SERIE 8:Probabilites solutions

EXERCICE 1

a) $B \cap C = \{2\}$ $A \cup B = \{2, 4, 6\}$ $B \cup C = \{1; 2, 4; 6\}$

$C \cap A = \{2\}$ $\bar{A} \cup D = \{1, 2, 3, 4, 5, 6\}$ $B \cap \bar{C} = \{4; 6\}$.

b) $A \cap E = \emptyset$

$D \cup E = \Omega$

$D \cap E = \emptyset$

c) $P(A) = \frac{1}{6}$, $P(B) = \frac{3}{6} = \frac{1}{2}$; $P(C) = \frac{2}{6} = \frac{1}{3}$; $P(D) = \frac{4}{6} = \frac{2}{3}$, $P(E) = \frac{2}{6} = \frac{1}{3}$

$P(A \cap B) = \frac{1}{6}$ $P(A \cup B) = P(B) = \frac{3}{6} = \frac{1}{2}$

EXERCICE 2

$P(O) = 0.8$ $P(C) = 0.4$ $P(O \cap C) = 0.3$

a) $P(O \cap \bar{C}) = P(O) - P(O \cap C) = 0.5$

b) $P(O \cup C) = P(O) + P(C) - P(O \cap C) = 0.8 + 0.4 - 0.3 = 0.9$

c) $P(\bar{O} \cap \bar{C}) = P(\overline{O \cup C}) = 1 - P(O \cup C) = 1 - 0.9 = 0.1$

EXERCICE 3

$\Omega = \{$ (F,F,F,F), (F,FFP), (FFPP), (FPPP), (P,P,P,P),
(FFPF), (FPFF), (PFFF), (FPFP), (FPPF), (P,FPF)
(PPFF), (PFPP), (PPFP), (PPPF), (PFFP)

a) $P(F1) = \frac{4}{16} = \frac{1}{4} = 25\%$

b) $P(F2) = \frac{6}{16} = 37.5\%$

c) $P(F0) + P(F1) = \frac{1}{16} + \frac{4}{16} = \frac{5}{16} = 31.3\%$

d) $P(F \geq 2) = 1 - P(F=0) - P(F=1) = 1 - \frac{1}{16} - \frac{4}{16} = 68.7$

e) $P(\times FF \times) = \frac{1}{16} = 6.25\%$

EXERCICE 4

R \ V	1	2	3	4	5	6
1	(1,1)	(1,2)	1;3	(1;4)	(1;5)	(1;6)
2	2;1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4;1	4,2	4;3	4;4	4,5	4,6
5	5;1	5,2	5,3	5,4	5,5	5,6
6	6;1	6,2	6,3	6,4	6,5	6,6

a) (3,5) $P((3;5) = \frac{1}{36} = 2.8\%$.

b) $P((3,5) \cup (5,3)) = \frac{2}{36} = 5.5\%$

c) $P((1,1) \cup (2,2) \ldots \cup (6,6)) = \frac{6}{36} = \frac{1}{6} = 16,7\%$

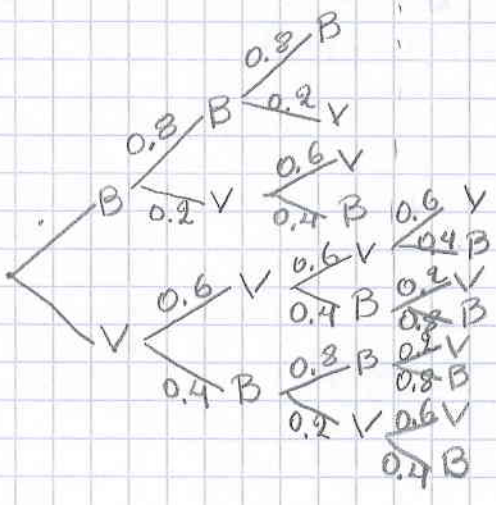
d) $P((2,6) \cup (6;2) \cup (4,4) \cup (3;5) \cup (5;3)) = \frac{5}{36} = 13,9\%$

e) $P(\text{pair}) = \frac{18}{36} = 50\%$

f) $P(F) = \frac{11}{36} = 30.6\%$

g) $P(\bar{F}) = 1 - \frac{11}{36} = 69.4\%$

EXERCICE 5



a) $P(BBB) + P(BVB) =$

$0.8^2 + 0.2 \cdot 0.4 = 0.72$

b) $P(VVVB) + P(VVBV) +$

$P(VVBB) + P(VBBV) +$

$+ P(VBBB) + P(VBVV) +$

$+ P(VBVB) =$

$= 0.6^2 \cdot 0.4 + 0.6 \cdot 0.4 \cdot 0.2 +$

$+ 0.6 \cdot 0.4 \cdot 0.8 + 0.4 \cdot 0.8 \cdot 0.2$

$+ 0.4 \cdot 0.8^2 + 0.4 \cdot 0.2 \cdot 0.6 +$

$0.4 \cdot 0.2 \cdot 0.4 = 0,784$

EXERCICE 6

$P(A)=0.6 \quad P(B)=0.3 \quad P(C)=0.1$

$P(D/A)=0.02 \quad P(D/B)=0.03 \quad P(D/C)=0.04$

$$P(C/D)=\frac{P(C\cap D)}{P(D)}=\frac{P(D/C)\cdot P(C)}{P(D)}=$$

$$=\frac{P(D/C)\cdot P(C)}{P(D/C)\cdot P(C)+P(D/A)\cdot P(A)+P(D/B)\cdot P(B)}=$$

$$=\frac{0.04\cdot 0.1}{0.04\cdot 0.1+0.02\cdot 0.6+0.03\cdot 0.3}=0.16$$

EXERCICE 7

$P(R/V)=\frac{1}{4} \quad P(R/B)=\frac{2}{5} \quad P(B)=\frac{2}{3} \quad P(V)=\frac{1}{3}$

$P(V/\bar{R})=?$

$$P(V/\bar{R})=\frac{P(V\cap\bar{R})}{P(\bar{R})}=\frac{P(\bar{R}/V)\cdot P(V)}{P(\bar{R})}$$

$$P(\bar{R})=1-P(R)=1-P(R/V)\cdot P(V)-P(R/B)\cdot P(B)=$$

$$=1-\frac{1}{4}\cdot\frac{1}{3}-\frac{2}{5}\cdot\frac{2}{3}=1-\frac{1}{12}-\frac{4}{15}=\frac{60-5-16}{60}=$$

$$=\frac{39}{60}=0.65$$

EXERCICE 8

A.P AC X Y

a) $P(A)=\dfrac{\binom{1}{1}\binom{3}{1}}{\binom{4}{2}}=\dfrac{\frac{3!}{1!2!}}{\frac{4!}{2!2!}}=\dfrac{3}{\frac{3\cdot 4}{2}}=\dfrac{1}{2}$

b) $P(B)=\dfrac{\binom{2}{2}\binom{2}{0}}{\binom{4}{2}}=\dots=\dfrac{1}{6}$

c) $P(2A/AP)=\dfrac{P(2A\cap AP)}{P(AP)}=\dfrac{\frac{1}{6}}{\frac{1}{2}}=\dfrac{2}{6}=\dfrac{1}{3}$

d) $P(AP/1\text{ ou }2As)=\dfrac{P(AP\cap(1\text{ ou }2))}{P(1\text{ ou }2)}=\dfrac{3/6}{5/6}=\dfrac{3}{5}$

$$P(1\text{ ou }2)=P(1As)+P(2As)=\frac{\binom{2}{1}\binom{2}{1}}{\binom{4}{2}}+\frac{\binom{2}{2}\binom{2}{2}}{\binom{4}{2}}=\frac{2\cdot 2}{6}+\frac{1}{6}=\frac{5}{6}$$

4-

$P(AP \cap 1 \text{ ou } 2) = P(AP \cap 1As) + P(AP \cap 2As) = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6}$

$P(AP) \cap 2As) = \frac{\binom{2}{2}\binom{2}{0}}{\binom{4}{2}} = \frac{1 \cdot 1}{6} = \frac{1}{6}$

$P(AP) \cap 1As) = \frac{\binom{1}{1}\binom{2}{1}}{\binom{4}{2}} = \frac{2}{6} = \frac{1}{3}$

EXERCICE 9

$P(V) = 0.4 \Rightarrow P(\bar{V}) = 0.6$

$P(V_1 \cap V_2) = 0.15$

Soit $X \to V \quad Y \to F$

$P(V) = 0.4 \Rightarrow \frac{X}{X+Y} = 0.4 \Leftrightarrow X = 0.4X + 0.4Y \Leftrightarrow$

$0.6X = 0.4Y \Leftrightarrow Y = 1.5X$

$P(V_1 \cap V_2) = \frac{X}{X+Y} \cdot \frac{X-1}{X+Y-1} = 0.15 \Leftrightarrow$

$\Leftrightarrow X(X-1) = 0.15(X+Y)(X+Y-1) \Leftrightarrow$

$\Leftrightarrow X^2 - X = 0.15(2.5X)(2.5X-1) \Leftrightarrow$

$\Leftrightarrow 0.0625X^2 - 0.625X = 0 \Leftrightarrow$

$\Leftrightarrow X(0.0625X - 0.625) = 0 \Leftrightarrow X = 0$ imp.

$\underline{X = 10}$ ✓

$Y = 1.5 \cdot 10 = \underline{15} = Y$

b) $P(x \geq 1) = 1 - P(x=0) = 1$ x nb of V

$= 1 - 0.1978 = 0.802$

$P(x=0) = \frac{15}{25} \cdot \frac{14}{24} \cdot \frac{13}{23} = 0.1978$

EXERCICE 10

$p = \frac{1}{6}$ $\quad n = 5$ $\quad 1-p = \frac{5}{6}$ $\quad X = \text{nb de } "6"$

a) $P(X=5) = \binom{5}{5}\left(\frac{1}{6}\right)^5 \cdot \left(\frac{5}{6}\right)^0 = \frac{1}{6^5} = \frac{1}{7776}$

b) $P(X=0) = \binom{5}{0}\left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^5 = \left(\frac{5}{6}\right)^5$

c) $P(X \geq 1) = 1 - P(X=0) = 0.598$

d) $P(X \geq 1) \geq 0.995 \Leftrightarrow 1 - P(X=0) \geq 0.995 \Leftrightarrow 0.005 \geq P(X=0) \Leftrightarrow$

$\Leftrightarrow 0.005 \geq \binom{n}{0}\left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^n \Leftrightarrow \log 0.005 \geq n \log\left(\frac{5}{6}\right) \Leftrightarrow$

$\Leftrightarrow -2.3 \geq n(-0.08) \Leftrightarrow n \geq 29.05 \Rightarrow n = 30$

EXERCICE 11

1. a) $\frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot 3 = \frac{75}{216} = \frac{25}{72}$

b) $P(X \geq 2) = P(X=2) + P(X=3) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot 3 + \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{15}{216} + \frac{1}{216} =$

$= \frac{16}{216} = \frac{2}{27}$

c) Tous les cas possibles pour la somme sont

3 - 1 fois
4 - 2 "
5 - 3 "
6 → 4 "
7 → 5 "
8 → 6 "
9 → 6 "
10 → 6 "
11 → 6 "
12 → 6 "
13 → 6 "
14 → 5 "
15 → 4 "
16 → 3 "
17 → 2 "
18 → 1 "

$P(\text{somme pair}) = \frac{33}{66} = \frac{1}{2}$

2. $n=?$ $p=$ obtenir somme 10 $= \frac{3}{36} = \frac{1}{12}$

$X=$ nb de somme 10

$P(X \geq 1) \geq 0.75 \Leftrightarrow 1 - P(X=0) \geq 0.75 \Leftrightarrow 0.25 \geq P(X=0) \Leftrightarrow$

$\Leftrightarrow 0.25 \geq \binom{n}{0}\left(\frac{1}{12}\right)^0 \cdot \left(\frac{11}{12}\right)^n \Leftrightarrow \log 0.25 \geq n \log \frac{11}{12} \Leftrightarrow$

$\Leftrightarrow -0.6 \geq n(-0.04) \Leftrightarrow n \geq 15.8 \Leftrightarrow n=16$

EXERCICE 12

$P(F)=0,5 \quad P(H)=0,5$

$P(P/F)=0,05 \quad P(P/H)=0,01$

1) $P(P) = P(P/F) \cdot P(F) + P(P/H) \cdot P(H) =$

$= 0.05 \cdot 0.5 + 0.01 \cdot 0.5 = 0.03$

2) $n=4$ $X=$ nb qui souffre de poly

$P(X=1) = \binom{4}{1}(0.01)^1 \cdot (0.99)^3 = 0.04.$

3) $n=?$ $p=0,05$

$P(X \geq 1) \geq 0.999 \Leftrightarrow 1-P(X=0) \geq 0.999 \Leftrightarrow 0.001 \geq P(X=0)$

$\Leftrightarrow 0.001 \geq \binom{n}{0} 0,05^0 \, 0,95^n \Leftrightarrow \log 0.001 \geq n \log 0.95$

$\Leftrightarrow -3 \geq n(-0,022) \Leftrightarrow n \geq 134.7 \Rightarrow n=135.$

4) $P(M)=0.35 \quad P(\bar{M})=0.65$

$P(P) = P(P/M) \cdot P(M) + P(P/\bar{M}) P(\bar{M}) =$

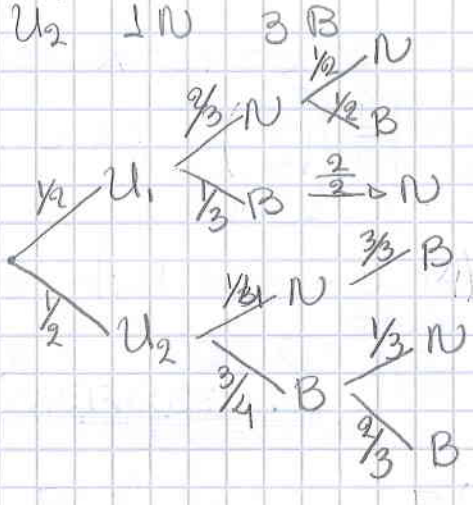
$= 0.01 \cdot 0.35 + 0.05 \cdot 0.65 = 0.036$

5) $P(\bar{M}/P) = \frac{P(\bar{M} \cap P)}{P(P)} = \frac{P(P/\bar{M}) \cdot P(\bar{M})}{P(P)} = \frac{0.05 \times 0.65}{0.036} =$

$= 0,9$

EXERCICE 13

U_1	2 N	1 B
U_2	1 N	3 B



a) $P(A) = P(U_1 \cap N) + P(U_1 \cap B \cap N) + P(U_2 \cap B \cap N) =$

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = \frac{4+4+3}{24} = \frac{11}{24}$

b) $P(B) = P(U_1 \cap N \cap N) + P(U_2 \cap B \cap B) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{2}{3} =$

$= \frac{1}{6} + \frac{1}{4} = \frac{2+3}{12} = \frac{5}{12}$

c) $P(2N/1B) = \frac{P(2N \cap 1B)}{P(1B)} = \frac{P(U_1 \cap B \cap N) + P(U_2 \cap B \cap N)}{P(U_1 \cap B \cap N) + P(U_2 \cap B \cap N) + P(U_2 \cap B \cap B)}$

$= \frac{\frac{1}{2} \cdot \frac{1}{3} \cdot + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{3}}{\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{2}{3}} = \frac{\frac{1}{6} + \frac{1}{8}}{\frac{1}{6} + \frac{1}{8} + \frac{1}{4}} = \frac{\frac{4+3}{24}}{\frac{4+3+6}{24}} = \frac{7}{13}$

d) $P(U_1/2N) = \frac{P(U_1 \cap 2N)}{P(2N)} = \frac{P(U_1 \cap N \cap N) + P(U_1 \cap B \cap N)}{P(U_1 \cap N \cap N) + P(U_1 \cap B \cap N) + P(U_2 \cap B \cap N)}$

$= \frac{\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{3}} = \frac{\frac{1}{6} + \frac{1}{6}}{\frac{1}{6} + \frac{1}{6} + \frac{1}{8}} = \frac{\frac{2}{6}}{\frac{2}{6} + \frac{1}{8}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{8}} = \frac{\frac{1}{3}}{\frac{8+3}{24}} =$

$= \frac{\frac{1}{3}}{\frac{11}{24}} = \frac{24}{3 \cdot 11} = \frac{8}{11}$

EXERCICE 14

1) 9 4B 5N. X = nb des B

a) $p = \frac{4}{9}$

$n = 4$ $\quad P(X=4) = \binom{4}{4}\left(\frac{4}{9}\right)^4\left(\frac{5}{9}\right)^0 = \left(\frac{4}{9}\right)^4$

b) $P(X \geq 1) = 1 - P(X=0) = 1 - \left(\frac{5}{9}\right)^4$

c) $P(X=2) = \binom{4}{2}\left(\frac{4}{9}\right)^2\left(\frac{5}{9}\right)^2 = 6 \cdot \frac{16 \cdot 25}{81^2} = 0.366$

d) Y = nb noires.

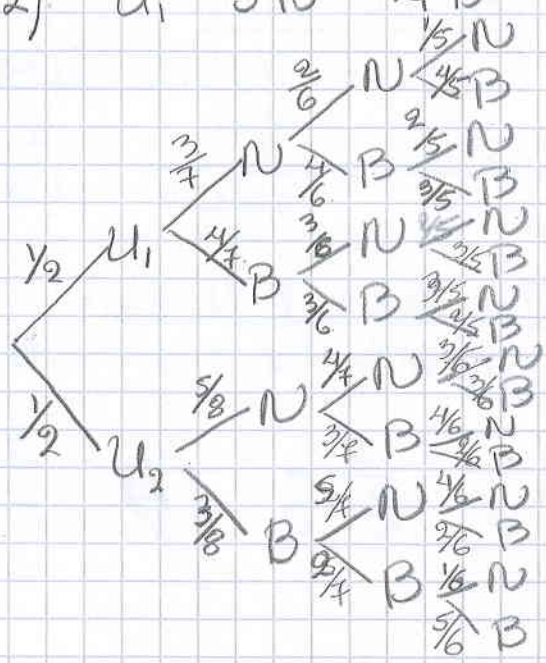
$P(Y \geq 1) \geq 0.99 \Leftrightarrow 1 - P(Y=0) \geq 0.99 \Leftrightarrow$

$\Leftrightarrow 0.01 \geq P(Y=0) \Leftrightarrow 0.01 \geq \binom{n}{0}\left(\frac{4}{9}\right)^n\left(\frac{5}{9}\right)^0 \Leftrightarrow$ ($\binom{n}{0} = 1$, $\left(\frac{5}{9}\right)^0 = 1$)

$\Leftrightarrow 0.01 \geq \left(\frac{4}{9}\right)^n \Leftrightarrow \log 0.01 \geq n \log\left(\frac{4}{9}\right) \Leftrightarrow$

$\Leftrightarrow -2 \geq n(-0.35) \Leftrightarrow n \geq 5.7 \Rightarrow \underline{n = 6}$.

2) U_1 3N 4B $\qquad U_2$ 5N 3B



a) $P(A) = 1 - P(U_1 NNN) - P(U_1 BBB) -$

$- P(U_2 NNN) - P(U_2 BBB) =$

$= 1 - \frac{1}{2}\cdot\frac{3}{7}\cdot\frac{2}{6}\cdot\frac{1}{5} - \frac{1}{2}\cdot\frac{4}{7}\cdot\frac{3}{6}\cdot\frac{2}{5} -$

$- \frac{1}{2}\cdot\frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} - \frac{1}{2}\cdot\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{5}{6} =$

$= 1 - \frac{1}{70} - \frac{2}{35} - \frac{5}{56} - \frac{5}{112} =$

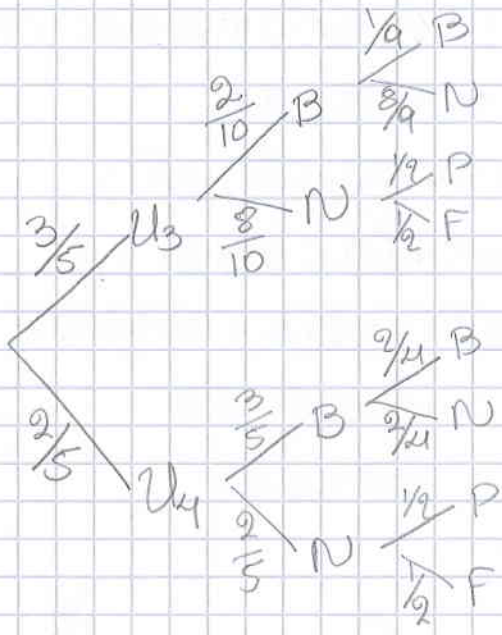
$= 0.8$

b) $P(B) = P(U_1 BBN) + P(U_2 BBN) = \frac{1}{2}\cdot\frac{4}{7}\cdot\frac{3}{6}\cdot\frac{3}{5} + \frac{1}{2}\cdot\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{1}{6} =$

$= \frac{3}{35} + \frac{1}{112} = 0.095$

3) 3. U_3 2B 8N

2 U_4 3B 2N



a) $P(A) = P(U_3 BB) + P(U_4 BB) =$

$= \frac{3}{5} \cdot \frac{2}{10} \cdot \frac{1}{9} + \frac{2}{5} \cdot \frac{3}{5} \cdot \frac{2}{4} =$

$= \frac{1}{75} + \frac{3}{25} = \frac{1+9}{75} = \frac{10}{75}$

b) $P(B) = P(U_3 N) + P(U_4 N) =$

$= \frac{3}{5} \cdot \frac{8}{10} + \frac{2}{5} \cdot \frac{2}{5} = \frac{12}{25} + \frac{4}{25} = \frac{16}{25}$

c) $P(N/U_4) = \frac{P(N \cap U_4)}{P(U_4)} =$

$= \frac{P(U_4 BN) + P(U_4 NN)}{P(U_4 B) + P(U_4 N)} =$

$= \frac{\frac{2}{5} \cdot \frac{3}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{2}{5}}{\frac{2}{5} \cdot \frac{3}{5} + \frac{2}{5} \cdot \frac{2}{5}} = \frac{\frac{3}{25} + \frac{4}{25}}{\frac{6}{25} + \frac{4}{25}} = \frac{7}{10}$

EXERCICE 15

a) $P(G) = 0.6 = p$ $\quad 1-p = 0.4$ $\quad X =$ nb des G

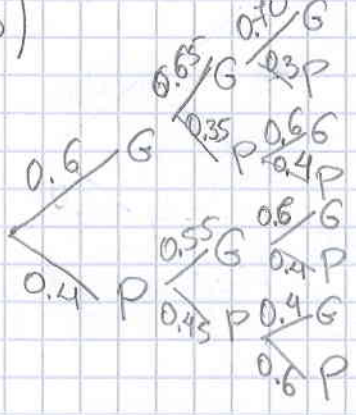
$n = 3$

$\rightarrow P(X=0) = \binom{3}{0} (0.6)^0 \cdot (0.4)^3 = 0.064$

$\rightarrow P(X \geq 2) = P(X=2) + P(X=3) = \binom{3}{2} 0.6^2 \cdot 0.4^1 + \binom{3}{3} 0.6^3 \cdot 0.4^0 =$

$= 0.432 + 0.216 = 0.648$

b)



$\rightarrow P(3G) = 0.6 \cdot 0.65 \cdot 0.7 = 0.273$

$\rightarrow P(G3) = P(GGG) + P(GPG) + P(PGG) + P(PPG)$

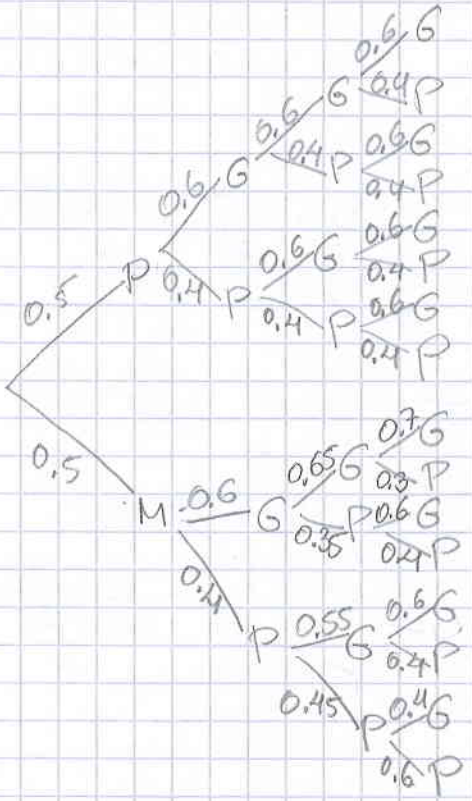
$= 0.6 \cdot 0.65 \cdot 0.7 + 0.6 \cdot 0.35 \cdot 0.6 +$

$+ 0.4 \cdot 0.55 \cdot 0.6 + 0.4 \cdot 0.45 \cdot 0.4 =$

$= 0.273 + 0.126 + 0.132 + 0.072 =$

0.603

c)



$$P(P/G3) = \frac{P(P \cap G3)}{P(G3)} = \frac{0.3}{0.6015} = 0.49$$

$$P(P \cap G3) = (0.6^3 + 0.6^2 \cdot 0.4 + 0.4 \cdot 0.6^2 + 0.4^2 \cdot 0.6) \cdot 0.5$$

$$= 0.6 \cdot 0.5 = 0.3$$

$$P(G3) = P(P \cap G3) + P(M \cap G3) =$$

$$= 0.6 \cdot 0.5 + (0.6 \cdot 0.65 \cdot 0.7 +$$

$$+ 0.6 \cdot 0.35 \cdot 0.6 + 0.4 \cdot 0.55 \cdot 0.6 +$$

$$+ 0.4 \cdot 0.45 \cdot 0.4) \cdot 0.5 =$$

$$= 0.3 + 0.3015 = 0.6015$$

EXERCICE 16

4Q → 5R. 1 correcte

1) Pour toute question $p(J) = \frac{1}{5}$ $p(\bar{J}) = \frac{4}{5}$

Pour la première question $P(J) = \frac{1}{5}$

Pour les restes on a J ou $\bar{J}$.

Donc $P(1\text{ere juste}) = \frac{1}{5}\left(P(X=0) + P(X=1) + P(X=2) + P(X=3)\right)$

où X = nb des réponses justes $p = \frac{1}{5}$ $n = 3$

$$\frac{1}{5}\left(\binom{3}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^3 + \dots + \binom{3}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^0\right) =$$

$$= \frac{1}{5}\left(\underbrace{\frac{4^3}{5^3} + 3 \cdot \frac{4^2}{5^3} + 3 \cdot \frac{4}{5^3} + \left(\frac{1}{5}\right)^3}_{1 \qquad \binom{4}{1}}\right) = \frac{1}{5} = 0.2$$

2) $P(1\text{ juste}) = \frac{1}{5} \cdot \left(\frac{4}{5}\right)^3 \cdot 4 = 0.4096$

3) $p = \frac{1}{5}$ $1-p = \frac{4}{5}$ $n = 4$

$P(X=2) = \binom{4}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = 6\,\frac{1}{5^2} \cdot \frac{16}{5^2} = 0.1536$

4) $P(X \geq 2) = 1 - P(X<2) = 1 - P(X=0) - P(X=1) = 1 - \binom{4}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 - \binom{4}{1}\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^3$

$= 1 - \left(\frac{4}{5}\right)^4 - 4 \cdot \frac{4^3}{5^4} = 1 - 2 \cdot \left(\frac{4}{5}\right)^4 = 0.1808$

Deuxième partie

5) $P(\bar{A}\bar{B}) = 0.12 * 0.25 = 0.03$

6) ~~0.03~~ $P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B}) = \frac{12}{100} + \frac{25}{100} - 0.03$

7) $P(\bar{A} \cup \bar{B} / R) = \frac{P(1 \text{ test} \cap R)}{P(R)} = \frac{P(\bar{A}B) + P(A\bar{B})}{P(R)} =$

$= \frac{0.12 \cdot 0.75 + 0.88 \cdot 0.25}{0.12 \cdot 0.75 + 0.88 \cdot 0.25 + 0.12 \cdot 0.25} = \frac{0.09 + 0.22}{0.09 + 0.22 + 0.03} =$

$= \frac{0.31}{0.34} = 0.912.$

8) p = probabilité d'un candidat est admissible =

$= 0.88 \cdot 075 = 0.66$

$n = ?$ $P(X \geq 1) \geq 0.999 \Leftrightarrow 1 - P(X=0) \geq 0.999 \Leftrightarrow$

$0.001 \geq P(X=0) \Leftrightarrow 0.001 \geq \binom{n}{0} 0.66^0 \, 0.34^n \Leftrightarrow$

$\Leftrightarrow 0.001 \geq 0.34^n \Leftrightarrow -3 \geq n \log 0.34 \Leftrightarrow$

$\Leftrightarrow n \geq 6.4 \Rightarrow \underline{n = 7}$

EXERCICE 17.

$P(D) = 0.4 \Rightarrow P(\bar{D}) = 0.6$

$P(P/D) = 0.94 \quad P(P/\bar{D}) = 0.08$

A) A1) $P(\bar{D} \cap \bar{P}) = P(\bar{P}/\bar{D}) \cdot P(\bar{D}) = 1 - P(P/\bar{D}) \cdot P(\bar{D}) =$

$= 1 - 0.08 \cdot 0.6 = 0.952$

A2) $P(P) = P(P \cap D) + P(P \cap \bar{D}) = P(P/D) \cdot P(D) + P(P/\bar{D}) \cdot P(\bar{D})$

$= 0.94 \cdot 0.4 + 0.08 \cdot 0.6 = 0.424$

A3) $P(\bar{D}/P) = \frac{P(P \cap \bar{D})}{P(P)} = \frac{P(P/\bar{D}) \cdot P(\bar{D})}{P(P)} = \frac{0.08 \cdot 0.6}{0.424} = 0.113$

A4) $P(P/\bar{D}) + P(\bar{P}/D) = \frac{8}{100} + 1 - P(P/D) = 0.08 + 1 - 0.94 = 0.14$

B) $P(\bar{P}) = 1 - 0.424 = 0.576$

$P = 0.424 \quad 1 - P = 0.576$

$X =$ nb des positifs

B1) $P(X=2) = \binom{3}{2} 0.424^2 \cdot 0.576 = 0.31$

B2) $P(X \geq 1) = 1 - P(X=0) = 1 - \binom{3}{0} 0.424^0 \cdot 0.576^3 =$

$= 1 - 0.191 = 0.809$

EXERCICE 18

1) a) $P(X=0) = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{1}{4}$

b) $P(X \geq 1) = 1 - P(X=0) = 1 - \frac{1}{4} = \frac{3}{4}$

c) $P(X=1) = \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot 2 + \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot 2 = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$

2) $P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{6}}{\frac{5}{12}} = \frac{12}{5 \cdot 6} = \frac{2}{5}$

1 fois

$P(C)$ = la cible est atteinte $= P(A \cap C) + P(B \cap C) =$

$= \frac{1}{4_2} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{3}{4} = \frac{1}{6} + \frac{1}{4_1} = \frac{5}{12}$

3) $n=5 \quad p = \frac{1}{3}$

a) $P(X \geq 1) = 1 - P(X=0) = 1 - \binom{5}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^5 = 1 - \left(\frac{2}{3}\right)^5 = 0.868$

b) $P(X \geq 1) = 1 - \binom{10}{0} \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^{10} = 0.983$

c) $P(X \geq 1) = 1 - \left(\frac{2}{3}\right)^n$

d) $P(X \geq 1) \geq 0.99 \Leftrightarrow 0.01 \geq \left(\frac{2}{3}\right)^n \Rightarrow -2 \geq n(-0.176) \Rightarrow$

$\Leftrightarrow n \geq 12$

EXERCICE 19

$P(C) = 0.65 \quad n = 20$

a) X = nb des incapables $p = 0.35 \quad 1-p = 0.65$

$P(X \geq 1) = 1 - P(X=0) = 1 - \binom{20}{0} (0.35)^0 (0.65)^{20} = 1 - 0.65^{20} =$

$= 0.9998$

b) $P(X=13) = \binom{20}{13} (0.65)^{13} (0.35)^7 = 0.1844$

c) $P(X<15) = 1 - P(X=15) - P(X=16) + P(X=17) - P(X=18) - P(X=19)$

$- P(X=20) = 0.7546$

EXERCICE 20

$p = P(F) = \frac{1}{2}$ $\quad n = 20$

a) $P(X=8) = \binom{20}{8}\left(\frac{1}{2}\right)^8 \cdot \left(\frac{1}{2}\right)^{12} = 0,12$

b) $P(X=9) = \binom{20}{9}\left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{11} = 0,16$

c) $P(X=10) = \binom{20}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10} = 0,1762$

d) $P(7 < X < 13) = P(X=8) + P(X=9) + P(X=10) + P(X=11) + P(X=12)$

$= 0,737$

EXERCICE 21

1) $P(G) = \frac{3}{4}$ $\quad P(\bar{G}) = \frac{1}{4}$

a) $n = 15$ $\quad p = \frac{3}{4}$

$P(X \geq 13) = P(X=13) + P(X=14) + P(X=15) =$

$= \binom{15}{13}\left(\frac{3}{4}\right)^{13} \cdot \left(\frac{1}{4}\right)^2 + \binom{15}{14}\left(\frac{3}{4}\right)^{14}\left(\frac{1}{4}\right) + \binom{15}{15}\left(\frac{3}{4}\right)^{15}\left(\frac{1}{4}\right)^0 = 0,236$

b) $P(X \geq 1) \geq 0,998 \Leftrightarrow 1 - 0,998 \geq P(X=0) \Leftrightarrow$

$\Leftrightarrow 0,002 \geq \binom{n}{0}\left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^n \Leftrightarrow \log 0,002 \geq n \log 0,25$

$\Leftrightarrow -2,7 \geq n(-0,6) \Leftrightarrow n \geq 4,48 \Rightarrow \underline{n \geq 5}$

2) c) $P(c) = \frac{3}{4} \cdot \frac{5}{6} = \frac{5}{8}$

d) $P(\bar{V}) = \frac{3}{4} \cdot \frac{1}{6} + \frac{1}{4} \cdot \frac{5}{6} = \frac{1}{8} + \frac{5}{24} = \frac{8}{24} = \frac{1}{3}$

e) $\frac{1}{4}\cdot\frac{5}{6}\cdot\frac{1}{4}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{5}{6}$

$+ \frac{1}{4}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{5}{6}$

$+ \frac{3}{4}\cdot\frac{1}{6}\cdot\frac{1}{4}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{5}{6}$

$+ \frac{3}{4}\cdot\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{5}{6}$ $\quad = \frac{5}{72}$

g) $P(S/1t) = \frac{P(S \cap 1t)}{P(1t)} = \frac{\frac{5}{8}}{\frac{2}{3}} = \frac{15}{16}$.

$P(1t) = P(S \cap 1t) + P(J \cap 1t) = \frac{3}{4}\cdot\frac{5}{6} + \frac{1}{4}\cdot\frac{1}{6} = \frac{5}{8} + \frac{1}{24} = \frac{16}{24} = \frac{2}{3}$

EXERCICE 22

1. a) $P(4A) = \frac{4}{36}\cdot\frac{3}{35}\cdot\frac{2}{34}\cdot\frac{1}{33} = 0.000017$

b) $P(3A) = \frac{\binom{4}{3}\cdot\binom{32}{1}}{\binom{36}{4}} = \frac{4\cdot 32}{58905} = 0.0022 = 0.22\%$

c) $P(2A) = \frac{\binom{4}{2}\binom{32}{2}}{\binom{36}{4}} = 5.05\%$

d) $P(1A) = \frac{\binom{4}{1}\binom{32}{3}}{\binom{36}{4}} = \frac{4\cdot 4960}{58905} = 0.3368 = 33.68\%$

e) $P(0A) = 1 - \frac{\binom{4}{0}\cdot\binom{32}{4}}{\binom{36}{4}} = \frac{35960}{58905} = 0.6105 = 61.05\%$

f) $P(A \geq 1) = 1 - P(0A) = 1 - 0.6105 = 38.95\%$.

g) $P(A \geq 1 / 1\bar{A}) = \frac{P(A \geq 1 \cap 1\bar{A})}{P(1\bar{A})} = \frac{\frac{32}{36} - P(0A)}{\frac{32}{36}} = \frac{\frac{32}{36} - 0.6105}{\frac{32}{36}} =$

$= 0.3132$

2. $\mu = 0\cdot 0.6105 + 1\cdot 0.3368 + \quad = 0.44$

EXERCICE 23

a) $P(x=2) = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 2\cdot\frac{1}{2^2} = \frac{1}{2}$

$P(x=3) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = 2\cdot\frac{1}{2^3} = \frac{1}{2^2}$

$P(x=4) = \frac{1}{2^3}$

$P(x=n) = \frac{1}{2}n-1$

b) $P(X>6) = 1 - P(X=2) - P(X=3) - P(X=4) - P(X=5) - P(X=6)$

$= 1 - \frac{1}{2} - \frac{1}{2^2} - \frac{1}{2^3} - \frac{1}{2^4} - \frac{1}{2^5} =$

$= 1 - \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}\right) =$

$= 1 - \frac{1}{2}\,\frac{1 - \frac{1}{2^5}}{1 - \frac{1}{2}} = 1 - 1 + \frac{1}{2^5} = \frac{1}{2^5}$?

$P(X<7) = P(X=2) + \dots + P(X=6) =$

$= \frac{1}{2} + \dots + \frac{1}{2^5} = \frac{1}{2}\left(1 + \frac{1}{2} + \dots + \frac{1}{2^4}\right) = \frac{1}{2}\,\frac{1 - \frac{1}{2^5}}{1 - \frac{1}{2}} =$

$= \frac{31}{32}$

$P(X<n) = P(X=2) + \dots + P(X=n-1) =$ ($\frac{1}{2^{n-2}}$)

$= \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{n-3}}\right) = \frac{1}{2}\,\frac{1 - \frac{1}{2^{n-2}}}{1 - \frac{1}{2}} = 1 - \frac{1}{2^{n-2}}$

c) $P(X<n) \geq 0.99 \Leftrightarrow 1 - \frac{1}{2^{n-2}} \geq 0.99 \Leftrightarrow 0.01 \geq \frac{1}{2^{n-2}} \Leftrightarrow$

$\Leftrightarrow (n-2)\log 2 \geq \log \frac{1}{0.01} \Leftrightarrow (n-2)\,0.3 \geq 2 \Leftrightarrow n-2 \geq 6.67$

$\Leftrightarrow n \geq 8.67 \Rightarrow n \geq 9$

d) $P(X=2k) = P(X=2) + P(X=4) + P(X=6) + \dots =$

$= \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \dots$

$= \frac{1}{2}\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \dots\right) = \frac{1}{2}\,\frac{1}{1 - \frac{1}{2^2}} = \frac{1}{2}\,\frac{4}{3} = \frac{2}{3}$

EXERCICE 24

Première partie

1 - 1 2 - 2 3 - 3 4 - 4

a)

X	1	2	3	4
$P(X=i)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

b) $\mu = 1 \cdot \frac{1}{10} + 2 \cdot \frac{2}{10} + 3 \cdot \frac{3}{10} + 4 \cdot \frac{4}{10} = \frac{1}{10}\left(1 + 4 + 9 + 16\right) = \frac{30}{10} = 3$

▲ $n=4$

c) $P(1234) = \frac{1}{10}\cdot\frac{2}{10}\cdot\frac{3}{10}\cdot\frac{4}{10} = \frac{24}{10^4}$

d) $P(DN) = \frac{1}{10}\cdot\frac{2}{10}\cdot\frac{3}{10}\cdot\frac{4}{10}\cdot 4! = \frac{24}{10^4}\cdot 4!$

e) $P(34) = \left(\frac{3}{10}\right)^2\cdot\left(\frac{4}{10}\right)^2\cdot\frac{4!}{2!2} = \frac{9\cdot 16\cdot 6}{10^4} = 0.0864$

f) $P(4\geq 2) = 1 - P(4=0) - P(4=1) = 1 - 0.13 - 0.35 = 0.5244$

$P(4=0) = \left(\frac{6}{10}\right)^4 \qquad P(4=1) = \left(\frac{6}{10}\right)^3\cdot\frac{4}{10}\cdot 4$

▲ Deuxième partie

$N = \frac{n(n+1)}{2}$

g) $P(X=1) = \frac{2}{n(n+1)} \qquad P(X=2) = \frac{2\cdot 2}{n(n+1)} = \frac{4}{n(n+1)}$

$P(X=3) = \frac{3\cdot 2}{n(n+1)} = \frac{6}{n(n+1)}$

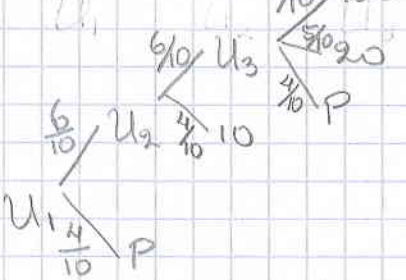
$P(X=k) = \frac{2\cdot k}{n(n+1)}$

h) $\mu = 1\cdot P(X=1) + 2\,P(X=2) + \ldots + n\,P(X=n) =$

$= 1\cdot\frac{2}{n(n+1)} + 2\cdot\frac{2\cdot 2}{n(n+1)} + 3\cdot\frac{2\cdot 3}{n(n+1)} + \ldots + n\cdot\frac{2\cdot n}{n(n+1)} =$

$= \frac{2}{n(n+1)}\left(1 + 2^2 + 3^2 + \ldots + n^2\right) = \frac{2}{n(n+1)}\cdot\frac{n(n+1)(2n+1)}{6} = \frac{2n+1}{3}$

EXERCICE 25



a) $P(10) = \frac{6}{10}\cdot\frac{4}{10} = 0.24$

$P(20) = \frac{6}{10}\cdot\frac{6}{10}\cdot\frac{5}{10} = 0.18$

$P(100) = \frac{6}{10}\cdot\frac{6}{10}\cdot\frac{1}{10} = 0.036$

$P(P) = \frac{4}{10} + \frac{6}{10}\cdot\frac{6}{10}\cdot\frac{4}{10} = 0.81$

b) $P(U_3/P) = \frac{P(U_3 \cap P)}{P(P)} = \frac{\frac{6}{10}\cdot\frac{6}{10}\cdot\frac{4}{10}}{\frac{6}{10}\cdot\frac{6}{10}\cdot\frac{4}{10} + \frac{4}{10}} = \frac{0.144}{0.544} = 0.265$

c) $P(X \geq 20) = 1 - P(10) - P(P) = 1 - \frac{6}{10} \cdot \frac{4}{10} - \frac{6}{10} \cdot \frac{6}{10} \cdot \frac{5}{10} =$

$= 1 - 0.24 - 0.18 = 0.58$

d) $n = 30 \quad p = \frac{6}{10} \cdot \frac{6}{10} \cdot \frac{1}{10} = 0.036$. X = nb de fois qu'il a gagné 100 CHF

$P(X=0) = \binom{30}{0} 0.036^0 \cdot (1-0.036)^{30} =$

$= (0.964)^{30} = 0.333$

$P(X \geq 1) = 1 - P(X=0) = 1 - 0.333 = 0.667$

e) n ?

$P(X \geq 1) \geq 0.9 \Leftrightarrow 1 - P(X=0) \geq 0.9 \Leftrightarrow 0.1 \geq P(X=0) \Leftrightarrow$

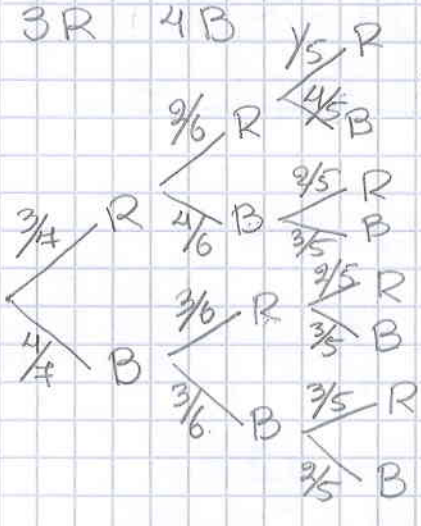
$\Leftrightarrow 0.1 \geq \binom{n}{0} 0.333^0 \cdot 0.667^n \Leftrightarrow 0.1 \geq 0.667^n \Leftrightarrow$

$\log 0.1 \geq n \log 0.667 \Leftrightarrow -1 \geq n(-0.176) \Leftrightarrow$

$\Leftrightarrow n \geq 5.6 \Rightarrow \underline{n \geq 6}$

EXERCICE 26

3R 4B



a) $P(RRR) + P(BBB) = \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} + \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} = \frac{1}{7}$

b) $P(RRR) + P(RRB) + P(RBR) + P(BRR) =$

$= \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} + \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{4}{5} + \frac{3}{7} \cdot \frac{4}{6} \cdot \frac{2}{5} + \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} =$

$= \frac{1}{35} + \frac{4}{35} + \frac{4}{35} + \frac{4}{35} = \frac{13}{35}$

c) $P(2B / 2dR) = \frac{P(2B \cap 2dR)}{P(2dR)} = \frac{\frac{6}{35}}{\frac{3}{7}} = \frac{6 \cdot 7}{8 \cdot 5 \cdot 7} = \frac{2}{5}$

$P(2B \cap 2dR) = P(BRB) = \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{3}{5} = \frac{6}{35}$

$P(2dR) = \frac{3}{7} \cdot \frac{2}{6} + \frac{4}{7} \cdot \frac{3}{6} = \frac{1}{7} + \frac{2}{7} = \frac{3}{7}$

▶ 5 fiches

d) $P(3R \cap 2B) = \dfrac{\binom{3}{3}\binom{4}{2}}{\binom{7}{5}} = \dfrac{6}{21} = \dfrac{2}{7} = 0.2857$

e) $P(RBRBR) + P(BRBRB) = \frac{3}{7}\cdot\frac{4}{7}\cdot\frac{3}{7}\cdot\frac{4}{7}\cdot\frac{3}{7} + \frac{4}{7}\cdot\frac{3}{7}\cdot\frac{4}{7}\cdot\frac{3}{7}\cdot\frac{4}{7} =$

$= \left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^3 + \left(\frac{4}{7}\right)^3\left(\frac{3}{7}\right)^2 = \left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^2\left(\frac{3}{7}+\frac{4}{7}\right) = \left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^2 = 0.06$

▶ f) $P(X=1) = \frac{3}{7}$

$P(X=2) = P(BR) = \frac{4}{7}\cdot\frac{3}{7} = \frac{12}{49}$

$P(X=3) = P(BBR) = \left(\frac{4}{7}\right)^2\cdot\frac{3}{7} = \frac{48}{343}$

$P(X=n) = P(\underbrace{BB\ldots B}_{n-1}R) = \left(\frac{4}{7}\right)^{n-1}\cdot\frac{3}{7}$,

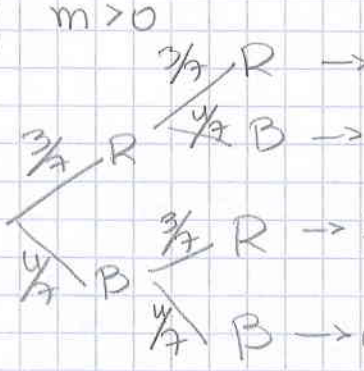
g) $P(X=2k) = P(X=2) + P(X=4) + P(X=6) + \ldots =$

$= \frac{4}{7}\cdot\frac{3}{7} + \left(\frac{4}{7}\right)^3\cdot\frac{3}{7} + \left(\frac{4}{7}\right)^5\cdot\frac{3}{7} + \ldots =$

$= \frac{3}{7}\left(\frac{4}{7} + \left(\frac{4}{7}\right)^3 + \left(\frac{4}{7}\right)^5 + \ldots\right) = \frac{3}{7}\cdot\frac{4}{7}\left(1 + \left(\frac{4}{7}\right)^2 + \left(\frac{4}{7}\right)^4 + \ldots\right) =$

$= \frac{12}{49}\cdot\left(\frac{1}{1-\left(\frac{4}{7}\right)^2}\right) = \frac{12}{7^2}\cdot\frac{1}{\frac{7^2-4^2}{7^2}} = \frac{7^2\cdot 3\cdot 4}{7^2}\cdot\frac{1}{7^2-4^2} = \frac{12}{33} = \frac{4}{11}$

h) $m > 0$



0 blue	1 blue	2 blue
i	0	1	2
$P(X=i)$	$\left(\frac{3}{7}\right)^2$	$\frac{3}{7}\cdot\frac{4}{7}\cdot 2$	$\left(\frac{4}{7}\right)^2$
.	m^3	$\frac{m}{2}$	0

$4 = m^3\cdot P(X=0) + \frac{m}{2}P(X=1) + 0\cdot P(X=2) =$

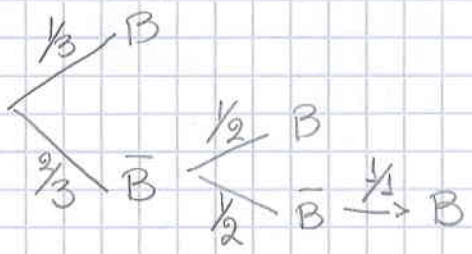
$= m^3\cdot\frac{9}{49} + \frac{m}{2}\cdot\frac{24}{49} + 0\cdot\frac{16}{49} = \frac{3}{49}m\left(3m^2+4\right) \geq m$

$\Leftrightarrow \frac{3}{49}\cdot(3m^2+4) \geq 1 \Leftrightarrow 3m^2+4 \geq \frac{49}{3} \Leftrightarrow$

$\Leftrightarrow 3m^2 \geq \frac{49}{3} - \frac{12}{3} \Rightarrow 3m^2 \geq \frac{37}{3} \Rightarrow m^2 \geq \frac{37}{9} \Leftrightarrow$

$m \geq \frac{\sqrt{37}}{3}$

EXERCICE 27



a) $P(X=1) = \frac{1}{3}$

b) $P(X=2) = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

c) $P(X=3) = \frac{2}{3} \cdot \frac{1}{2} \cdot 1 = \frac{1}{3}$

d) $P(X=1) = \frac{1}{3}$

e) $P(X=2) = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$

f) $P(X=3) = \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} = \frac{4}{27}$

g) $P(X=n) = \left(\frac{2}{3}\right)^{n-1} \cdot \frac{1}{3}$

h) $\left(\frac{2}{3}\right)^{n-1} \cdot \frac{1}{3} \leq \frac{1}{100} \Leftrightarrow \left(\frac{2}{3}\right)^{n-1} \leq \frac{3}{100} \Leftrightarrow (n-1)\log\frac{2}{3} \leq \log\frac{3}{100} \Leftrightarrow$

$\Leftrightarrow (n-1)(-0,1761) \leq (-1,523) \Leftrightarrow (n-1) \geq 8,65 \Leftrightarrow n \geq 9,65$

$\Rightarrow \underline{n \geq 10}$

i) $P(X = 2k+1) = P(X=1) + P(X=3) + P(X=5) + \ldots =$

$= \frac{1}{3} + \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^4 \cdot \frac{1}{3} + \ldots$

$= \frac{1}{3}\left(1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^4 + \ldots\right) =$

$= \frac{1}{3} \cdot \frac{1}{1 - \left(\frac{2}{3}\right)^2} = \frac{1}{3} \cdot \frac{9}{9-4} = \frac{3}{5}$

▲ $P(E) = \frac{1}{10}$

j) $P(X=1) + P(X=2) = P(X=1 \cap E) + P(X=1 \cap \bar{E}) + P(X=2 \cap E) + P(X=2 \cap \bar{E})$

$= P(X=1/E) \cdot P(E) + P(X=1/\bar{E}) \cdot P(\bar{E}) + P(X=2/E) \cdot P(E) + P(X=2/\bar{E}) \cdot P(\bar{E}) =$

$= \frac{1}{3} \cdot \frac{1}{10} + \frac{1}{3} \cdot \frac{9}{10} + \frac{2}{9} \cdot \frac{1}{10} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{9}{10} =$

$= \frac{1}{30} + \frac{9}{30} + \frac{1}{45} + \frac{3}{10} = \frac{1}{30} + \frac{3}{10} + \frac{1}{45} + \frac{3}{10} = \frac{3+27+2+27}{90} =$

$= \frac{59}{90}$

k) $P(E/3\,\text{Essai}) = \frac{P(E \cap 3\,\text{Essai})}{P(3\,\text{Essai})} = \frac{P(3\,\text{Essai}/E)\cdot P(E)}{P(3\,\text{Es}/E)\cdot P(E) + P(3\,\text{Es}/\bar{E})\cdot P(\bar{E})} =$

$= \frac{\frac{2}{27}\cdot\frac{1}{10}}{\frac{2}{27}\cdot\frac{1}{10} + \frac{1}{8}\cdot\frac{9}{10}} = \frac{\frac{2}{27\cdot 5}}{\frac{2}{27} + \frac{3}{10}} = \frac{\frac{2}{27\cdot 5}}{\frac{4+81}{270}} = \frac{4}{85}$