SERIE 2: Geometrie Espace

EXERCICE 1

Π : $A(1;1;6)$ $B(-2;-5;3)$ $C(5;0;2)$

1. $\vec{AB} = \begin{pmatrix}-3\\-6\\-3\end{pmatrix} \parallel \begin{pmatrix}1\\2\\1\end{pmatrix}$ $\vec{AC} = \begin{pmatrix}4\\-1\\-4\end{pmatrix}$

$$\Pi : \begin{cases} x = 1 + t + 4k \\ y = 1 + 2t - k \\ z = 6 + t - 4k \end{cases}$$

2. $D(3;1;4) \overset{?}{\in} \Pi$ $\begin{cases} 3 = 1 + t + 4k \\ 1 = 1 + 2t - k \\ 4 = 6 + t - 4k \end{cases} \Leftrightarrow \begin{cases} 2 = t + 4k & ① \\ 0 = 2t - k & ② \\ -2 = t - 4k & ③ \end{cases}$

① + ③ $0 = 2t \Rightarrow t = 0$ ① $\Rightarrow 2 = 4k \Rightarrow k = 1/2$.

On remplace en ② $0 = 0 - \frac{1}{2}$ ↯ $\Rightarrow D \notin \Pi$

$E(-11;4;18) \overset{?}{\in} \Pi$ $\begin{cases} -11 = 1 + t + 4k & ① \\ 4 = 1 + 2t - k & ② \\ 18 = 6 + t - 4k & ③ \end{cases}$

① + ③ $7 = 7 + 2t \Rightarrow t = 0$.   ① $\Rightarrow -11 = 1 + 4k \Rightarrow -12 = 4k \Rightarrow k = -3$

On remplace en ②
$4 = 1 + 0 + 3$ ✓ $\Rightarrow E \in \Pi$

EXERCICE 2

a) $A(4;3;1)$ $\vec{r} = \begin{pmatrix}1\\0\\-1\end{pmatrix}$ $\vec{s} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$

$$\Pi : \begin{cases} x = 4 + t + 2k & ① \\ y = 3 + k & ② \\ z = 1 - t - k & ③ \end{cases}$$

① + ③ $x + z = 5 + k$
② $+$ $-y = -3 - k$

$x - y + z = 2$

b) $A(1;-1;1)$ $B(1;0;1)$ $C(2;2;0)$

$\vec{AB} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$$\Pi : \begin{cases} x = 1 + k & (1) \\ y = -1 + t + 2k & (2) \\ z = 1 - k & (3) \end{cases} \qquad (1)+(3) \quad x + z = 2$$

c) $A(1;1;0)$ $d : \begin{cases} x = 3 + \lambda \\ y = 2\lambda \\ z = 2 \end{cases}$

Vecteurs directeurs : $\vec{d} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.

Pour trouver encore un vecteur on trouve un point quelconque de d par exemple pour $t = 0$ $B(3;0;2)$

$\vec{AB} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

$$\Pi : \begin{cases} x = 1 + t + 3k & \times(-2) \\ y = 1 + 2t - k \\ z = 2t + 2k & \times(-1) \end{cases} \Leftrightarrow \begin{cases} -2x = -2 - 2t - 6k & (1) \\ y = 1 + 2t - k & (2) \\ -z = -2t - 2k & (3) \end{cases}$$

$(1)+(2)$ $-2x + y = -1 - 7k$ $*(3)$
$(2)+(3)$ $y - z = 1 - 3k$ $*(-7)$

$$\begin{cases} -6x + 3y = -3 - 21k \\ -7y + 7z = -7 + 21k \end{cases}$$

$$-6x - 4y + 7z = -10 \quad \Pi$$

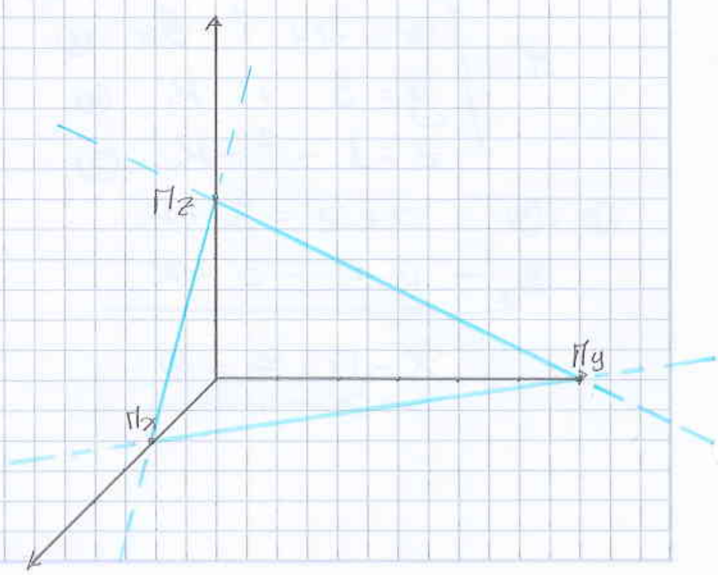
Exercice 3

1) $\Pi : 3x + y + 2z - 6 = 0$

$\Pi_x : y = z = 0$ $\Pi_x(2;0;0)$

$\Pi_y : x = z = 0$ $\Pi_y(0;6;0)$

$\Pi_z : x = y = 0$ $\Pi_z(0;0;3)$

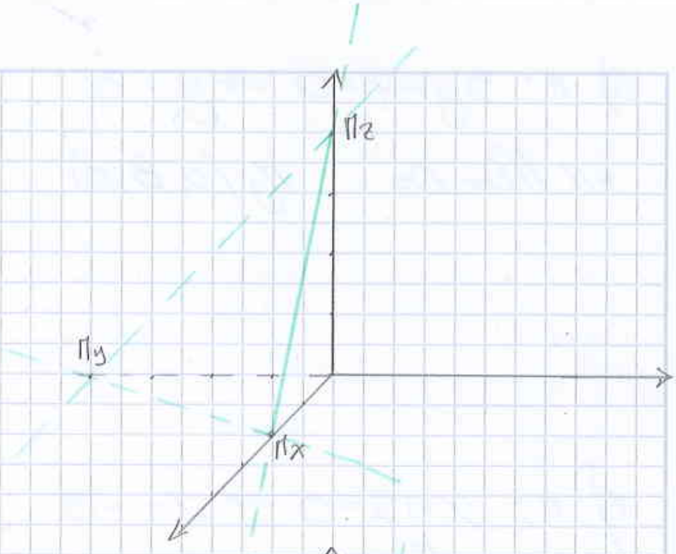


2) $\Pi : 2x - y + z - 4 = 0$

$\Pi_x : y = z = 0 \quad \Pi_x\,(2; 0; 0)$

$\Pi_y : x = z = 0 \quad \Pi_y\,(0; -4; 0)$

$\Pi_z : x = y = 0 \quad \Pi_z\,(0; 0; 4)$

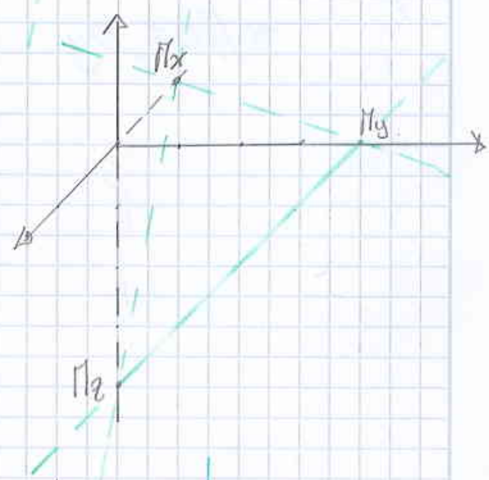


3) $\Pi : 2x - y + z + 4 = 0$

$\Pi_x \,(y = z = 0 \quad \Pi_x\,(-2; 0; 0).$

$\Pi_y : x = z = 0. \quad \Pi_y\,(0; 4; 0).$

$\Pi_z : y = x = 0 \quad \Pi_z\,(0; 0; -4)$

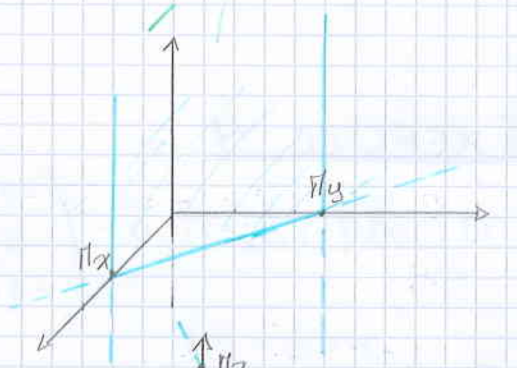


4) $\Pi : 5x + 4y - 10 = 0$

$\Pi_x : y = z = 0 \quad \Pi_x\,(2; 0; 0).$

$\Pi_y : x = z = 0 \quad \Pi_y\,(0; \frac{5}{2}; 0).$

$\Pi_z : y = x = 0.$ Impossible $\Rightarrow$ $\Pi \parallel Oz$.

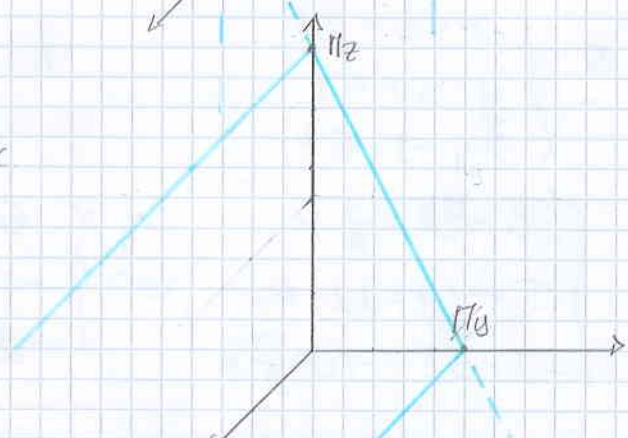


5) $\Pi : 2y + z - 5 = 0.$

$\Pi_x : y = z = 0.$ Impossible $\Rightarrow \Pi \parallel Ox$

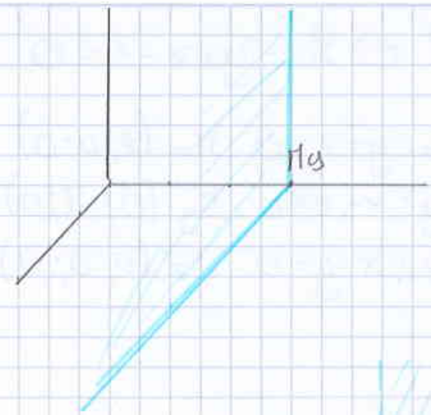
$\Pi_y : x = z = 0 \quad \Pi_y\,(0; \frac{5}{2}; 0).$

$\Pi_z : y = x = 0 \quad \Pi_z\,(0; 0; 5)$



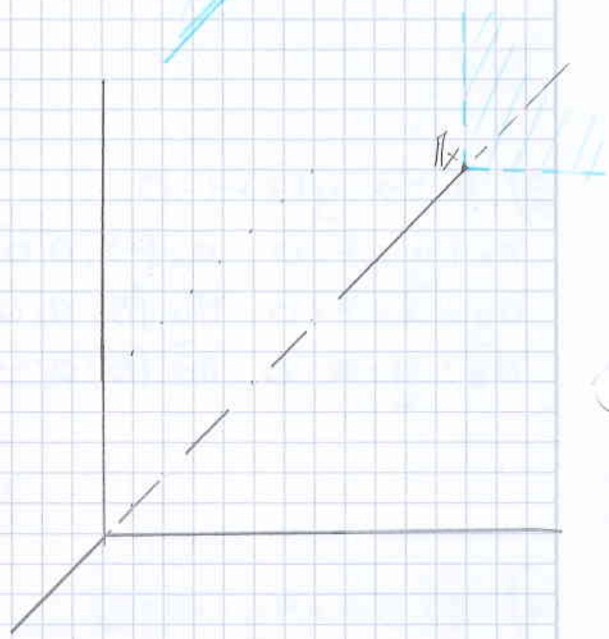
6) $\pi : 3y - 9 = 0 \Rightarrow y = 3$

$\pi // Ox, Oz \qquad \pi_y(0;3;0)$



7) $\pi : x + 12 = 0 \Rightarrow x = -12$

$\pi // Oz, Oy$



EXERCICE 4

1) $a : A(3;5;-1) \; \vec{a} = \begin{pmatrix} -2 \\ 4 \\ -6 \end{pmatrix}$ $\qquad b : B(0;11;-10) \; \vec{b} = \begin{pmatrix} 1/2 \\ -1 \\ 3/2 \end{pmatrix} // \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

$a : \begin{cases} x = 3 - 2t \\ y = 5 + 4t \\ z = -1 - 6t \end{cases} \qquad b : \begin{cases} x = \frac{1}{2}k \\ y = 11 - k \\ z = -10 + \frac{3}{2}k \end{cases}$

$\begin{cases} 3 - 2t = \frac{1}{2}k \\ 5 + 4t = 11 - k \\ -1 - 6t = -10 + \frac{3}{2}k \end{cases} \Leftrightarrow \begin{cases} 6 - 4t = k \\ -6 + 4t = -k \\ -2 - 12t = -12 + 3k \end{cases} \Leftrightarrow \begin{cases} k = 6 - 4t \quad *(-3) \\ 3k = 10 - 12t \end{cases}$

$\begin{cases} -3k = -18 + 12t \\ 3k = 10 - 12t \end{cases}$
$+$
$0 = -8 \;\; ↯$

$\Rightarrow$ Il n'y a pas de points communs $\Rightarrow$ // gauches

On observe que $\vec{a} = -4\vec{b} \Rightarrow a // b$

2) $a = \begin{cases} x = 1+2\lambda \\ y = -\lambda \\ z = 1+\lambda \end{cases}$ $\qquad b : \begin{cases} x = -1+3\mu \\ y = 1 \\ z = 2+\mu \end{cases}$

$\begin{cases} 1+2\lambda = -1+3\mu \\ -\lambda = 1 \\ 1+\lambda = 2+\mu \end{cases} \Leftrightarrow \begin{cases} 2\lambda = -2+3\mu \\ \lambda = -1 \\ \lambda = 1+\mu \end{cases} \quad -1 = 1+\mu \Rightarrow \mu = -2$

① $\Rightarrow -2 = -2-6$ ↯ $\Rightarrow \begin{cases} // \\ \text{gauches} \end{cases}$

$\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

On suppose que $\vec{a} = k \cdot \vec{b} \Rightarrow \begin{cases} 2 = 3k \\ -1 = k \cdot 0 \Rightarrow -1 = 0 \text{ ↯} \\ 1 = k \end{cases}$

$\Rightarrow \nexists k \Rightarrow \vec{a} \not\parallel \vec{b} \Rightarrow$ a, b sont gauches

3) $a : A(1;0;4) \quad \vec{a} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \qquad b : \begin{cases} x = -6\lambda \\ y = -1-3\lambda \\ z = 2+12\lambda \end{cases}$

$a = \begin{cases} x = 1+2t \\ y = t \\ z = 4-4t \end{cases}$

$\begin{cases} 1+2t = -6\lambda \\ t = -1-3\lambda \\ 4-4t = 2+12\lambda \end{cases} \Leftrightarrow \begin{cases} 2t = -1-6\lambda \\ t = -1-3\lambda \\ 2t = 1-6\lambda \end{cases} \Leftrightarrow \begin{cases} 2t = -1-6\lambda & ① \\ t = -1-3\lambda & ② \\ 2t = 1-6\lambda & ③ \end{cases}$

① − ③ $\quad 0 = -2$ ↯ $\Rightarrow \begin{cases} // \\ \text{gauches} \end{cases}$

$\vec{a} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -6 \\ -3 \\ 12 \end{pmatrix} // \begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix} // \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \Rightarrow \vec{a} // \vec{b} \Rightarrow$

a // b

4) $a : \begin{cases} x = 1+2\lambda \\ y = 3-\lambda \\ z = 1+\lambda \end{cases} \qquad b : \begin{cases} x = 4-\mu \\ y = 3-\mu \\ z = 4-2\mu \end{cases}$

$\begin{cases} 1+2\lambda = 4-\mu \\ 3-\lambda = 3-\mu \\ 1+\lambda = 4-2\mu \end{cases} \Leftrightarrow \begin{cases} \mu = 3-2\lambda \\ \mu = \lambda \\ 2\mu = 3-\lambda \end{cases} \quad 3-2\lambda = \lambda \Rightarrow \lambda = 1 = \mu$

$2 = 3-1$ ✓

$\Rightarrow$ sécantes $\qquad I(-6;-2;14)$

EXERCICE 5

1) $d: \begin{cases} x = 2-2\lambda \\ y = 1+\lambda \\ z = 2\lambda \end{cases}$ $\pi: x+2y-2z-6=0$

$d \to \pi$: $(2-2\lambda)+2(1+\lambda)-2\cdot 2\lambda-6=0 \Leftrightarrow$

$\Leftrightarrow 2-\cancel{2\lambda}+2+\cancel{2\lambda}-4\lambda-6=0 \Leftrightarrow$

$\Leftrightarrow -4\lambda-4=0 \Leftrightarrow -4=4\lambda \Leftrightarrow \lambda=-1$

Donc d coupe le plan au le point

$A(4;0;-2)$ (en remplaçant en d).

2) d: $A(3;1;2)$ $B(-1;2;0)$ $\pi: x+2y-z-3=0$

$\vec{AB}=\begin{pmatrix}-4\\1\\-2\end{pmatrix}$ $d=\begin{cases} x=3-4t \\ y=1+t \\ z=2-2t \end{cases}$

$d \to \pi$ $3-4t+2(1+t)-(2-2t)-3=0 \Leftrightarrow$

$\Leftrightarrow \cancel{3}-\cancel{4t}+\cancel{2}+\cancel{2t}-\cancel{2}+\cancel{2t}-\cancel{3}=0 \Leftrightarrow 0=0$

Alors il y a une infinité de points communs $\Rightarrow d \in \pi$

3) $d: \begin{cases} x=1+5\lambda \\ y=1-3\lambda \\ z=3 \end{cases}$ $\pi: \begin{cases} x=1-\mu+2\sigma & (1) \\ y=1+\mu-\sigma & (2) \\ z=-3-2\mu-\sigma & (3) \end{cases}$

Equation cartésienne de π.

$(1)+(2)$ $x+y=2+\sigma$

$2\times(2)$ $2y=2+2\mu-2\sigma$

$+$ $z=-3-2\mu-\sigma$

$2y+z=-1-3\sigma$ $\times 3$

$+$ $3x+3y=6+3\sigma$

$\boxed{3x+5y+z=5}$

$d \to \pi \quad 3(1+5\lambda)+5(1-3\lambda)+3=5 \Leftrightarrow$

$\Leftrightarrow 3+\cancel{15\lambda}+5-\cancel{15\lambda}+3=5$

$8 = 2$ ↯ Impossible $\Rightarrow d \parallel \pi$

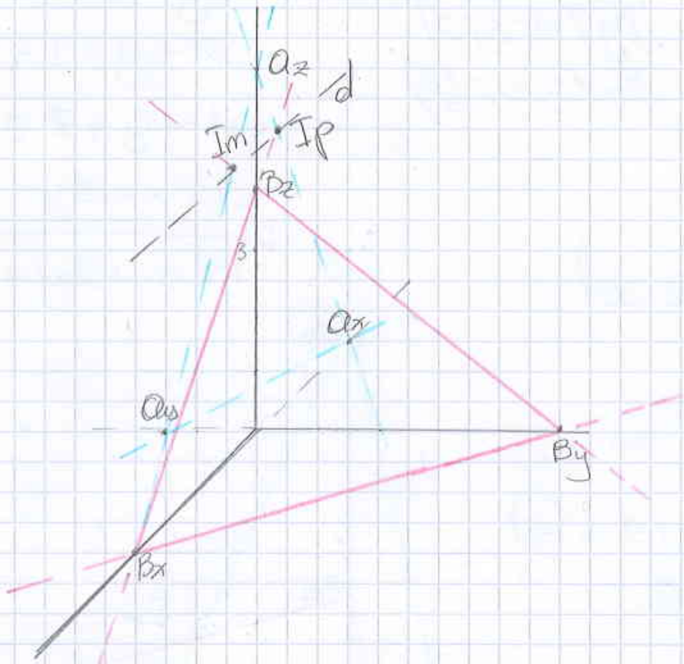
EXERCICE 6

1) $\alpha: -2x-4y+z-6=0 \qquad \beta: 5x+4y+5z-20=0$

$\alpha_x: (-3;0;0) \qquad \beta_x\ (4;0;0)$

$\alpha_y: (0;-\frac{3}{2};0) \qquad \beta_y\ (0;5;0)$

$\alpha_z: (0;0;6) \qquad \beta_z\ (0;0;4)$



$\alpha \cap \beta$ dans le mur

$x=0$

$-4y+z=6$

$+\quad 4y+5z=20$

$6z=26 \Rightarrow z=\frac{13}{3}$

$-4y+\frac{13}{3}=6 \Rightarrow -4y=\frac{18-13}{3}$

$\Rightarrow -4y=\frac{5}{3} \Rightarrow y=-\frac{5}{12}$

$I_{mur}\left(0;-\frac{5}{12};\frac{13}{3}\right)$

$\alpha \cap \beta$ dans le paroi

$y=0$

$\begin{cases}-2x+z=6\\5x+5z=20\end{cases} \Rightarrow$

$-\begin{cases}-2x+z=6\\x+z=4\end{cases}$

$-3x=2 \Rightarrow x=-\frac{2}{3}$

$-\frac{2}{3}+z=4 \Leftrightarrow z=4+\frac{2}{3}$

$\Leftrightarrow z=\frac{14}{3}$

$I_{paroi}=\left(-\frac{2}{3};0;\frac{14}{3}\right)$

$\overrightarrow{I_m I_p}=\begin{pmatrix}-2/3\\5/12\\1/3\end{pmatrix} \parallel \begin{pmatrix}-2\\5/4\\1\end{pmatrix} \parallel \begin{pmatrix}-8\\5\\4\end{pmatrix}$

$d: \begin{cases}x=-8t\\y=-\frac{5}{12}+5t\\z=\frac{13}{3}+4t.\end{cases}$

2) $\alpha: 3x+4y+2z-12=0$ $\quad \beta: P(2;1;-2)\ Q(-1;5;2)$
$R(0;3;2)$

$\beta:\ \vec{PQ}=\begin{pmatrix}-3\\4\\4\end{pmatrix} \quad \vec{PR}=\begin{pmatrix}-2\\2\\4\end{pmatrix} \parallel \begin{pmatrix}-1\\1\\2\end{pmatrix}$

$\beta: \begin{cases} x=2-3t-k & (1)\\ y=1+4t-k & (2)\\ z=-2+4t+2k & (3)\end{cases}$

$(1)-(2): x-y=1-7t$

$2\times(2) \quad 2y=2+8t-2k$
$(3) \quad z=-2+4t+2k$
$+$

$2y+z=12t \quad *7$
$x-y=1-7t \quad *12$

$\begin{cases}14y+7z=84t\\ 12x-12y=12-84t\end{cases}$
$+$

$12x+2y+7z=12 \ :\beta$

$\alpha_x(4;0;0) \quad \beta_x(1;0;0)$
$\alpha_y(0;3;0) \quad \beta_y(0;6;0)$
$\alpha_z(0;0;6) \quad \beta_z(0;0;\frac{12}{7})$

$\alpha\cap\beta$ dans le sol
$z=0$

$\begin{cases}3x+4y=12\\ 12x+2y=12\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases}3x+4y=12 \quad *(-2)\\ 6x+y=6\end{cases}$

$\begin{cases}-6x-8y=-24\\ 6x+y=6\end{cases}$

$-7y=-18$

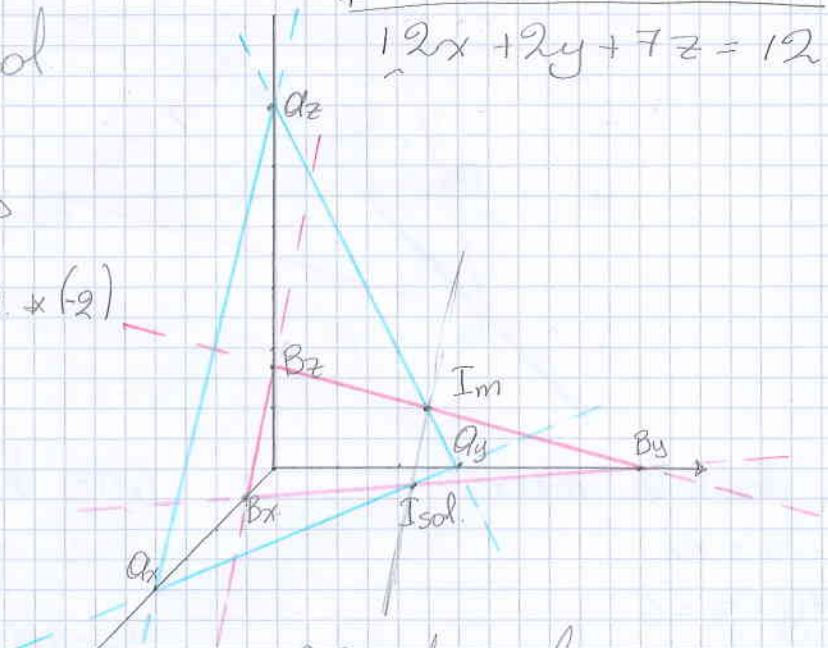
$y=\frac{18}{7}$

$6x+\frac{18}{7}=6 \Leftrightarrow$

$\Leftrightarrow 6x=\frac{42-18}{7} \Leftrightarrow$

$\Leftrightarrow 6x=\frac{24}{7} \Rightarrow x=\frac{24}{42}=\frac{4}{7}$

$I_{sol}\left(\frac{4}{7};\frac{18}{7};0\right)$



$\alpha\cap\beta$ dans le mur $x=0$

$\begin{rcases}4y+2z=12\\ 2y+7z=12\end{rcases} \Leftrightarrow \begin{aligned}2y+z&=6\\ -\ \ 2y+7z&=12\end{aligned} \ -$

$-6z=-6 \Rightarrow z=1$

$2y+1=6 \Rightarrow 2y=5 \Rightarrow$
$y=\frac{5}{2}$

$I_{mur}\left(0;\frac{5}{2};1\right)$

$\vec{I_{sol}I_{mur}} \begin{pmatrix}-\frac{4}{7}\\ -\frac{1}{14}\\ 1\end{pmatrix}$

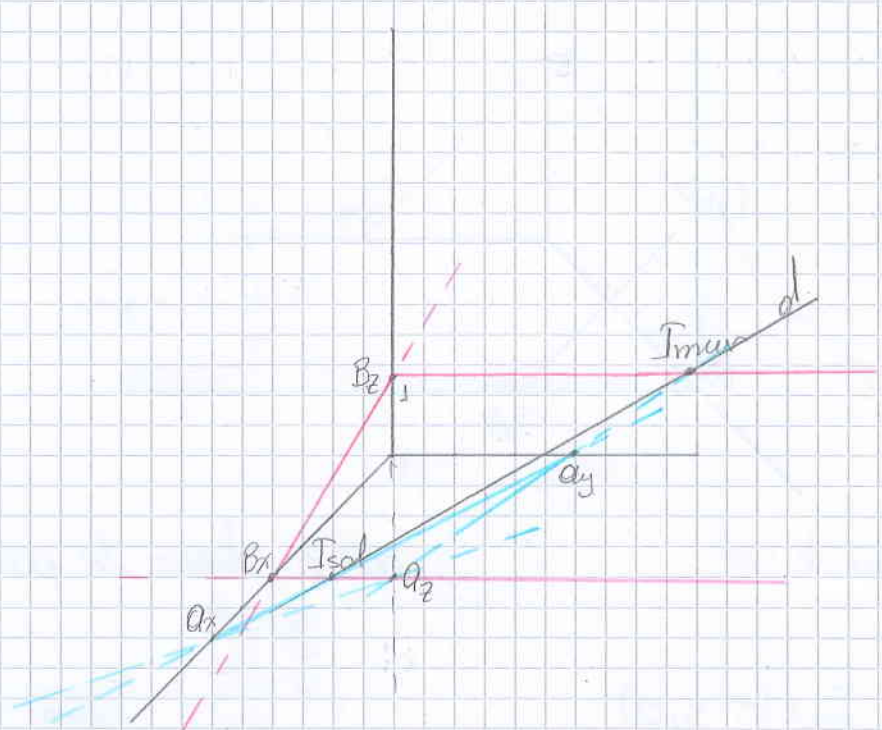
$d: \begin{cases}x=-\frac{4}{7}t\\ y=\frac{5}{2}-\frac{1}{14}t\\ z=1+t\end{cases}$

3) $\alpha: x+2y-3z-6=0$ $\qquad$ $\beta: x+3z-4=0$

$\alpha_x: (6;0;0)$ $\qquad$ $\beta_x (4;0;0)$

$\alpha_y: (0;3;0)$ $\qquad$ β_y impossible

$\alpha_z: (0;0;-2)$ $\qquad$ $\beta_z \left(0;0;\frac{4}{3}\right)$



$\alpha \cap \beta$ dans le sol

$z=0$

$\begin{cases} x+2y=6 \\ x=4 \end{cases} \Rightarrow 2y=2 \Rightarrow y=1$ $\quad$ $I_{sol}(4;1;0)$

$\alpha \cap \beta$ dans le mur

$x=0$

$2y-3z=6 \Leftrightarrow 2y-4=6 \Leftrightarrow 2y=10 \Leftrightarrow y=5$

$3z=4 \Rightarrow z=\frac{4}{3}$

$I_{mur}\left(0;5;\frac{4}{3}\right)$

$\overrightarrow{I_s I_m} = \begin{pmatrix} -4 \\ 4 \\ 4/3 \end{pmatrix} \parallel \begin{pmatrix} -1 \\ 1 \\ 1/3 \end{pmatrix} \parallel \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$

$$\boxed{d = \begin{cases} x = 4-3t \\ y = 1+3t \\ z = t \end{cases}}$$

4) $\alpha : x - 5 = 0$

$\alpha_x (5;0;0)$

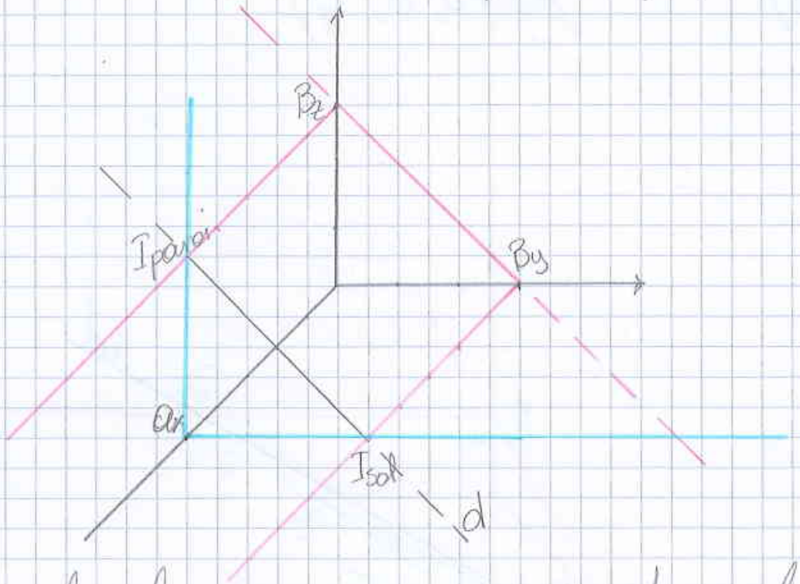
α_y, α_z Impossible

$\beta : y + z - 3 = 0$

β_x Impossible

$\beta_y : (0;3;0)$

$\beta_z : (0;0;3)$



$\alpha \cap \beta$ dans le sol

$z = 0$

$\begin{cases} x = 5 \\ y = 3 \end{cases}$ $I_{sol} (5;3;0)$

$\overrightarrow{I_S I_P} = \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix}$

$\alpha \cap \beta$ dans le paroi

$y = 0$

$\begin{cases} x = 5 \\ z = 3 \end{cases}$ $I_{paroi} (5;0;3)$

$$d = \begin{cases} x = 5 \\ y = 3 - 3t \\ z = 3t \end{cases}$$

EXERCICE 7

$a: \begin{cases} x = 1+2\lambda \\ y = -\lambda \\ z = 1+2\lambda \end{cases} \quad \vec{a} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \qquad b: \begin{cases} x = -1+\mu \\ y = 1-2\mu \\ z = 2-\mu \end{cases} \quad \vec{b} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

1) Facilement on regarde que $\vec{a} \not\parallel \vec{b}$ donc

a, b sont $\begin{cases} \text{sécantes} \\ \text{gauches} \end{cases}$

$\begin{cases} 1+2\lambda = -1+\mu & (1) \\ -\lambda = 1-2\mu & (2) \\ 1+2\lambda = 2-\mu & (3) \end{cases}$

$(1)+(3) \Rightarrow 2+4\lambda = 1 \Rightarrow 4\lambda = -1 \Rightarrow$
$\Rightarrow \lambda = -1/4$
$(3): 1 - 2\left(\frac{1}{4}\right) = 2-\mu \Leftrightarrow$
$\Leftrightarrow 1 - \frac{1}{2} - 2 = -\mu \Leftrightarrow -\frac{3}{2} = -\mu \Rightarrow$
$\Leftrightarrow \mu = \frac{3}{2}$

On remplace en (2):

$\frac{1}{4} = 1 - 2 \cdot \frac{3}{2} \Rightarrow \frac{1}{4} = -2$ ↯ Impossible

$\Rightarrow$ Il n'existe pas de points communs

$\Rightarrow$ a, b sont gauches

2) plan α: point $A(1;0;1)$
vecteurs: $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

$\alpha: \begin{cases} x = 1+2t+k & (1) \\ y = -t-2k & (2) \\ z = 1+2t-k & (3) \end{cases}$

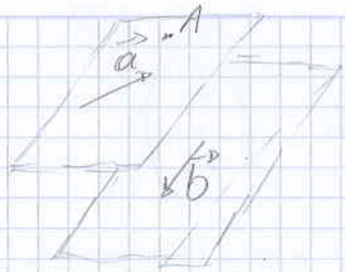
$(1)+(3) \quad x+z = 2+4t$

$+\begin{cases} 2\times(1) & 2x = 2+4t+2k \\ (2) & y = -t-2k \end{cases}$

$\begin{cases} 2x+y = 2+3t & \times(-4) \\ x+z = 2+4t & \times(3) \end{cases}$

$\Rightarrow \begin{cases} -8x-4y = -8-12t \\ 3x+3z = 6+12t \end{cases}$

$\boxed{-5x-4y+3z=-2} \quad \underline{\underline{\alpha}}$



Comme $\alpha // \beta$ on a :

$\beta : -5x - 4y + 3z + c = 0$

Mais le point $B(-1;1;2) \in \beta$

Donc : $-5\cdot(-1) - 4\cdot 1 + 3\cdot 2 + c = 0 \Leftrightarrow$

$\Leftrightarrow 5 - 4 + 6 + c = 0 \Leftrightarrow c = -7$

D'où $\boxed{\beta : -5x - 4y + 3z - 7 = 0}$

EXERCICE 8

$\pi : 6x + 4y + 3z - 24 = 0 \qquad A(1;3;2)$

1) $A \in \pi : \; 6\cdot 1 + 4\cdot 3 + 3\cdot 2 - 24 = 6 + 12 + 6 - 24 = 0$ ✓

2) $\pi_x : z = y = 0 \Rightarrow x = 4 \quad \pi_x(4;0;0)$
$\pi_y : x = z = 0 \Rightarrow y = 6 \quad \pi_y(0;6;0)$
$\pi_z : x = y = 0 \Rightarrow z = 8 \quad \pi_z(0;0;8)$

$h // \pi_x\pi_y$

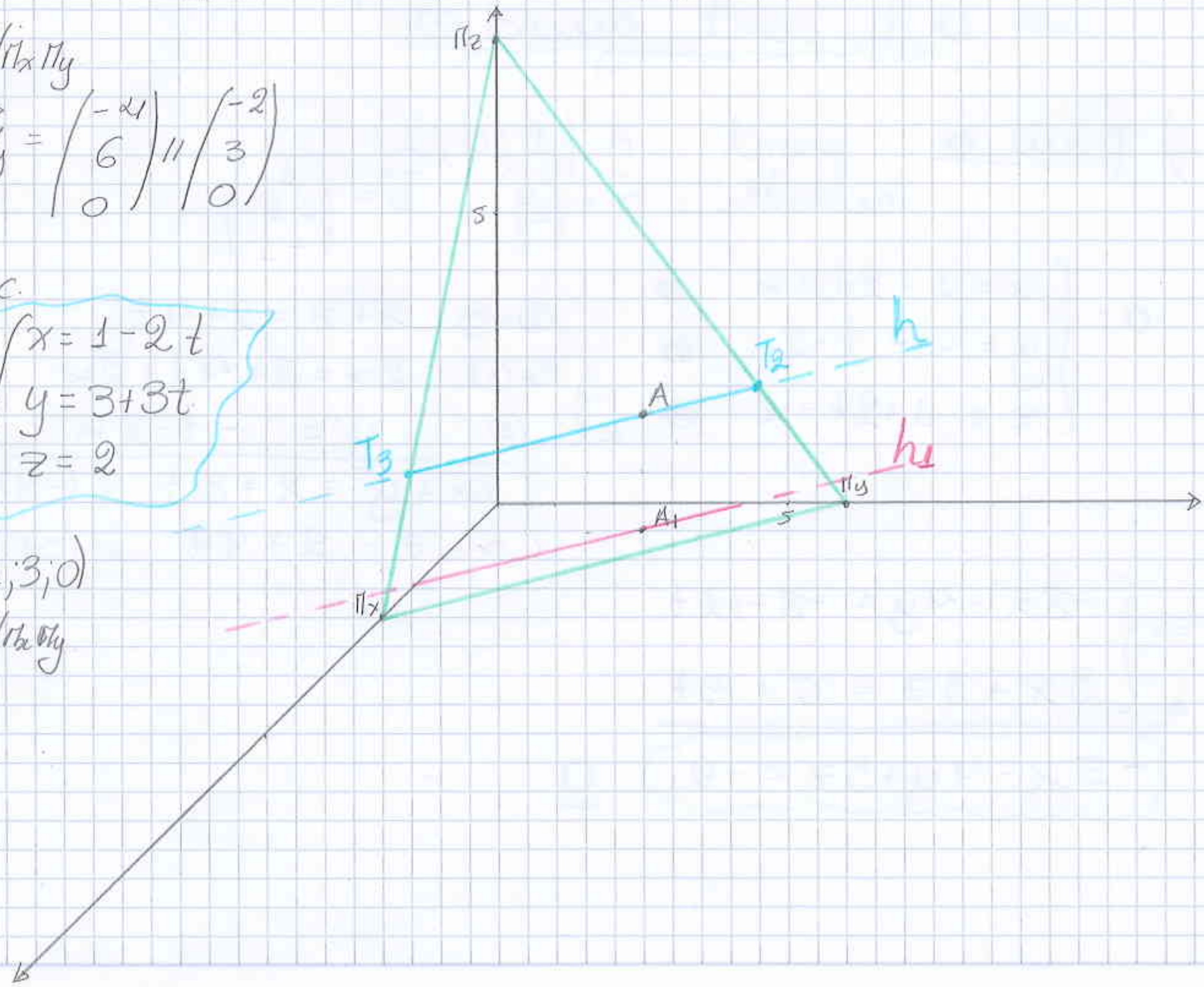
$\overrightarrow{\pi_x\pi_y} = \begin{pmatrix} -4 \\ 6 \\ 0 \end{pmatrix} // \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$

donc

$$\boxed{h : \begin{cases} x = 1 - 2t \\ y = 3 + 3t \\ z = 2 \end{cases}}$$

$A_1(1;3;0)$

$h_1 // \pi_x\pi_y$



EXERCICE 9

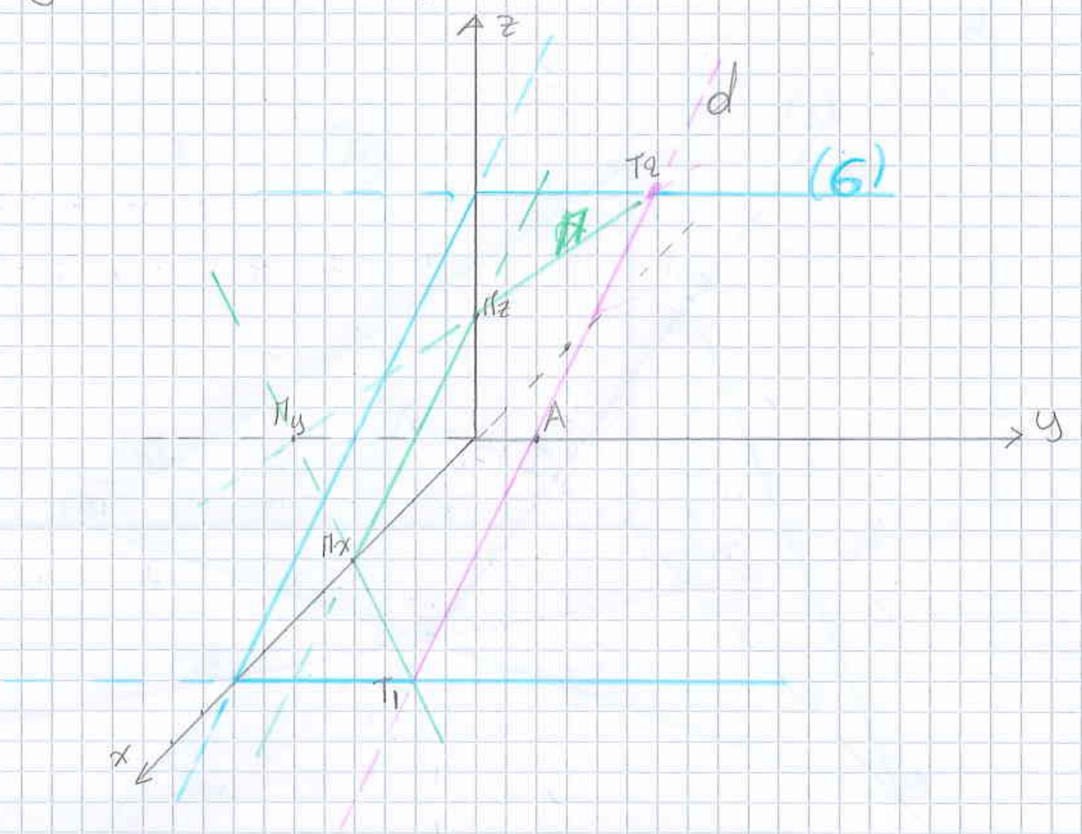
$\Pi : 3x - 4y + 6z - 12 = 0 \qquad A(4;3;2) \in \Pi$

Vérification : $12 - 12 + 12 - 12 = 0$

1) $\Pi_x : y = z = 0 \Rightarrow x = 4 \quad \Pi_x(4;0;0)$

$\Pi_y : x = z = 0 \Rightarrow y = -3 \quad \Pi_y(0;-3;0)$

$\Pi_z : x = y = 0 \Rightarrow z = 2 \quad \Pi_z(0;0;2)$



La droite d // au plan $xOz \Rightarrow d \,//\, \Pi_x\Pi_z$

$\overrightarrow{\Pi_x\Pi_z} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix} \parallel \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \qquad d : \begin{cases} x = 4 - 2t \\ y = 3 \\ z = 2 + t \end{cases}$

2) $A(4;3;2) \quad \vec{d} = \overrightarrow{\Pi_x\Pi_z} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$ Encore un vecteur est le vecteur de l'axe $Oy : \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

donc $6 : \begin{cases} x = 4 - 2t \\ y = 3 + k \\ z = 2 + t \end{cases}$

EXERCICE 10

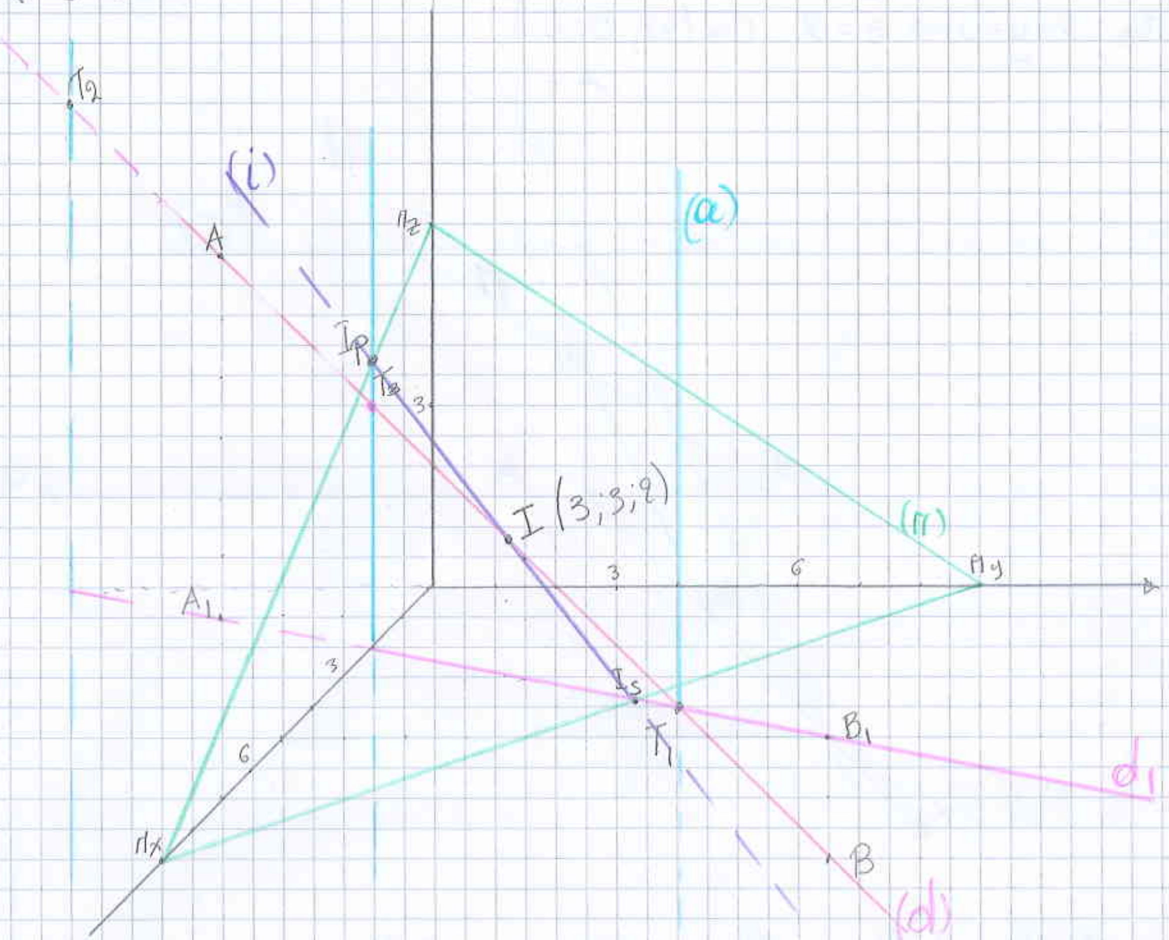
a) $\Pi : 2x+2y+3z-18=0$

$d : A(1;-3;6) \quad B(5;9;-2)$

1) $\Pi_x : (9;0;0) \quad A_1(1;-3;0) \quad B_1(5;9;0)$

$\Pi_y(0;9;0)$

$\Pi_z(0;0;6)$



(d): $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 12 \\ -8 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ $\quad d : \begin{cases} x = 1+t \\ y = -3+3t \\ z = 6-2t \end{cases}$

(α) : $A(1;-3;6)$ deux vecteurs : $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ et $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\alpha : \begin{cases} x = 1+t \\ y = -3+3t \\ z = 6-2t+k \end{cases}$ $\quad \times(-3) \quad \begin{array}{r} -3x = -3-3t \\ y = -3+3t \\ \hline -3x+y = -6 \end{array}$ (α)

(I_s) : $(z=0)$ $\quad 2x+2y=18 \Rightarrow x+y=9$

$\begin{array}{r} - \quad -3x+y=-6 \\ \hline 4x = 15 \end{array} \Rightarrow x = \frac{15}{4}$

$x = 9 - \frac{15}{4} \Rightarrow y = \frac{21}{4}$

$I_S \left(\frac{15}{4} ; \frac{21}{4} ; 0\right)$

$(I_P)\ (y=0)\quad \left.\begin{array}{l} 2x+3z=18 \\ -3x = -6 \end{array}\right\} \Rightarrow \begin{array}{l} 3z = 18-4 \Rightarrow 3z=14 \Rightarrow z=\frac{14}{3} \\ x=2 \end{array}$

$I_P \left(2 ; 0 ; \frac{14}{3}\right)$

$\overrightarrow{I_S I_P} = \begin{pmatrix} 2-\frac{15}{4} \\ -\frac{21}{4} \\ \frac{14}{3} \end{pmatrix} = \begin{pmatrix} -7/4 \\ -21/4 \\ 14/3 \end{pmatrix}$

$(i) \begin{cases} x = 2 - \frac{7}{4}k \\ y = -\frac{21}{4}k \\ z = \frac{14}{3} + \frac{14}{3}k \end{cases}$

$I!\ \left.\begin{array}{l} 1+t = 2-\frac{7}{4}k \\ -3+3t = -\frac{21}{4}k \\ 6-2t = \frac{14}{3}+\frac{14}{3}k \end{array}\right\} \Rightarrow \left.\begin{array}{l} 4+4t = 8-7k \\ -12+12t = -21k \\ 18-6t = 14+14k \end{array}\right\} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 4t = 4-7k & \times(-3) \\ 12t = 12-21k & ② \\ -6t = -4+14k & \times(2) \end{cases}$

$\begin{array}{r} -12t = -12+21k \\ + \quad 12t = 12-21k \\ \hline 0=0 \end{array}$

$\begin{array}{r} 12t = 12-21k \\ -12t = -8+28k \\ \hline 0 = 4+7k \end{array} \Rightarrow k = -4/7$

② $4t = 4-7k$

$k \rightarrow:\ 4t = 4+4 \Rightarrow$

$4t = 8 \Rightarrow t = 8/4 = 2$

On remplace en d le $t = 2$

$\boxed{I = (3 ; 3 ; 2)}$

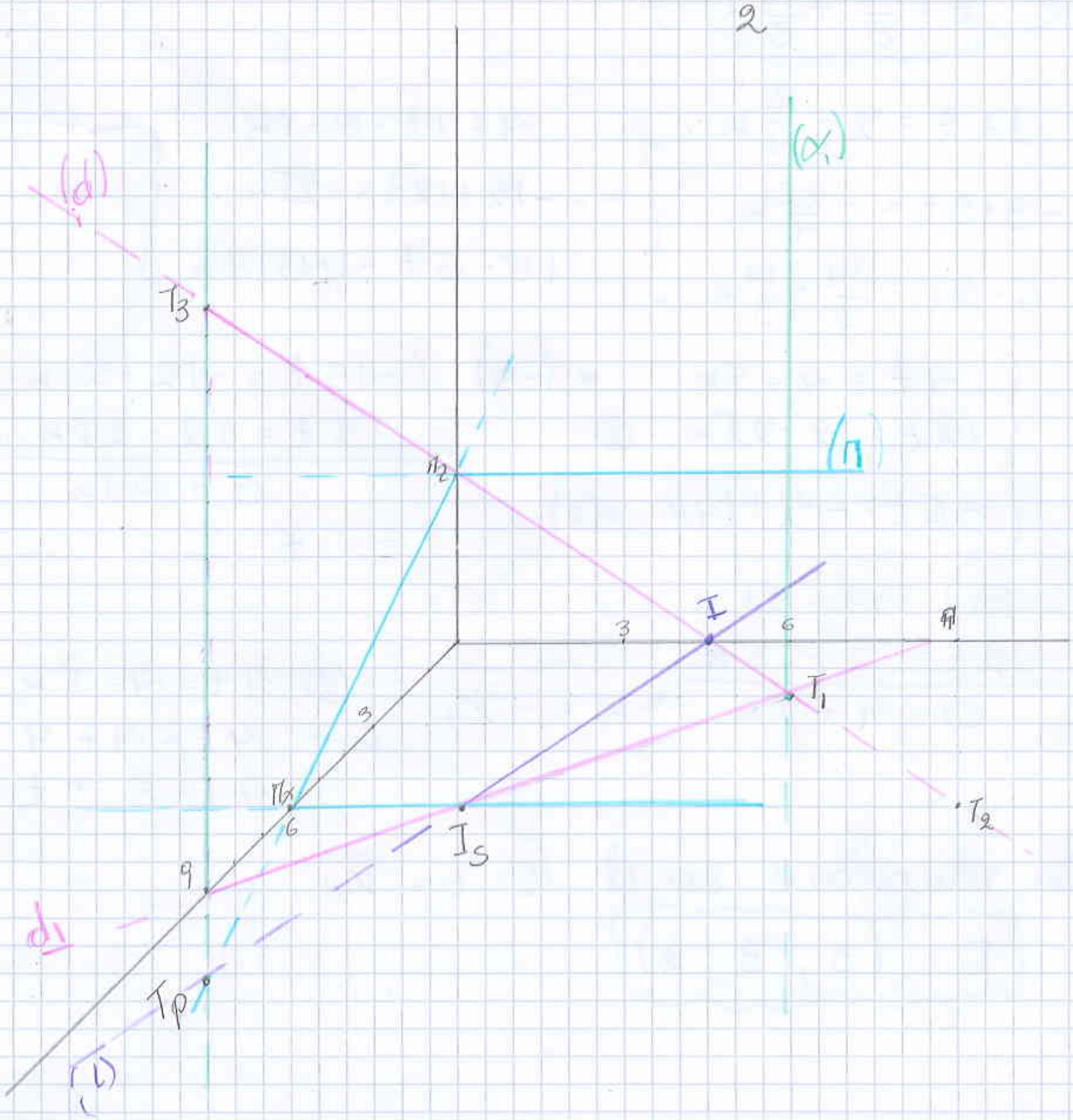
b) $\pi : x+2z-6=0$ $\quad d:\begin{cases} x=6+2\lambda \\ y=3-2\lambda \\ z=6+3\lambda \end{cases}$

$\pi_x : z=y=0 \quad \pi_x(6;0;0)$
$\pi_y : z=x=0 \quad$ Impossible $\quad \pi \parallel O_y$
$\pi_z : x=y=0 \quad \pi_z(0;0;3)$

$T_1 : z=0 \Rightarrow 6+3\lambda=0 \Leftrightarrow \lambda=-2 \Rightarrow \begin{matrix} x=2 \\ y=7 \end{matrix} \quad T_1(2;7;0)$

$T_2 : x=0 \Rightarrow 6+2\lambda=0 \Leftrightarrow \lambda=-3 \Rightarrow \begin{matrix} y=9 \\ z=-3 \end{matrix} \quad T_2(0;9;-3)$

$T_3 : y=0 \Rightarrow 3-2\lambda \Leftrightarrow \lambda=\frac{3}{2} \Rightarrow \begin{matrix} x=9 \\ z=\frac{21}{2} \end{matrix} \quad T_3(9;0;\frac{21}{2})$



(α): Point $A\,(6;3;6)$

Vecteurs: $\begin{pmatrix}2\\-2\\3\end{pmatrix} /\!/ d \quad \begin{pmatrix}0\\0\\1\end{pmatrix} /\!/ Oz$

$$\alpha : \left\{\begin{array}{l} x = 6+2t \\ y = 3-2t \\ z = 6+3t+k \end{array}\right\} + \quad \boxed{x+y=9} \; (\alpha)$$

(I_s): $z=0$ $\quad \left\{\begin{array}{l} x=6 \\ x+y=9 \end{array}\right. \quad \left\{\begin{array}{l} x=6 \\ y=3 \end{array}\right. \quad I_s\,(6;3;0)$

(I_p): $y=0$ $\quad \left\{\begin{array}{l} x+2z=6 \\ x=9 \end{array}\right. \Leftrightarrow \begin{array}{l} 2z=-3 \\ x=9 \end{array} \Leftrightarrow \left\{\begin{array}{l} z=-3/2 \\ x=9 \end{array}\right.$

$I_p\,(9;0;-3/2)$

(i) passe par I_s, I_p

$$\overrightarrow{I_sI_p} = \begin{pmatrix}3\\-3\\-3/2\end{pmatrix} /\!/ \begin{pmatrix}1\\-1\\-1/2\end{pmatrix} /\!/ \begin{pmatrix}2\\-2\\-1\end{pmatrix}$$

$$(i) \left\{\begin{array}{l} x = 6+2t \\ y = 3-2t \\ z = -t \end{array}\right. \qquad (d) \left\{\begin{array}{l} x = 6+2\lambda \\ y = 3-2\lambda \\ z = 6+3\lambda \end{array}\right.$$

$$(I): \left.\begin{array}{l} 6+2t = 6+2\lambda \\ 3-2t = 3-2\lambda \\ -t = 6+3\lambda \end{array}\right\} \quad \left.\begin{array}{l} -2t = -2\lambda \\ t = -6-3\lambda \end{array}\right\} \Leftrightarrow \begin{array}{l} t=\lambda \\ t=-6-3t \end{array} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} t=\lambda \\ 4t=-6 \end{array}\right. \Rightarrow \left\{\begin{array}{l} t=\lambda \\ t=-3/2 \end{array}\right. \Rightarrow t=\lambda=-3/2$$

En remplaçant en (i) $I\,(3;6;\frac{3}{2})$

c) $\Pi: \; z - 3 = 0$

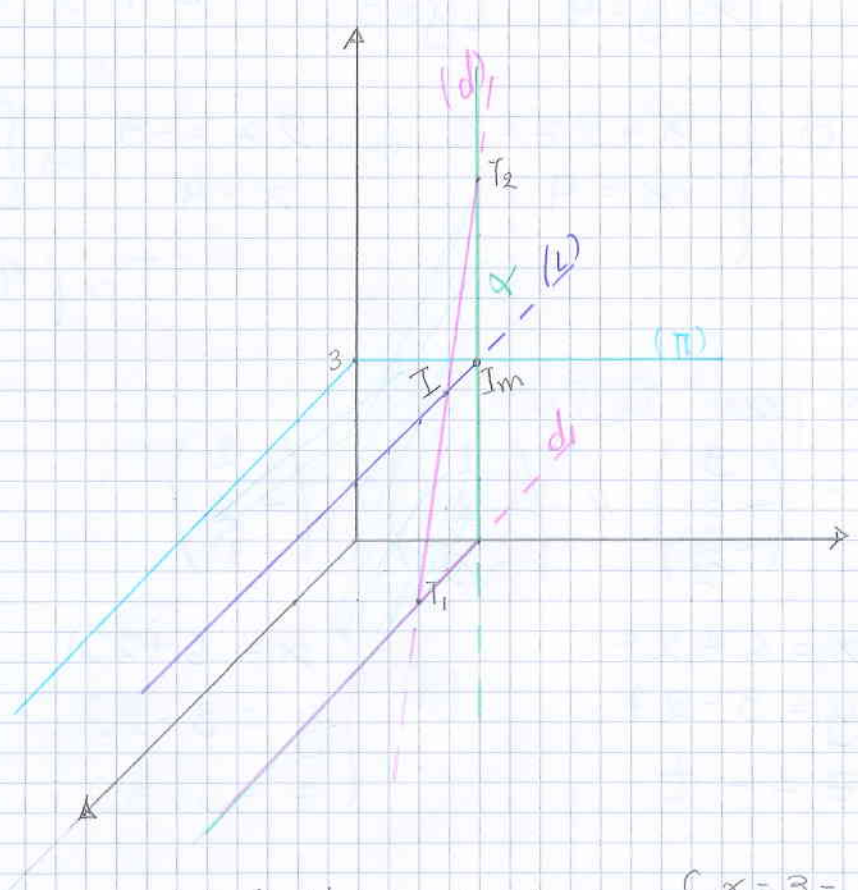
$$d = \begin{cases} x = 3 - \lambda \\ y = 2 \\ z = -3 + 3\lambda \end{cases}$$

Π_x : Impossible
Π_y : Impossible
Π_z $(0;0;3)$

$T_1: z = 0 \Rightarrow \lambda = 1 \Rightarrow \begin{matrix} x = 2 \\ y = 2 \end{matrix} \quad T_1(2;2;0)$

$T_2: x = 0 \Rightarrow \lambda = 3 \Rightarrow \begin{matrix} y = 2 \\ z = 6 \end{matrix} \quad T_2(0;2;6)$

T_3: $y = 0$ Impossible



(α): $A(3;2;-3)$ $\vec{d} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\alpha = \begin{cases} x = 3 - t \\ y = 2 \\ z = -3 + 3t + k \end{cases}$

$\alpha: \quad y = 2$

$Im: \begin{cases} z = 3 \\ y = 2 \end{cases} \quad \left.\begin{matrix} Im = (0;2;3) \\ i \,//\, Ox \,//\, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{matrix}\right\} \quad i = \begin{cases} x = t \\ y = 2 \\ z = 3 \end{cases} \qquad d = \begin{cases} x = 3 - \lambda \\ y = 2 \\ z = -3 + 3\lambda \end{cases}$

$I: \left.\begin{matrix} t = 3 - \lambda \\ 3 = -3 + 3\lambda \end{matrix}\right\} \Leftrightarrow \begin{cases} t = 3 - \lambda \\ 6 = 3\lambda \end{cases} \Leftrightarrow \begin{cases} t = 3 - \lambda \\ \lambda = 2 \end{cases} \Leftrightarrow \begin{cases} t = 1 \\ \lambda = 2 \end{cases}$

Donc $I(1;2;3)$