frac{1}{3}; \dots \right\}$$

## EXERCICE 3

$$A = \{0; 2; 4; 6; 8; \dots\} = \{2n : n \in \mathbb{N}\}$$

$$B = \{8; 12; 16; \dots; 80\} = \{4n : n \in \mathbb{N} \text{ et } 2 \leq n \leq 20\}$$

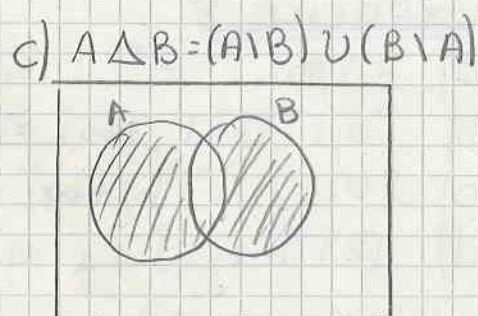
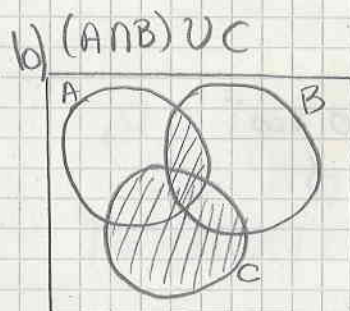
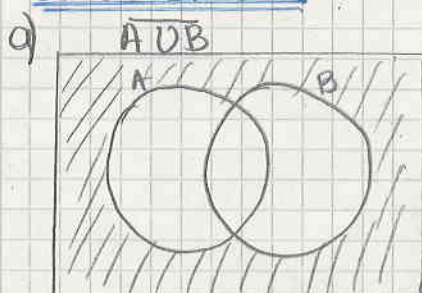
$$C = \{\dots; -3; -2; -1; 1; 2; 3; \dots\} = \mathbb{Z}^*$$

$$D = \{1; 8; 27; 64; 125; \dots\} = \{n^3 : n \in \mathbb{N}^*\}$$

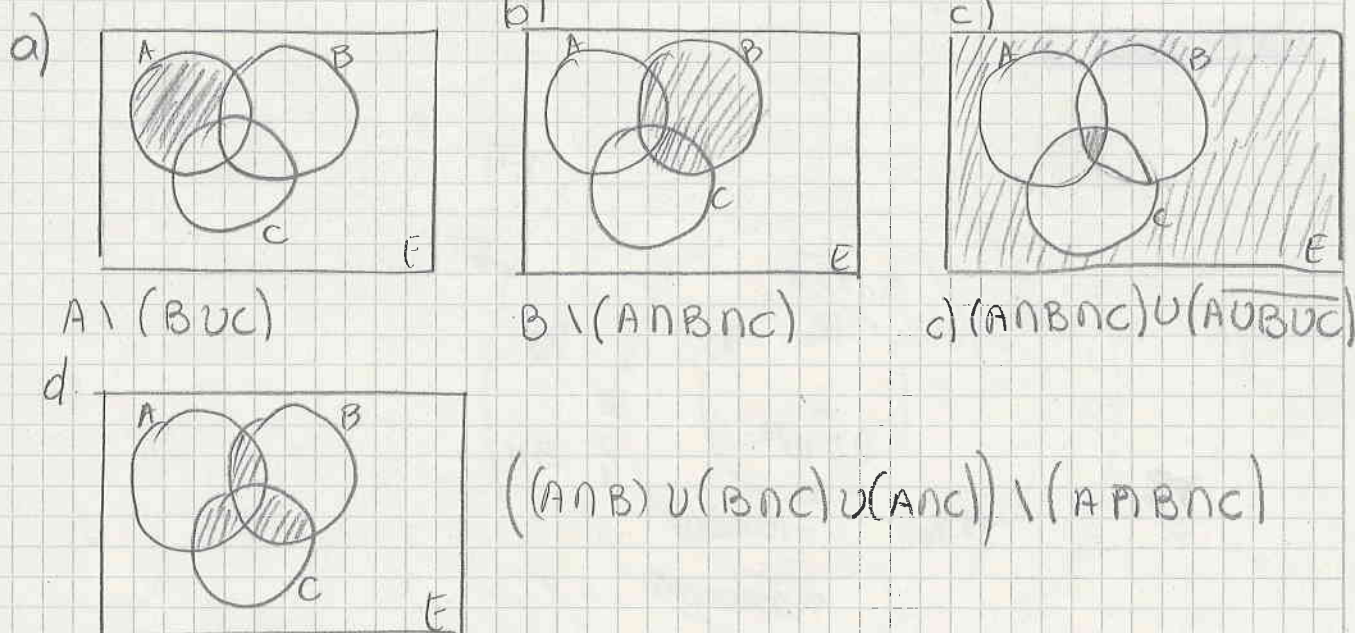
$$E = \left\{ 0; \frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5}; \dots \right\} = \left\{ \frac{n}{n+1} : n \in \mathbb{N} \right\}$$

$$F = \{-1; 4; -9; 16; -25; \dots\} = \{(-1)^n \cdot n^2 : n \in \mathbb{N}^*\}$$

## EXERCICE 4



## EXERCICE 5



## EXERCICE 6

- a)  $\mathbb{Q} \subset \mathbb{R}$  vraie  
 b)  $\mathbb{N} \in \mathbb{Z}$  vraie  
 c)  $[0; 1] \subset \mathbb{Q}$  fausse  
 d)  $a \in \{a, b\}$  vraie

## EXERCICE 7

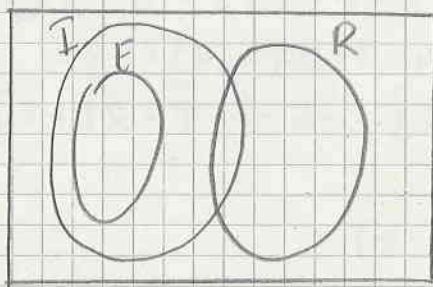
- a)  $A = \{x : -4 \leq x < 9\} = [-4; 9[$   
 b)  $B = \{x : x+1 > 0\} = \{x : x > -1\} = ]-1; +\infty[$   
 c)  $C = A \cap B = ]-1; 9[$   
 d)  $D = \{x : x^2 < 16\} = ]-4; 4[$   
 e)  $E = \{x : \sqrt{x-1} \text{ existe dans } \mathbb{R}\} = \{x : x \geq 1\} = [1; +\infty[$   
 f)  $F = \{x : |x| \geq 8\} = ]-\infty; -8] \cup [8; +\infty[$   
 g)  $G = \mathbb{R} = ]-\infty; +\infty[$   
 h)  $H = \{n\} = [n; n]$   
 i)  $I = \{x : x \geq -3 \text{ et } x \leq 3\} = [-3; 3]$

## EXERCICE 8

- a)  $\mathbb{Z} \cap \mathbb{N} = \mathbb{Z}_+$   
 b)  $[0; 1] \cup ]1; +\infty[ = [0; +\infty[ = \mathbb{R}_+$   
 c)  $[0; 1] \cap ]1; +\infty[ = \emptyset$   
 d)  $\mathbb{R} \cap [-1; 1] = [-1; 1]$

EXERCICE 9

I : les triangles isocèles  
 E : les triangles équilatéraux  
 R : les triangles rectangles



$$I \cap E = E$$

$$E \cap R = \emptyset$$

$I \cap R$  : les triangles isocèles et rectangles

EXERCICE 10

- a)  $3 \cdot 4 + 5 \cdot 12 - 6 \cdot 2 = 12 - 60 + 12 = -60$   
 b)  $8 \cdot 6 - 4 : 4 + 1 \cdot 2 = 48 - 1 + 2 = 49$   
 c)  $25 - 10 : 5 + 5 - 1 \cdot 5 \cdot 12 - 6 \cdot 2 = 25 - 2 + 5 - 60 = 12 = -44$   
 d)  $150 - (100 + 50) : 5 = 150 - 150 : 5 = 150 - 30 = 120$   
 e)  $\{100 - [50 - (40 - 9)]\} \cdot 2 = \{100 - [50 - 31]\} \cdot 2 = (100 - 19) \cdot 2 = 162$   
 f)  $(3 \cdot 4 + 5) \cdot 12 - 6 \cdot 2 = (17) \cdot 12 - 12 = 16 \cdot 12 = 192$   
 g)  $(-6 + 3 \cdot 2) : [4 + 2 \cdot (-2)] = \text{impossible}$   
 h)  $(-5 + 5) : (3 - 2) = 0$   
 i)  $[-3 - (-2)] : (-13 - (-13)) = \text{impossible}$   
 j)  $\{[-1 - (-2)] : (-1)\} \cdot (-2) = \{(-1 + 2) : (-1)\} \cdot (-2) = (-1) \cdot (-2) = 2$

EXERCICE 11

- a)  $3 \cdot (3 + 4) - 2 \cdot (6 - 3) = 3 \cdot 7 - 2 \cdot 3 = 21 - 6 = 15$   
 b)  $150 - (100 - 50) : 5 = 150 - 50 : 5 = 150 - 10 = 140$   
 c)  $20 - (3 + (10 - 7) \cdot 3) = 20 - (3 + 9) = 20 - 12 = 8$   
 d)  $-3 \cdot (-5) + (-2) = 15 - 2 = 13$   
 e)  $2 \cdot (-3) - 5(-2) = -6 + 10 = 4$   
 f)  $(-6 + 3 \cdot 2) : (7 + 2(-2)) = 0$   
 g)  $-6 + 4 : (-2) + 2 = -6 - 2 + 2 = -6$   
 h)  $(-[-1 + (-2)] : (-3)) \cdot (-2) = (+3 : (-3)) \cdot (-2) = 2$   
 i)  $3 - (-2) - (-1) \cdot (-4) = 3 + 2 - 4 = 1$   
 j)  $(18 - (21 - (24 - 27))) \cdot 2 = (18 - (21 + 3)) \cdot 2 = (18 - 24) \cdot 2 = -12$

EXERCISE 12

$$a) -\{6 - [43 - (17 + 18) - 4] - (-22 + 2)\} - (6 - 4 - 2) \\ = -[6 - (43 - 35 - 4)] - (-20) - 0 = -((6 - 4) + 20) = -22$$

$$b) -19 - [-13 - (-21)(-2) + 1] - [(2)(-3) - (-9)] = \\ = -19 - (-13 - 42 + 1) - (6 + 9) = -19 + 54 - 15 = 20$$

$$c) -5 + 2 - 3 - \{-1 - 4 - [2 - 14 - (-4 - 7 + 8) - 19] + 7\} = \\ = -6 - \{-5 - [-12 - (-3) - 19] + 7\} = -6 - \{-5 - 21 + 7\} = - \\ -6 + 19 = 13$$

$$d) \{-12 - [-24 - (-15) - (126)] - 3\}(-2) = \\ = \{-12 - [-24 + 15 - 126] - 3\}(-2) = \\ = \{-12 + 135 - 3\}(-2) = 120(-2) = -140$$

EXERCISE 13

$$a) \frac{65}{180} : \frac{50}{13} = \frac{65}{39} = \frac{5}{3}$$

$$b) \frac{11}{6} : \frac{77}{36} = \frac{11}{6} \cdot \frac{36}{77} = \frac{6}{7}$$

$$c) \frac{4}{9} + \frac{9}{18} + \frac{5}{54} = \frac{24 + 27 + 5}{54} = \frac{56}{54} = \frac{28}{27}$$

$$d) 2 + \frac{9}{14} + \frac{5}{6} + \frac{4}{7} = \frac{84 + 27 + 35 + 24}{42} = \frac{170}{42} = \frac{85}{21}$$

EXERCISE 14

$$a) (3+7) \cdot 3+7 = 37$$

$$b) 3 + (7-3) \cdot 7 = 31$$

EXERCISE 15

$$\begin{array}{r|l} 128 & 2 \\ 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

$$\begin{array}{r|l} 168 & 2 \\ 84 & 2 \\ 42 & 2 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array}$$

$$128 = 2^7$$

$$168 = 2^3 \cdot 3 \cdot 7$$

$$\text{ppmc}(128, 168) = 2^7 \cdot 3 \cdot 7 = 2688$$

$$\text{pgdc}(128, 168) = 2^3 = 8$$

EXERCISE 16

$$a) \begin{array}{r|l} 168 & 128 \\ -128 & 1 \\ \hline 40 & \end{array} \rightarrow \begin{array}{r|l} 128 & 40 \\ -120 & 3 \\ \hline 8 & \end{array} \rightarrow \begin{array}{r|l} 40 & 8 \\ -40 & 5 \\ \hline 0 & \end{array} \rightarrow \text{pgdc}(128, 168) = 8$$

$$b) \frac{168}{128} = \frac{21}{16}$$

EXERCICE 17

a)  $n = 0,2828\dots$

b)  $n = 2,5281$

(-)  $100n = 28,2828\dots$

-  $10000n = 25281,5281$

$99n = 28 \Leftrightarrow n = \frac{28}{99}$

$9999n = 25279$   
 $\Leftrightarrow n = \frac{25279}{9999}$

c)  $n = 4,67999$

(-)  $\begin{cases} 100n = 467,999 \\ 1000n = 4679,999 \end{cases}$

$999n = 4212 \Leftrightarrow n = \frac{4212}{999} = \frac{156}{37}$

EXERCICE 18

$n+8$  est divisible par 8  $\Leftrightarrow 8$  est divisible par  $n$   
 $\Leftrightarrow n \in \{1; 2; 4; 8\}$

EXERCICE 19

a)  $\frac{5}{7} : \frac{2}{7} = \frac{5}{2}$

b)  $\frac{2}{3} + \frac{1}{9} = \frac{5}{9}$

c)  $2 : \frac{1}{5} = 10$

d)  $2 + \frac{1}{5} = \frac{11}{5}$

e)  $2 - \frac{1}{5} = \frac{9}{5}$

EXERCICE 20

a)  $\frac{1}{48} \cdot \frac{2}{5} = \frac{1}{20}$

b)  $\frac{1}{2} \cdot \frac{9}{2} = \frac{9}{4}$

EXERCICE 21

a)  $-\frac{-1}{-4} \cdot 4 = -1$

b)  $\frac{3}{2} \cdot \frac{4}{155} = \frac{2}{5}$

c)  $\frac{2}{5} + \frac{3}{4} - \frac{3}{2} = \frac{8+15-30}{20} = \frac{-7}{20}$

d)  $\left(\frac{14}{6} - \frac{2}{9}\right) : \frac{38}{9} = \left(\frac{7}{3} - \frac{2}{9}\right) \cdot \frac{9}{38} = \frac{21-2}{38} \cdot \frac{9}{38} = \frac{19}{38} = \frac{1}{2}$

e)  $\frac{\frac{1}{2} + \frac{1}{4}}{1 - \frac{2}{5}} = \frac{\frac{2+1}{4}}{\frac{5-2}{5}} = \frac{3}{4} \cdot \frac{5}{3} = \frac{5}{4}$

f)  $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}} = 1 + \frac{1}{1 + \frac{1}{\frac{3}{2}}} = 1 + \frac{1}{1 + \frac{2}{3}} = 1 + \frac{1}{\frac{5}{3}} = 1 + \frac{3}{5} = \frac{8}{5}$

EXERCICE 22

$$a) \frac{4}{5} : \frac{5}{6} + 1 + \frac{2}{3} : \frac{1}{5} = \frac{4}{5} \cdot \frac{6}{5} + 1 + \frac{2}{3} \cdot \frac{5}{1} = \frac{24}{25} + 1 + \frac{10}{3} =$$

$$= \frac{72 + 75 + 250}{75} = \frac{397}{75}$$

$$b) \left( \frac{1}{21} - \frac{5}{7} \right) : \frac{3}{14} = \left( \frac{1}{21} - \frac{15}{21} \right) \cdot \frac{14}{3} = -\frac{14}{21} \cdot \frac{14}{3} = -\frac{28}{9}$$

$$c) \frac{1}{2} - \left\{ \frac{2}{3} - \left[ \frac{3}{4} + \left( \frac{4}{5} - \frac{5}{6} \right) \right] \right\} = \frac{1}{2} - \left\{ \frac{2}{3} - \left[ \frac{3}{4} + \left[ \frac{24-25}{30} \right] \right] \right\}$$

$$= \frac{1}{2} - \left\{ \frac{2}{3} - \left[ \frac{3}{4} - \frac{1}{30} \right] \right\} = \frac{1}{2} - \left\{ \frac{2}{3} - \left[ \frac{45-2}{60} \right] \right\} = \frac{1}{2} - \left\{ \frac{2}{3} - \frac{43}{60} \right\} =$$

$$= \frac{1}{2} - \left[ \frac{40-43}{60} \right] = \frac{1}{2} + \frac{3}{60} = \frac{1}{2} + \frac{1}{20} = \frac{10+1}{20} = \frac{11}{20}$$

$$d) \frac{1}{2} + \left\{ \frac{2}{3} + \left[ \frac{3}{4} - \left( \frac{4}{5} + \frac{5}{6} \right) \right] \right\} = \frac{1}{2} + \left\{ \frac{2}{3} + \left[ \frac{3}{4} - \left( \frac{24+25}{30} \right) \right] \right\}$$

$$= \frac{1}{2} + \left\{ \frac{2}{3} + \left[ \frac{3}{4} - \frac{49}{30} \right] \right\} = \frac{1}{2} + \left\{ \frac{2}{3} + \frac{45-98}{60} \right\} = \frac{1}{2} + \frac{2}{3} - \frac{53}{60} =$$

$$= \frac{30+40-53}{60} = \frac{17}{60}$$

$$e) \frac{1}{2} - \left\{ \frac{1}{3} - \left[ \frac{2}{3} - \left( \frac{1}{6} - \frac{1}{2} \right) \right] \right\} = \frac{1}{2} - \left\{ \frac{1}{3} - \left[ \frac{2}{3} - \left( -\frac{1}{3} \right) \right] \right\} =$$

$$= \frac{1}{2} - \left\{ \frac{1}{3} - \left[ \frac{2}{3} + \frac{1}{3} \right] \right\} = \frac{1}{2} - \left\{ \frac{1}{3} - \frac{3}{3} \right\} = \frac{1}{2} - \left( -\frac{2}{3} \right) = \frac{1}{2} + \frac{2}{3} =$$

$$= \frac{3+4}{6} = \frac{7}{6}$$

$$f) \frac{3 + \frac{1}{5}}{3 - \frac{2}{3} - \frac{1}{5}} = \frac{\frac{15}{5}}{\frac{45-10+3}{15}} = \frac{\frac{16}{5}}{\frac{38}{15}} = \frac{16}{5} \cdot \frac{15}{38} = \frac{24}{19}$$

$$g) \frac{1 - \left[ \frac{7}{6} - \left( \frac{1}{3} + \frac{1}{6} \right) \right]}{1 - \left[ \frac{7}{6} - \left( \frac{1}{3} - \frac{1}{6} \right) \right]} = \frac{1 - \left[ \frac{7}{6} - \frac{3}{6} \right]}{1 - \left[ \frac{7}{6} - \frac{1}{6} \right]} = \frac{1 - \frac{4}{6}}{1 - \frac{6}{6}} = \frac{1 - \frac{2}{3}}{0} \text{ impos}$$

$$h) 2 + \frac{2}{2 + \frac{2}{2 - \frac{1}{2}}} = 2 + \frac{2}{2 + \frac{2}{\frac{3}{2}}} = 2 + \frac{2}{2 + \frac{4}{3}} = 2 + \frac{2 \cdot 3}{10} =$$

$$= 2 + \frac{3}{5} = \frac{13}{5}$$