

LDDR - Niveau 1 : Calcul - Littéral

EXERCICE 23

a) $a^2 \cdot a^3 \cdot a^5 = a^9$

b) $x^{-2} \cdot x^{12} = x^{-15} = \frac{1}{x^{15}}$

c) $\frac{z^{-7}}{z^{-4}} = z^{-3} = \frac{1}{z^3}$

d) $\frac{a^3}{a^{-9}} = a^{12}$

e) $y^{2-4} \cdot y^{6-2} = y^2$

f) $\frac{2^{3n}}{2^{2n}} = \frac{2^n}{2} = 2^{n-1}$

EXERCICE 24

a) $(-x)^3 \cdot x^5 = -x^3 x^5 = -x^8$

b) $\frac{1}{3^5} = 3^{-5}$

c) $2^n + 2^n = 2 \cdot 2^n = 2^{n+1}$

d) $\frac{a^{n+1}}{a^{n-1}} = a^2$

e) $\frac{9y^5 \cdot (y^2)^3}{(3y)^2 \cdot y^9} = \frac{9y^5 y^6}{9y^2 y^9} = \frac{y^{11}}{y^{11}} = 1$

f) $\frac{(-x)^{2n}}{x} = \frac{x^{2n}}{x} = x^{2n-1}$

g) $\frac{1}{x^{-5}} = x^5$

h) $\frac{(x^3 y)^2 \cdot y^4}{(x^2 y)^3} : y = \frac{x^6 y^2 \cdot y^4}{x^6 y^3} \cdot \frac{1}{y} = \frac{x^6 y^6}{x^6 y^4} = y^2$

i) $x^{5-n} \cdot \frac{x^{n-2}}{x^3} = x^{5-n+n-2-3} = x^0 = 1$

j) $\frac{a^4 (a^2)^{-1}}{(a^2 \cdot a^{-3})^{-1}} = \frac{a^4 \cdot a^{-2}}{a^{-2} a^3} = \frac{a^2}{a} = a$

k) $(z^{-3})^{-n} = z^{3n}$

l) $\frac{x^{-11}}{x^{-12}} = x$

m) $\frac{y^{-n}}{y^{-(n+1)}} = y$

EXERCICE 25

a) $(-3x^2)^3 = -27x^6$

b) $((-2xy^2)^4)^3 = (-2xy^2)^{12} = 2^{12} x^{12} y^{24}$

c) $(t^{x-4})^2 \cdot (t^{6-x})^2 = t^{2x-8+12-2x} = t^4$

d) $\frac{54a^3 x^5 y^2}{18a^{-4} x y^8} = 3a^7 x^4 y^{-6}$

e) $\left(\frac{x^3}{y^4}\right)^6 \cdot \left(\frac{x^2 b^{-3}}{y^{-1}}\right)^{-3} = \frac{x^{18}}{y^{24}} \cdot \frac{x^{-6} b^9}{y^3} = \frac{x^{12} b^9}{y^{27}} = x^{12} b^9 y^{-27}$

f) $\frac{(a^2 b)^3 \cdot a^4}{(a \cdot b^3)^3} = \frac{a^6 b^3 a^4}{a^3 b^9} = a^7 b^{-6}$

g) $\frac{a^{-2} b^{-10}}{a^7 b^{-8}} \cdot \frac{a^4 b^{-3}}{(a^{-3} b)^2} = \frac{a^{-2} b^{-10} a^4 b^{-3}}{a^7 b^{-8} a^{-6} b^{-2}} = a^{-11} b^{-3}$

$$h) (x+a)^2 : (x+a)^{-3} \cdot (x+a)^4 = \frac{(x+a)^2 \cdot (x+a)^4}{(x+a)^3} = (x+a)^3$$

$$i) \frac{(x^4 y^3)^n}{(x^9 y^4)^{2n-1}} = \frac{x^{4n} y^{3n}}{x^{4n-2} y^{8n-4}} = x^2 y^{-5n+4}$$

$$j) \frac{(27a^6b^3)^{n+1}}{(3a^2b)^{3n+3}} = \frac{3^{3n+3} a^{6n+6} b^{3n+3}}{3^{3n+3} a^{6n+6} b^{3n+3}} = 1$$

$$k) \left(\frac{a^{-3}}{a^{-4}} \right)^2 \cdot \left(\frac{a^3}{a^4} \right)^{-2} = \frac{a^{-6}}{a^{-8}} \cdot \frac{a^{-6}}{a^{-8}} = a^4$$

$$l) \frac{(8a^7b^4)^3}{(4a^5b^2)^4} = \frac{2^6 a^{21} b^{12}}{2^8 a^{20} b^8} = 2^{-2} a b^4$$

EXERCISE 26

$$a) \sqrt[4]{81} = 3 \quad b) \sqrt[3]{1000} = 10 \quad c) \sqrt[3]{0.001} = 0.1 \quad d) \sqrt[4]{13^4} = 13$$

EXERCISE 27

$$\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2} \quad \sqrt{1875} = \sqrt{75 \cdot 25} = \sqrt{3 \cdot 25 \cdot 25} = 25\sqrt{3}$$

$$\sqrt{200} = \sqrt{2 \cdot 100} = 10\sqrt{2} \quad \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

EXERCISE 28

$$a) \sqrt{800} = \sqrt{4 \cdot 2 \cdot 100} = 2 \cdot 10\sqrt{2} = 20\sqrt{2}$$

$$b) \sqrt{27} \cdot \sqrt{3} = \sqrt{3 \cdot 9} \cdot \sqrt{3} = 3\sqrt{3} = 9$$

$$c) \frac{2}{\sqrt{20}} = \frac{2\sqrt{20}}{20} = \frac{2 \cdot 2\sqrt{5}}{20} = \frac{2\sqrt{5}}{10} = \frac{\sqrt{5}}{5}$$

$$d) \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

$$e) \frac{1}{3-\sqrt{2}} = \frac{3+\sqrt{2}}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{3+\sqrt{2}}{9-2} = \frac{3+\sqrt{2}}{7}$$

$$f) \frac{\sqrt{7}}{\sqrt{7}-\sqrt{5}} = \frac{\sqrt{7}(\sqrt{7}+\sqrt{5})}{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})} = \frac{\sqrt{7}^2 + \sqrt{35}}{7-5} = \frac{7+\sqrt{35}}{2}$$

$$g) \frac{(\sqrt{5}-1)^2}{\sqrt{5}-3} = \frac{(5-2\sqrt{5}+1)(\sqrt{5}+3)}{(\sqrt{5}-3)(\sqrt{5}+3)} = \frac{(6-2\sqrt{5})(\sqrt{5}+3)}{5-9} =$$

$$= \frac{2(3-\sqrt{5})(3+\sqrt{5})}{-4} = \frac{2(9-5)}{-4} = \frac{2 \cdot 4}{-4} = -2$$

EXERCICE 29

a) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

b) $\frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

c) $\frac{\sqrt{8}}{2\sqrt{3}} = \frac{2\sqrt{2} \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{6}}{3}$

d) $\frac{\sqrt{5}}{3\sqrt{15}} = \frac{\sqrt{5}}{3\sqrt{3}\sqrt{5}} = \frac{\sqrt{3}}{9}$

e) $\frac{\sqrt{12}}{3\sqrt{3}} = \frac{\sqrt{3} \cdot 2}{3\sqrt{3}} = \frac{2}{3}$

f) $\frac{\sqrt{27}}{\sqrt{15}} = \frac{3\sqrt{3}}{\sqrt{3}\sqrt{5}} = \frac{3\sqrt{5}}{5}$

EXERCICE 30

a) $\frac{4}{3-\sqrt{3}} = \frac{4(3+\sqrt{3})}{9-3} = \frac{4(3+\sqrt{3})}{6} = \frac{2(3+\sqrt{3})}{3}$

b) $\frac{\sqrt{7}}{6+\sqrt{7}} = \frac{\sqrt{7}(6-\sqrt{7})}{36-7} = \frac{6\sqrt{7}-7}{29}$

c) $\frac{18}{4-\sqrt{7}} = \frac{18(4+\sqrt{7})}{16-7} = \frac{18(4+\sqrt{7})}{9} = 2(4+\sqrt{7})$

d) $\frac{-\sqrt{2}}{\sqrt{2}-\sqrt{5}} = \frac{-\sqrt{2}(\sqrt{2}+\sqrt{5})}{2-5} = \frac{-2-\sqrt{10}}{-3} = \frac{2+\sqrt{10}}{3}$

e) $\frac{\sqrt{6}}{-\sqrt{3}-\sqrt{2}} = \frac{\sqrt{6}(-\sqrt{3}+\sqrt{2})}{3-2} = \frac{-\sqrt{18}+\sqrt{12}}{1} = -3\sqrt{2}+2\sqrt{3}$

f) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}-\sqrt{3})^2}{5-3} = \frac{5-2\sqrt{15}+3}{2} = \frac{8-2\sqrt{15}}{2} = 4-\sqrt{15}$

g) $\frac{7-5\sqrt{15}}{3-2\sqrt{15}} = \frac{(7-5\sqrt{15})(3+2\sqrt{15})}{9-4 \cdot 15} = \frac{21-15\sqrt{15}+14\sqrt{15}-10 \cdot 15}{-51} = \frac{-129-\sqrt{15}}{-51} = \frac{129+\sqrt{15}}{51}$

h) $\frac{7\sqrt{5}-5\sqrt{7}}{\sqrt{7}+\sqrt{5}} = \frac{(7\sqrt{5}-5\sqrt{7})(\sqrt{7}-\sqrt{5})}{7-5} = \frac{7\sqrt{35}-5 \cdot 7-7 \cdot 5+5\sqrt{35}}{2} = \frac{12\sqrt{35}-70}{2} = 6\sqrt{35}-35$

EXERCICE 31

a) $25^{1/2} = \sqrt{25} = 5$

b) $9^{3/2} = \sqrt{9^3} = 3^3 = 27$

c) $1024^{0.1} = 1024^{1/10} = \sqrt[10]{1024} = 2$

d) $(\frac{1}{9})^{-1/2} = 9^{1/2} = \sqrt{9} = 3$

f) $9^{3/4} 9^{-1/4} = \frac{9^3}{\sqrt[4]{9}} = \sqrt[4]{9^3} = \sqrt[4]{9^2} = 3$

EXERCICE 32

a) $5^{1/2} = \sqrt{5}$

b) $2^{4/5} = \sqrt[5]{2^4}$

c) $7^{3/40} = \sqrt[40]{7^3}$

d) $1024^{1/10} = \sqrt[10]{1024} = 2$

e) $0^{1/5} = 0$

f) $36^{3/2} = \sqrt{36^3} = 6^3$

g) $25^{0.5} = \sqrt{25} = 5$

h) $(\frac{1}{9})^{1/2} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

i) $(\frac{27}{9})^{2/3} = \sqrt[3]{3^2} = \sqrt[3]{9}$

j) $2^{-1/2} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

k) $7^{-1/3} = \frac{1}{\sqrt[3]{7}}$

l) $4^{-5/3} = \sqrt[3]{\frac{1}{4^5}} = \frac{1}{\sqrt[3]{4^5}}$

EXERCISE 33

$$\begin{aligned}
 a) \sqrt[4]{256} &= \sqrt[4]{5^4} = 5 & b) \sqrt{24} &= 2^{\frac{1}{2}} = 2^{\frac{2}{4}} = 4 & c) \sqrt[4]{4^2} &= \sqrt[4]{2^4} = 2 \\
 d) \sqrt[3]{7^8} &= 7^{\frac{8}{3}} & e) \sqrt[5]{2^{15}} &= 2^{\frac{15}{5}} = 2^3 = 8 & f) \frac{\sqrt{7}}{\sqrt[3]{7}} &= 7^{\frac{1}{2} - \frac{1}{3}} = 7^{\frac{1}{6}} \\
 g) \sqrt[6]{\sqrt[4]{2^{10}}} &= \sqrt[6]{2^{\frac{10}{4}}} = \sqrt[6]{2^{\frac{5}{2}}} = 2^{\frac{5}{2} \cdot \frac{1}{6}} = 2^{\frac{5}{12}}
 \end{aligned}$$

EXERCISE 34

$$\begin{aligned}
 a) \sqrt[4]{27} &= 27^{\frac{1}{4}} & b) \sqrt[3]{a^n} &= a^{\frac{n}{3}} & c) \frac{1}{\sqrt[5]{x^7}} &= x^{-\frac{7}{5}} \\
 d) \frac{1}{\sqrt[3]{27}} &= \frac{1}{3\sqrt{3}} = \frac{3^{-\frac{1}{2}}}{3} & e) \sqrt[4]{b^8} &= b^{\frac{8}{4}} = b^2 & f) \frac{1}{\sqrt[3]{a^{2n+1}}} &= a^{-\frac{2n+1}{3}} \\
 g) (\sqrt[5]{a^6})^{15} &= a^{\frac{6 \cdot 15}{5}} = a^{6 \cdot 3} = a^{18} \\
 h) (\sqrt{x^4})^{2n} &= x^{\frac{2n}{1}} = x^{2n} \\
 i) \sqrt[2n-1]{a^{4n-2}} &= a^{\frac{4n-2}{2n-1}} = a^2 \\
 j) (\sqrt[n]{a^p})^{nm} &= a^{\frac{p \cdot nm}{n}} = a^{pm}
 \end{aligned}$$

EXERCISE 35

$$\begin{aligned}
 a) \sqrt[4]{a^8} &= a^{\frac{8}{4}} = a^2 & b) \sqrt[n]{a^{3n}} &= a^{\frac{3n}{n}} = a^3 & c) \sqrt[2n]{x^n} &= x^{\frac{n}{2n}} = x^{\frac{1}{2}} \\
 d) (\sqrt[3]{x^2})^6 &= x^{\frac{12}{3}} = x^4 & e) (\sqrt[3]{a^n})^3 &= a^{\frac{3n}{3}} = a^n \\
 f) (\sqrt{a})^{2n} &= a^{\frac{2n}{2}} = a^n & g) \frac{\sqrt[6]{x^5}}{\sqrt{x} \sqrt[3]{x}} &= x^{\frac{5}{6} - \frac{1}{2} - \frac{1}{3}} = \\
 &= x^{\frac{5-3-2}{6}} = x^0 = 1 \\
 h) \frac{\sqrt[3]{a^4}}{\sqrt{a}} &= a^{\frac{4}{3} - \frac{1}{2}} = a^{\frac{8-3}{6}} = a^{\frac{5}{6}} \\
 i) \frac{\sqrt{9xy^2}}{3x^{\frac{1}{2}}\sqrt[5]{y^5}} &= \frac{3x^{\frac{1}{2}}y}{3x^{\frac{1}{2}}} \cdot y^{\frac{5}{5}} = y^1 = y
 \end{aligned}$$

EXERCISE 36

$$\begin{aligned}
 12^{100} \cdot 6^{-149} \cdot (\sqrt{1.5})^{100} &= \frac{12^{100}}{6^{149}} \cdot \left(\frac{3}{2}\right)^{50} = \frac{6^{100} \cdot 2^{100} \cdot 3^{50}}{6^{149} \cdot 2^{50}} \\
 &= \frac{2^{50} \cdot 3^{50}}{6^{49}} = \frac{6^{50}}{6^{49}} = 6
 \end{aligned}$$

EXERCICE 37

a) $3/x - 2/x + 5x - 2/x + 4/x - 3/x = 5x$

b) $5b^9 - b^9 - 4b^9 = 0$

c) $2ab - 3ab + 5ba - 4ab + ba = -9ab$

d) $(2b)^3 - b^3 - 2b^3 = 8b^3 - 3b^3 = 5b^3$

EXERCICE 38

a) $P(x) = 2x^3 + 1 \quad Q(x) = -2x^3 + 3x^2 - 7$

$$P(x) + Q(x) = 3x^2 - 6$$

$$P(x) - Q(x) = 4x^3 - 3x^2 + 8$$

$$P(x) \cdot Q(x) = -4x^6 + 6x^5 - 16x^3 + 3x^2 - 7$$

b) $P(x) = x^2 + x + 1 \quad Q(x) = x - 1$

$$P(x) + Q(x) = x^2 + 2x \quad P(x) - Q(x) = x^2 + 2$$

$$P(x) \cdot Q(x) = x^3 - 1$$

EXERCICE 39

$$A(2) = 55 \quad B(2) = -201 \quad C(2) = -10 \quad D(2) = 59$$

$$A(-3) = 0 \quad B(-3) = -96 \quad C(-3) = -120 \quad D(-3) = -138$$

EXERCICE 40

a) $4 - (5 + x) = 4 - 5 - x = -1 - x$

b) $-a \cdot b \cdot a(-a)(-b) = -a^3 b^2$

c) $-(- (a + 2b) + c) = -(-a - 2b + c) = a + 2b - c$

EXERCICE 41

$$x \neq 0 \quad -x^2 < 0 \quad \text{Vrai}$$

EXERCICE 42

a) $(3a + 2b + c) - (-3a - 2b - c) = 3a - 2b + c + 3a + 2b + c = 6a + 2c$

b) $4x^2 + 3y - (6x^2 - 2y) = -2x^2 + 5y$

c) $18x - [7x - (8x - y)] = 18x - 7x + 8x - y = 19x - y$

d) $25x - \{13x - [24x - (5x + 3y) - (7x - y)] + (24x - 2y)\} = 0$

EXERCISE 43

a) $2a \cdot (3a^2 + 7z) = 6a^3 + 14az$

b) $-3c^2 \cdot (-2x - 5c^2) = 6c^2x + 15c^4$

c) $(3x - 7)(2y - 5) = 6xy - 14y - 15x + 35$

d) $(-2x + 1)(y - 2) = -2xy + y + 4x - 2$

e) $(4y - 1)(4y^2 - 2y + 6) = 16y^3 - 4y^2 - 8y^2 + 2y + 24y - 6 =$
 $= 16y^3 - 12y^2 + 26y - 6$

f) $\underbrace{x(x+1)}_{x^2+x} (x^2 - x - 1) = x^4 - x^3 - x^2 + x^3 - x^2 - x = x^4 - 2x^2 - x$

g) $(2a+1)(3a-1)(2a-3) = 12a^3 + 2a^2 - 2a - 18a^2 - 3a + 3$
 $= 12a^3 - 16a^2 - 5a + 3$

h) $(a+b+c)(a-b-c) = a^2 - ab - ac + ab - b^2 - bc + ac - bc - c^2$
 $= a^2 - b^2 - c^2 - bc$

EXERCISE 44

a) $(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$

$(a-b)^2 = (a-b)(a-b) = a^2 - ba - ab + b^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 + ba - ab - b^2 = a^2 - b^2$

$(a+b)^3 = (a+b)^2(a+b) = (a^2 + 2ab + b^2)(a+b) =$
 $= a^3 + a^2b + 2a^2b + 2ab^2 + b^2a + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = (a-b)^2(a-b) = (a^2 - 2ab + b^2)(a-b) =$
 $= a^3 - a^2b - 2a^2b + 2ab^2 + b^2a - b^3 = a^3 - 3a^2b + 3ab^2 - b^3$

EXERCISE 45

a) $(4x + 7y)^2 = 16x^2 + 56xy + 49y^2$

b) $4(2x + y)^2 = 16x^2 + 16xy + 4y^2$

c) $(3x - 2y)^2 = 9x^2 - 12xy + 4y^2$

d) $(5x^4 - 3x)^2 = 25x^8 - 30x^5 + 9x^2$

e) $(xy^2z - 5)(xy^2z + 5) = x^2y^4z^2 - 25$

f) $(-x + 1)(-x - 1) = x^2 - 1$

g) $(2x + 3y)^3 = 8x^3 + 36x^2y + 54xy^2 + 27y^3$

h) $2(4x^3 + 4y)^3 = 686x^6 + 1176x^4y + 672x^2y^2 + 64y^3$

$$i) (3x-8)^3 = 27x^3 - 216x^2 + 576x - 512$$

$$j) (4x^2 - 3x^3)^3 = 64x^6 - 144x^7 + 108x^8 - 27x^9$$

EXERCISE 46

$$a) (-1 + 5x)^2 = 25x^2 - 10x + 1$$

$$b) (a+2b)(7a-5b) = 7a^2 + 9ab - 10b^2$$

$$c) 1 - (x+1) + 2(x-1) = 1 - x - 1 + 2x - 2 = x - 2$$

$$d) (x^2+1)^2 = x^4 + 2x^2 + 1$$

$$e) -2x(x-3)(4-2x^2) = 4x^4 - 12x^3 - 8x^2 + 24x$$

$$f) (x-1)(x^3+x^2+x+1) = x^4 - 1$$

$$g) (\sqrt{2} - \sqrt{5})^2 = 7 - 2\sqrt{10}$$

$$h) (-x-y)^2 = x^2 + 2xy + y^2$$

$$i) (2a+3)(2a-3) = 4a^2 - 9$$

$$j) 3(x+y)^2 = 3x^2 + 6xy + 3y^2$$

$$k) [3(x+y)]^2 = 9x^2 + 18xy + 9y^2$$

$$l) (x + \frac{1}{x})(x - \frac{1}{x}) = x^2 - \frac{1}{x^2}$$

EXERCISE 47

$$a) 3a - 12ab = 3a(1 - 4b)$$

$$b) 4a^2b - 2a^3c + 6av = 2a(2ab - a^2c + 3v)$$

$$c) 4b^3 + 12b^5 = 4b^3(1 + 3b^2)$$

$$d) 18abc + 30a^2b = 6ab(3c + 5a)$$

EXERCISE 48

$$a) xyz + x^2yz - xy^2z + 2xyz^2 = xyz(1 + x - y + 2z)$$

$$b) 3x^2 + 2xy + 6x + 4y = x(3x + 2y) + 2(3x + 2y) = (3x + 2y)(x + 2)$$

$$c) 1 - x + x^2 - x^3 + x^4 - x^5 = (1 - x)(1 + x^2 + x^4)$$

$$d) 2(x+1)^2 + 4(x+1) = 2(x+1)(x+1+2) = 2(x+1)(x+3)$$

$$e) (5x+2y) - 2x(2y+5x)^2 + 7(5x+2y) = (5x+2y)(1 - 2x(2y+5x) + 7) = (5x+2y)(1 - 4xy - 10x^2 + 7)$$

$$f) (2x-1)^2 - 3(2x-1)(x+2) + (x+4)(2x-1) =$$

$$= (2x-1)(2x-1-3x-6+x+4) = (2x-1)(-3) = -3(2x-1)$$

$$g) (2x-1)^2 + (2xy-y) = (2x-1)^2 + y(2x-1) =$$

$$= (2x-1)(2x-1+y)$$

$$h) (2x+2)^2 + (x+1) = 4(x+1)^2 + (x+1) = (x+1)(4x+4+1)$$

$$= (x+1)(4x+5)$$

EXERCICE 49

$$a) 64x^2 + \frac{16}{3}, x + \frac{1}{9} = \left(8x + \frac{1}{3}\right)^2$$

$$b) 9x^2 - 24x + 16 = (3x-4)^2$$

$$c) \left(\frac{1}{3}x + 6\right)^2 = \frac{1}{9}x^2 + 4x + 36$$

$$d) \left(\frac{1}{5}x + \frac{3}{4}\right)^2 = \frac{1}{25}x^2 + \frac{3}{10}x + \frac{9}{16}$$

EXERCICE 50

$$a) 4x^2 + 20x + 25 = (2x+5)^2$$

$$b) x^2 + 14x + 49 = (x+7)^2$$

$$c) x^2 - 20x + 100 = (x-10)^2$$

$$d) (3x+5)^2 = 9x^2 + 30x + 25$$

$$e) 64 + 48x^2 + 9x^4 = (8+3x^2)^2$$

$$f) 9x^2 - 12x + 4 = (3x-2)^2$$

$$g) x^2 - 16 = (x-4)(x+4)$$

$$h) (2x-6)^2 = 4x^2 - 24x + 36$$

EXERCICE 51

$$a) x^2 + 2xy + y^2 = (x+y)^2$$

$$b) 4x^2 + 4x + 1 = (2x+1)^2$$

$$c) 16z^2 - 8z + 1 = (4z-1)^2$$

$$d) x^2 + xy + \frac{y^2}{4} = \left(x + \frac{y}{2}\right)^2$$

$$e) uv^4 - 100u = u(v^4 - 100) = u(v^2 - 10)(v^2 + 10) =$$

$$= u(v - \sqrt{10})(v + \sqrt{10})(v^2 + 10)$$

$$f) x^3 + 2x^2y + xy^2 = x(x^2 + 2xy + y^2) = x(x+y)^2$$

EXERCISE 52.

- a) $x^2 + 5x + 6 = (x+3)(x+2)$
- b) $x^2 + 3x + 2 = (x+2)(x+1)$
- c) $x^2 + 7x + 12 = (x+3)(x+4)$
- d) $x^2 + x - 30 = (x+6)(x-5)$
- e) $x^2 + 15x + 56 = (x+7)(x+8)$
- f) $x^2 + 2x - 3 = (x-1)(x+3)$
- g) $x^2 + x - 12 = (x+4)(x-3)$
- h) $x^2 + x - 56 = (x+8)(x-7)$

EXERCISE 53

- a) $10ab + 15ac - 5a = 5a(2b + 3c - 1)$
- b) $1 - 16x^2 = (1 - 4x)(1 + 4x)$
- c) $49x^2 + 14xy + y^2 = (7x + y)^2$
- d) $a(x+y) + b(x+y) = (x+y)(a+b)$
- e) $x^2 - 18x + 9 = x^2 - 18x + 9$
- f) $x^2 + x - 12 = (x+4)(x-3)$
- g) $p^3 - 4p^2 + 4p = p(p-2)^2$
- h) $-z^2 - z + 42 = -(z+7)(z-6)$
- i) $5y^2z - 45z = 5z(y-3)(y+3)$
- j) $-\frac{1}{2}x^2 - \frac{7}{2}x - 5 = -\frac{1}{2}(x+2)(x+5)$
- k) $(x^2+1)(x-5) + 2x(x-5) = (x-5)(x^2+2x+1) = (x-5)(x+1)^2$
- l) $3bx^2 + 15bx - 42b = 3b(x-2)(x+7)$
- m) $(2x+1)(6x-2) + x(2x+1) = 2(x+1)(2x+1)$

EXERCISE 54

$$a) \frac{a}{2} - \frac{a}{3} + \frac{a}{6} = \frac{3a - 2a + a}{6} = \frac{2a}{6} = \frac{a}{3}$$

$$b) \frac{a}{b} \cdot \frac{b}{2a} = \frac{1}{2}$$

$$c) \frac{1}{4b} - \frac{1}{2b} = \frac{1-2}{4b} = -\frac{1}{4b}$$

$$d) \frac{x}{x+1} \cdot \frac{x+1}{x^2} = \frac{1}{x}$$

$$e) \frac{x-3}{1+2} \cdot \frac{15}{x-3} = \frac{3}{2}$$

$$f) \frac{a(x+1)}{9x} \cdot \frac{3}{2x+2} = \frac{a(x+1)}{3x \cdot 2(x+1)} = \frac{a}{6x}$$

$$g) \frac{5x+5y}{x} : 5 = \frac{5(x+y)}{x} \cdot \frac{1}{5} = \frac{x+y}{x}$$

$$h) \frac{e-1}{e+2} (e+2)^9 = (e-1)(e+2)^8$$

$$i) (x+5)^8 \cdot \frac{x}{(x+5)^9} = \frac{x}{x+5}$$

$$j) -\frac{2b^3}{5} \cdot \frac{20}{b^2} = -8b$$

$$k) -\frac{d}{7} \cdot \left(-\frac{d}{7}\right) = \frac{d^2}{49}$$

$$l) \frac{(x+1)^8}{x-1} \cdot \frac{(x-2)(x-4)}{(x-1)(x+1)} = \frac{(x+1)(x-2)}{x-1}$$

$$m) \frac{x+y}{2} - \frac{y-x}{3} = \frac{3(x+y) - 2(y-x)}{6} = \frac{3x+3y-2y+2x}{6} = \frac{5x+y}{6}$$

$$n) \frac{1}{x+1} + \frac{1}{x^2+1} = \frac{1}{x+1} + \frac{1}{(x-1)(x+1)} = \frac{x-1+1}{(x-1)(x+1)} = \frac{x}{(x-1)(x+1)}$$

$$o) 1 - \frac{1}{x-1} = \frac{x-1-1}{x-1} = \frac{x-2}{(x-1)(x-1)} = \frac{1}{x-1}$$

$$p) \frac{x}{x+p} + \frac{x}{x-y} = \frac{x(x-y) + x(x+y)}{(x+y)(x-y)} = \frac{x^2 - xy + x^2 + xy}{(x+y)(x-y)} = \frac{2x^2}{x^2 - y^2}$$

$$q) \frac{1}{x-y} - \frac{2y}{x^2-y^2} = \frac{x+y-2y}{(x+y)(x-y)} = \frac{x-y}{(x+y)(x-y)} = \frac{1}{x+y}$$

$$r) \frac{3(t+3)-2(t-3)-1^2}{(t-3)(t+3)} = \frac{3t+9-2t+6-1^2}{(t-3)(t+3)} = \frac{t+3}{(t-3)(t+3)} = \frac{1}{t-3}$$

$$s) \left(2 + \frac{1}{a}\right) : \left(1 + \frac{2}{a}\right) = \frac{2a+1}{a} : \frac{a+2}{a} = \frac{2a+1}{a+2}$$

$$t) \left(\frac{3}{x} - \frac{8}{x}\right) : \left(\frac{x}{2} + \frac{2}{x}\right) = -\frac{5}{x} : \frac{x^2+4}{2x} = -\frac{5}{x} \cdot \frac{2x}{x^2+4} = -\frac{10}{x^2+4}$$

$$u) \left(\frac{1}{a} + \frac{1}{b}\right) : \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{a+b}{ab} : \frac{a-b}{ab} = \frac{a+b}{ab} \cdot \frac{ab}{a-b} = \frac{a+b}{a-b}$$

EXERCICE 55

$$a) \frac{5a+15b}{4a+12b} = \frac{5(a+3b)}{4(a+3b)} = \frac{5}{4}$$

$$b) \frac{x^2-y^2}{ax+ay} = \frac{(x-y)(x+y)}{a(x+y)} = \frac{x-y}{a}$$

$$c) \frac{12ax^2+18a^2x}{9a^3+12ax+4x^2} = \frac{6ax(2x+3a)}{(3a+2x)^2} = \frac{6ax}{3a+2x}$$

$$d) \frac{2a^2-18}{4a^2+24a+36} = \frac{2(a-3)(a+3)}{4(a^2+6a+9)} = \frac{(a-3)(a+3)}{2(a+3)^2} = \frac{a-3}{2(a+3)}$$

$$e) \frac{ax-ay+bx-by}{x^2-2xy+y^2} = \frac{x(a+b)-y(a+b)}{(x-y)^2} = \frac{(a+b)(x-y)}{(x-y)^2} = \frac{a+b}{x-y}$$

$$f) \frac{xy^2+x-1-y^2}{x^2-2x+1} = \frac{y^2(x-1)+(x-1)}{(x-1)^2} = \frac{(x-1)(y^2+1)}{(x-1)^2} = \frac{y^2+1}{x-1}$$

$$g) \frac{9a^2+42ab+49b^2}{49b^2-9a^2} = \frac{(3a+7b)^2}{(7b-3a)(7b+3a)} = \frac{3a+7b}{7b-3a}$$

$$h) \frac{ab-ac+bc-a^2}{(a^2-c^2)(a-b)^2} = \frac{a(b-a)+c(b-a)}{(a-c)(a+c)(a-b)^2} = \frac{(b-a)(a+c)}{(a-c)(a+c)(a-b)^2} = \frac{b-a}{(a-c)(a-b)^2} = -\frac{1}{(a-c)(a-b)}$$

EXERCICE 5G

$$a) \frac{2-3x}{27x^3-18x^2} = \frac{2-3x}{9x^2(3x-2)} = \frac{-(3x-2)}{9x^2(3x-2)} = \frac{-1}{9x^2}$$

$$b) \frac{64a^2-9b^2}{5} : (16a+6b) = \frac{(8a-3b)(8a+3b)}{5} \cdot \frac{1}{2(8a+3b)} = \frac{8a-3b}{10}$$

$$c) \frac{x^2-4x-21}{x-7} = \frac{(x+3)(x-7)}{x-7} = x+3$$

$$d) \frac{2a^2c-4ac+2c}{4ac^2-4c^2} = \frac{2c(a^2-2a+1)}{4c^2(a-1)} = \frac{2c(a-1)^2}{4c^2(a-1)} = \frac{a-1}{2c}$$

$$e) \frac{2}{x-2} - \frac{1}{x+2} - \frac{8}{x^2-4} = \frac{2(x+2)-(x-2)-8}{(x-2)(x+2)} = \frac{2x+4-x+2-8}{(x-2)(x+2)} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$$

$$f) \frac{a(b-c)}{1-a} - \frac{b-c}{1-a} = \frac{(b-c)(a-1)}{-(a-1)} = \frac{b-c}{-1} = c-b$$

EXERCICE 5F

$$a) \frac{a^2-a}{a^2+4a+4} \cdot \frac{a^2+3a+2}{2-a} \cdot \frac{a^2-4}{a^2-1} = \frac{a(a-1)}{(a+2)^2} \cdot \frac{(a+1)(a+2)}{-(a-2)} \cdot \frac{(a-2)(a+2)}{(a-1)(a+1)} = -a$$

$$b) \frac{4x^2-12xy+9y^2}{3x+2y} \cdot \frac{9x^2-4y^2}{4x^2-9y^2} = \frac{(2x-3y)^2}{3x+2y} \cdot \frac{(3x-2y)(3x+2y)}{(2x-3y)(2x+3y)} = \frac{(2x-3y)(3x-2y)}{2x+3y}$$

$$c) \frac{x+y}{3} + \frac{x-y}{2} = \frac{2(x+y)+3(x-y)}{6} = \frac{2x+2y+3x-3y}{6} = \frac{5x-y}{6}$$

$$d) \frac{x-y}{xy} - \frac{z-y}{yz} + \frac{z-x}{xz} = \frac{z(x-y)-x(z-y)+y(z-x)}{xyz} = \frac{zx-zy-xz+xy+yz-yx}{xyz} = 0$$

$$e) \frac{a^2}{(a+b)^3} - \frac{2a}{(a+b)^2} + \frac{1}{a+b} = \frac{a^2-2a(a+b)+(a+b)^2}{(a+b)^3} = \frac{a^2-2a^2-2ab+a^2+2ab+b^2}{(a+b)^3} = \frac{b^2}{(a+b)^3}$$

$$\begin{aligned}
 d) \frac{1}{x^2-y^2} + \frac{1}{x+xy} - \frac{1}{2x^2-2xy} &= \frac{1}{(x-y)(x+y)} + \frac{1}{x(x+y)} - \frac{1}{2x(x+y)} = \\
 &= \frac{2x + 2(x-y) - (x-y)}{2x(x-y)(x+y)} = \frac{2x + 2x - 2y - x + y}{2x(x-y)(x+y)} = \\
 &= \frac{3x - y}{2x(x-y)(x+y)}
 \end{aligned}$$

$$g) \left(\frac{1}{a} - \frac{1}{b} \right) \cdot \frac{a^2 b^2}{a^2 - b^2} = \frac{-(a-b)}{ab} \cdot \frac{a^2 b^2}{(a-b)(a+b)} = \frac{-ab}{a+b}$$

$$\begin{aligned}
 h) \frac{x^2+xy}{x^2+y^2} \left(\frac{x}{x-y} - \frac{y}{x+y} \right) &= \frac{x(x+y)}{x^2+y^2} \left(\frac{x(x+y) - y(xy)}{(x-y)(x+y)} \right) \\
 &= \frac{x(x+y) \cdot (x^2+xy - xy + y^2)}{(x^2+y^2)(x-y)(x+y)} = \frac{x(x+y)(x^2+y^2)}{(x^2+y^2)(x-y)(x+y)} = \\
 &= \frac{x}{x-y}
 \end{aligned}$$