SERIE 1: Geometrie Espace

EXERCICE 1.

$\vec{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ -5 \\ 4 \end{pmatrix}$

1) Calculer $-3\vec{a} + 2\vec{b} = \begin{pmatrix} -9 \\ -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -10 \\ 8 \end{pmatrix} = \begin{pmatrix} -7 \\ -16 \\ 11 \end{pmatrix}$,

$\frac{1}{3}\vec{a} + \frac{5}{2}\vec{b} = \begin{pmatrix} 1 \\ 2/3 \\ -1/3 \end{pmatrix} + \begin{pmatrix} 5/2 \\ -25/2 \\ 10 \end{pmatrix} = \begin{pmatrix} 7/2 \\ -71/6 \\ 29/3 \end{pmatrix}$

2) $2\vec{a} - 4\vec{b} + 3\vec{c} = \vec{0} \Leftrightarrow \vec{c} = \frac{1}{3}(4\vec{b} - 2\vec{a}) \Rightarrow$

$= \frac{1}{3}\left[\begin{pmatrix} 4 \\ -20 \\ 16 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix}\right] = \frac{1}{3}\begin{pmatrix} -2 \\ -24 \\ 18 \end{pmatrix} = \begin{pmatrix} -2/3 \\ -8 \\ 6 \end{pmatrix}$

EXERCICE 2

1) a) $\vec{a} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}$

On suppose que $\vec{c} = \lambda\vec{a} + \mu\vec{b} \Leftrightarrow$

$\Leftrightarrow \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} = \lambda\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \Rightarrow \begin{array}{ll} 0 = -3\lambda + \mu & (1) \\ 5 = 2\lambda + \mu & (2) \\ 3 = \lambda - 2\mu & (3) \end{array}$

$$\begin{array}{ll} (1) & 0 = -3\lambda + \mu \\ -(2) & -5 = -2\lambda - \mu \\ \hline & -5 = -5\lambda \Leftrightarrow \underline{\lambda = 1} \end{array}$$

$(1) \Rightarrow 0 = -3 + \mu \Rightarrow \underline{\mu = 3}$

On remplace en (3) $3 = 1 - 6$ impossible

Donc on ne peut pas écrire $\vec{c}$ comme combinaison linéaire de $\vec{a}$ et $\vec{b}$

! Ne suffit pas. Il faut vérifier que $\vec{a} \nparallel \vec{b}$

On suppose $\vec{a} = \lambda\vec{b} \Leftrightarrow \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \Leftrightarrow \begin{array}{l} \lambda = -3 \\ \lambda = 2 \\ \lambda = 1/2 \end{array}$ Impossible

Donc $\vec{a}, \vec{b}, \vec{c}$ sont linéairement indépendants $\Rightarrow$ ils forment une base.

b) $\vec{a} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} -2 \\ 13 \\ 21 \end{pmatrix}$

Soit $\lambda, \mu \in \mathbb{R}$ tels que

$\vec{c} = \lambda \vec{a} + \mu \vec{b} \Leftrightarrow \begin{pmatrix} -2 \\ 13 \\ 2 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} \Leftrightarrow$

$\Leftrightarrow -2 = 3\lambda + \mu$

$13 = -2\lambda + 4\mu$

$2 = 4\lambda + 6\mu$

① $\times(-4)$ $\quad +8 = -12\lambda - 4\mu$

② $\quad + \quad 13 = -2\lambda + 4\mu$

$21 = -14\lambda \Leftrightarrow \lambda = -\frac{3}{2}$

① $-2 = -\frac{9}{2} + \mu \Rightarrow \mu = -2 + \frac{9}{2} \Rightarrow$

$\Rightarrow \mu = \frac{5}{2}$

On remplace en ③ $2 = 4\left(-\frac{3}{2}\right) + 6 \cdot \frac{5}{2} \Rightarrow$

$\Leftrightarrow 2 = -6 + 15$ ↯ Impossible

Soit $\vec{a} = m\vec{b} \Leftrightarrow \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} = m \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} \Leftrightarrow \begin{matrix} m = 3 \\ m = -\frac{1}{2} \\ m = \frac{2}{3} \end{matrix}$ ↯ Impos

Donc $\vec{a}, \vec{b}, \vec{c}$ forment une base

c) $\vec{a} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} 2 \\ -9 \\ 1 \end{pmatrix}$

Soit $k; \lambda \in \mathbb{R}$ tels que:

$\vec{c} = k\vec{a} + \lambda\vec{b} \Rightarrow \begin{cases} 2 = 4k + 2\lambda & ① \\ -9 = -3k + \lambda & ② \\ 1 = k + 5\lambda & ③ \end{cases}$

① $\quad 2 = 4k + 2\lambda$

② $\times(-2)$ $\quad 18 = 6k - 2\lambda$

$+$

$20 = 10k \Rightarrow k = 2$

① $\quad 2 = 8 + 2\lambda \Rightarrow \lambda = -3$

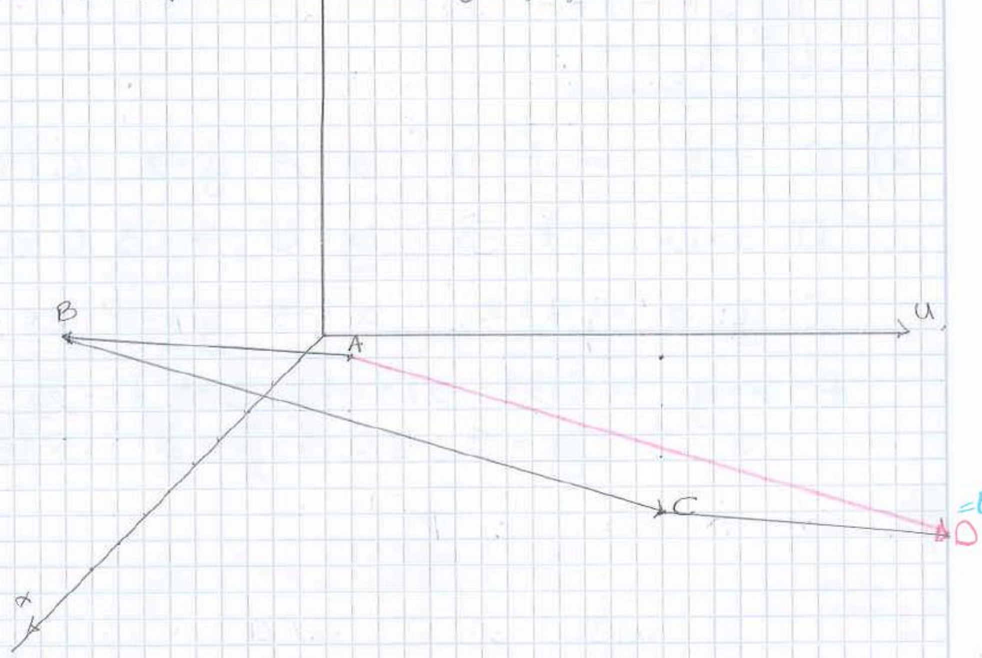
③ $\Rightarrow 1 \neq 2 - 15$ ↯ Impos

D'où $\vec{a}, \vec{b}, \vec{c}$ forment une base

EXERCICE 4

$A(3;2;1)$ $B(4;-3;2)$ $C(1;7;-3)$

1. $A_1(3;2;0)$ $B_1(4;-3;0)$ $C_1(1;7;0)$
$A_2(0;2;1)$ $B_2(0;-3;2)$ $C_2(0;7;-3)$
$A_3(3;0;1)$ $B_3(4;0;2)$ $C_3(1;0;-3)$



$\vec{AB} = \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} -2 \\ 5 \\ -4 \end{pmatrix}$ $\vec{A_1B_1} = \begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix}$ $\vec{A_2B_2} = \begin{pmatrix} 0 \\ -5 \\ 1 \end{pmatrix}$ $\vec{A_3B_3} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

3) $D(x_D, y_D, z_D)$

$$\vec{AD} = \vec{BC} \Rightarrow \begin{pmatrix} x_D - 3 \\ y_D - 2 \\ z_D - 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 10 \\ -5 \end{pmatrix} \Rightarrow \begin{cases} x_D = 0 \\ y_D = 12 \\ z_D = -4 \end{cases}$$

$$\vec{EC} = \vec{AB} \Rightarrow \begin{pmatrix} 1 - x_E \\ 7 - y_E \\ -3 - z_E \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix} \Leftrightarrow \begin{cases} x_E = 0 \\ y_E = 12 \\ z_E = -4 \end{cases}$$

$\Rightarrow E \equiv D$

EXERCICE 5

d : A(6;0;3) B(2;4;1)

1) $\vec{AB} = \begin{pmatrix} -4 \\ 4 \\ -2 \end{pmatrix} // \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ (On divise pour -2)

$d : \begin{cases} x = 6 + 2t \\ y = \quad -2t \\ z = 3 + t \end{cases}$

2) C : $z = 2 \Rightarrow t = -1$ $x = 4$ $y = 2$ $C(4;2;2)$

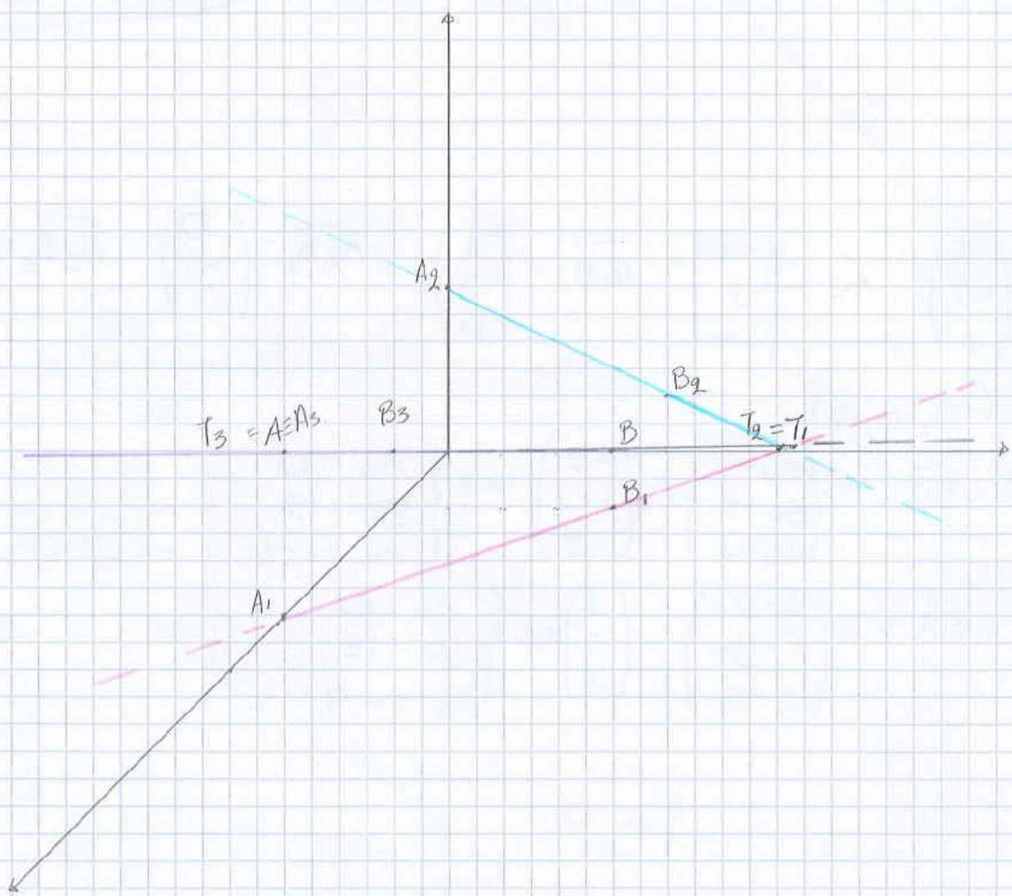
D : $x = 0 \Rightarrow t = -3$ $y = 0$ $z = 3$ $D(0;0;3)$

E : $y = -8 \Rightarrow t = 4$ $x = 14$ $y = -8$ $z = 7$ $E(14;-8;7)$

F : $x = y \Rightarrow 6 + 2t = -2t \Leftrightarrow 4t = -6 \Rightarrow t = -\frac{3}{2}$

$x = 3$ $y = 3$ $z = 3 - \frac{3}{2} = \frac{3}{2}$ $F(3;3;\frac{3}{2})$

3) T_1



EXERCICE 6

a) 1) d : A (4;2;3) B (2;4;3)

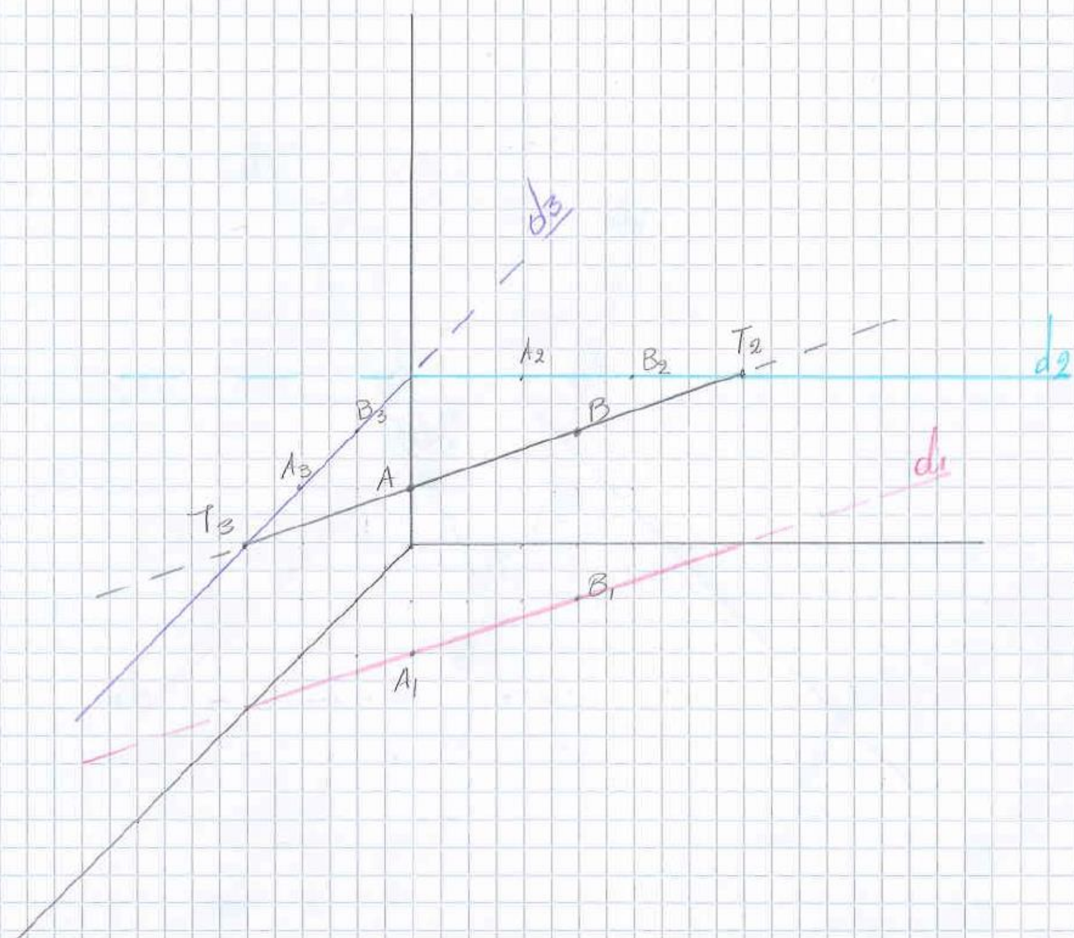
$\vec{AB} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} // \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ d: $\begin{cases} x = 4 - t \\ y = 2 + t \\ z = 3 \end{cases}$

2. T_1 : z = 0 Impossible ⇒ d // sol

T_2 : x = 0 ⇒ t = 4 y = 6 z = 3 T_2 (0;6;3)

T_3 : y = 0 ⇒ t = −2 x = 6 z = 3 T_3 (6;0;3)

3.



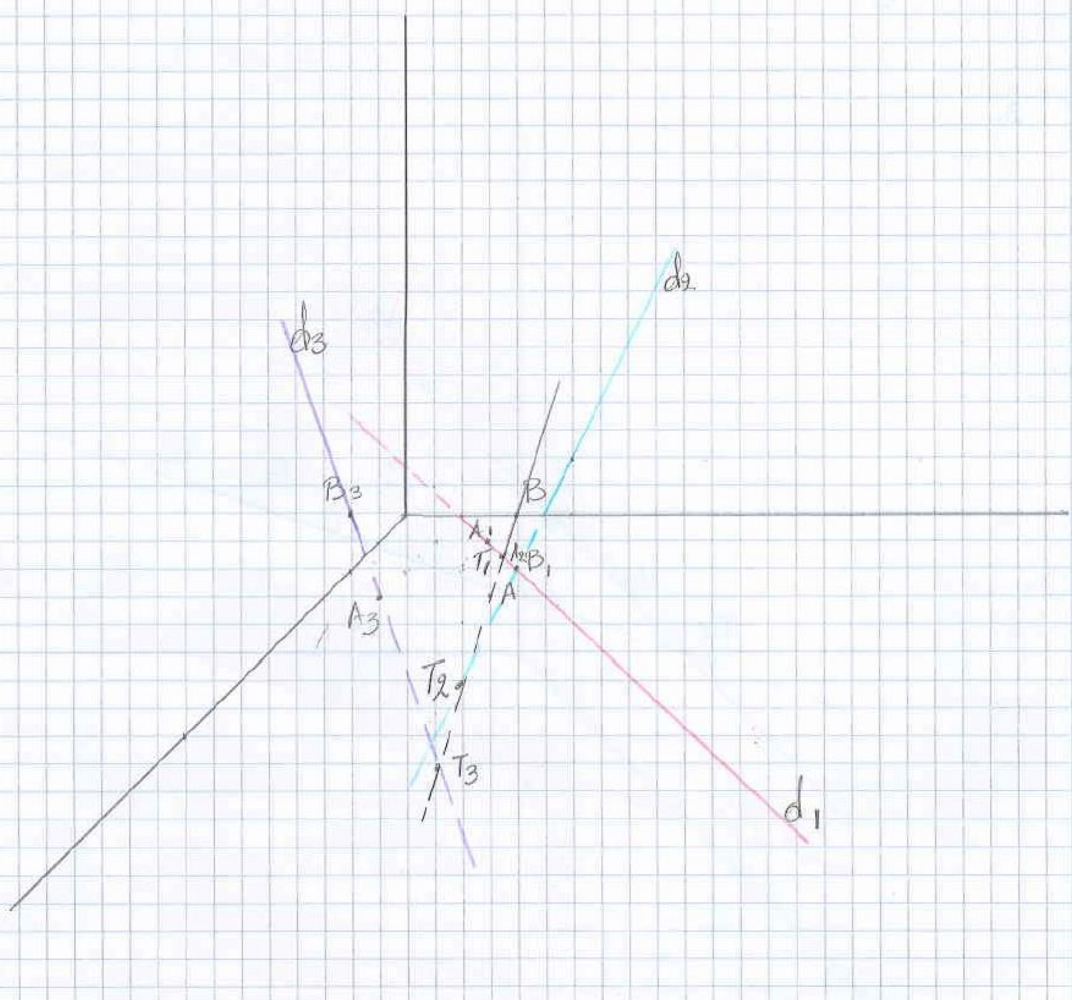
b) $d: A(1;2;-1) \quad B(2;3;1)$

1. $\vec{AB} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\quad d. \begin{cases} x = 1 + t \\ y = 2 + t \\ z = -1 + 2t \end{cases}$

2. $T_1: z = 0 \Rightarrow t = \frac{1}{2} \quad x = \frac{3}{2} \quad y = \frac{5}{2} \quad T_1\left(\frac{3}{2}; \frac{5}{2}; 0\right)$

$T_2: x = 0 \Rightarrow t = -1 \quad y = 1 \quad z = -3 \quad T_2(0; 1; -3)$

$T_3: y = 0 \Rightarrow t = -2 \quad x = -1 \quad z = -5 \quad T_3(-1; 0; -5)$



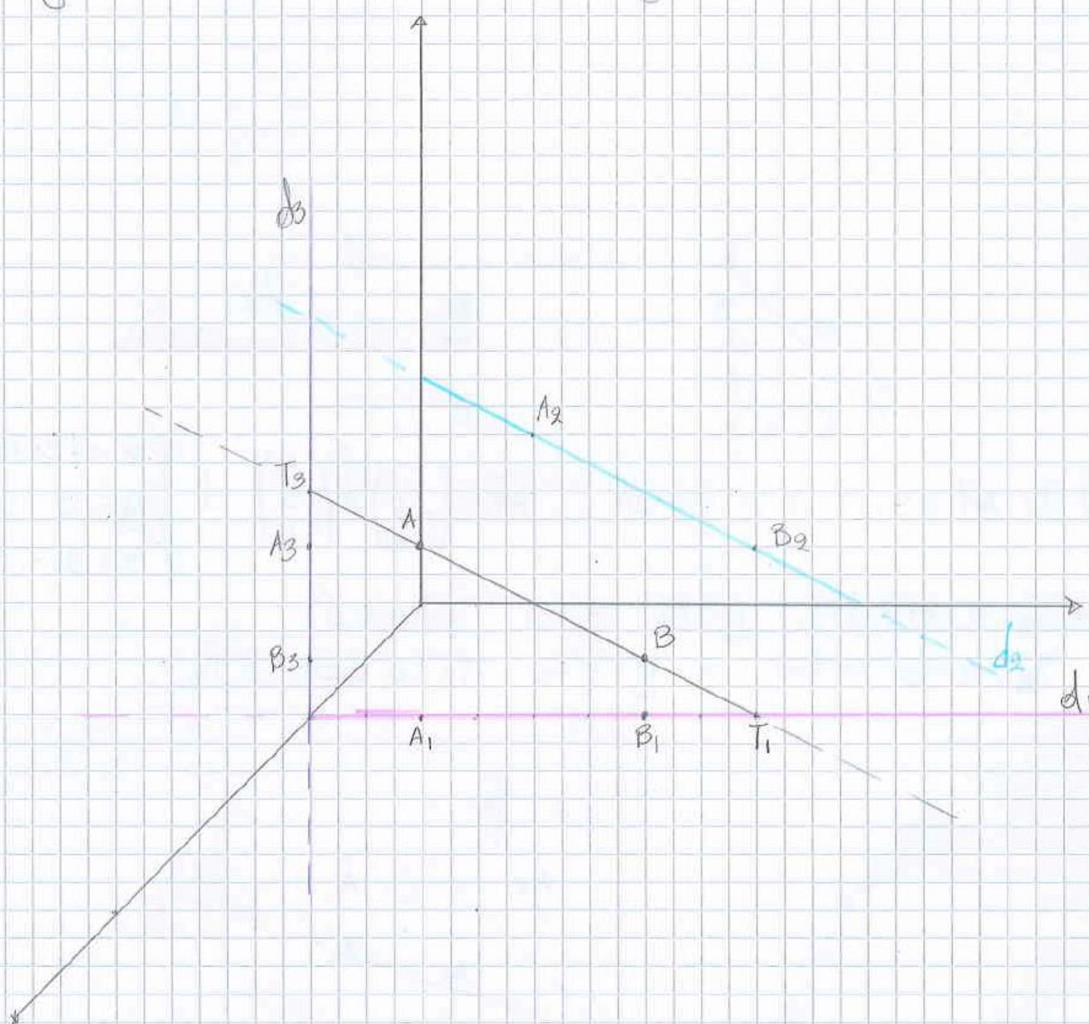
c) d : $A(4;2;3)$ $B(4;6;1)$

1. $\vec{AB} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} \parallel \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$ $\quad d: \begin{cases} x = 4 \\ y = 2 + 2t \\ z = 3 - t \end{cases}$

2. T_1 : $z = 0 \Leftrightarrow t = 3$ $\quad x = 4$ $\quad y = 8$ $\quad T_1(4;8;0)$

T_2 : $x = 0$ impossible $\Rightarrow d \parallel$ mur

T_3 : $y = 0 \Rightarrow t = -1$ $\quad x = 4$ $\quad y = 0$ $\quad z = 4$ $\quad T_3(4;0;4)$

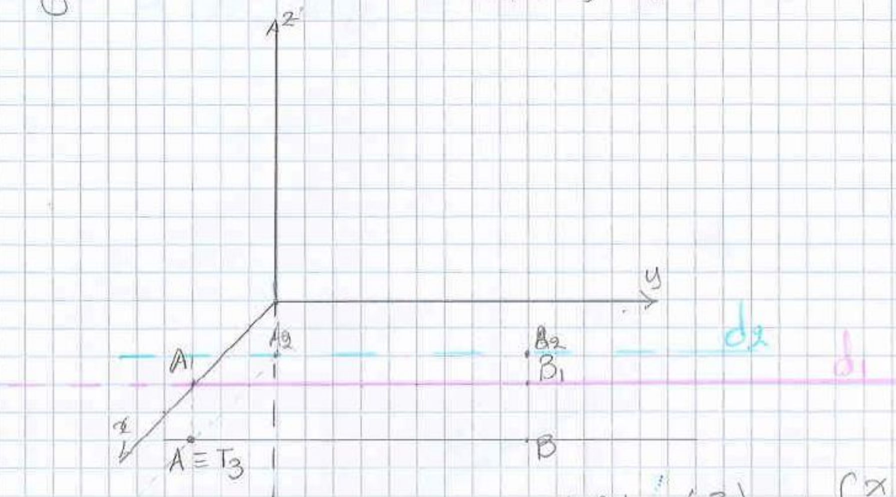


d) d: $A(3;0;-1)$ $B(3;6;-1)$

1. $\vec{AB} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} \parallel \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\quad d: \begin{cases} x = 3 \\ y = t \\ z = -1 \end{cases}$

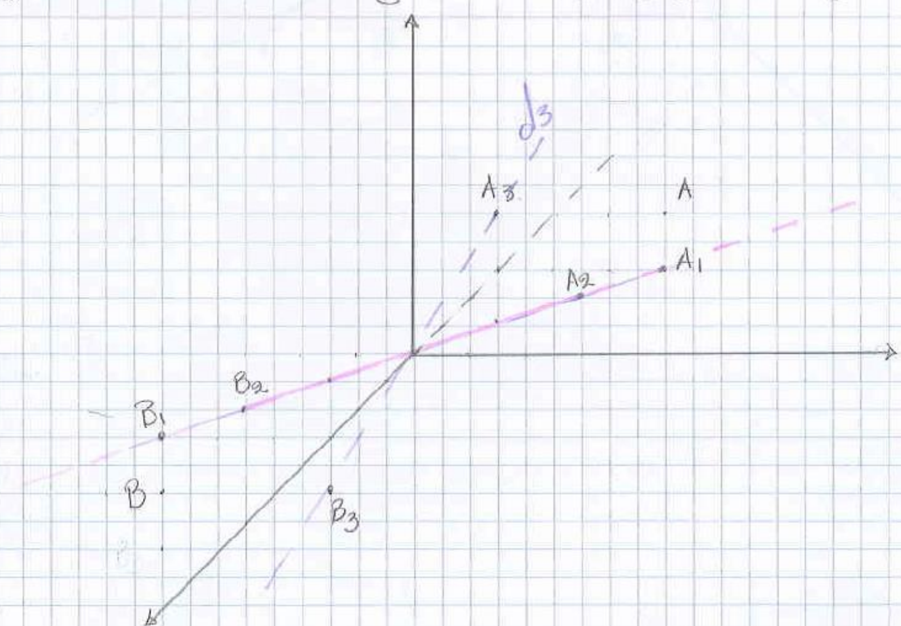
2. T_1: $z=0$ Impossible $\Rightarrow$ $d \parallel$ sol
T_2: $x=0$ Impossible $\Rightarrow$ $d \parallel$ mur
T_3: $y=0$ $\Rightarrow t=0$ $\quad T_3(3;0;-1)$

3.



e) d: $A(-3;3;1)$ $B(3;-3;-1)$ $\quad \vec{AB} = \begin{pmatrix} 6 \\ -6 \\ -2 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix}$ $\quad d: \begin{cases} x = -3+3t \\ y = 3-3t \\ z = 1-t \end{cases}$

2. T_2: $x=0 \Leftrightarrow t=1$ $\quad y=0$ $\quad z=0$ $\quad T_2(0,0,0)$.
T_1: $z=0 \Rightarrow t=1$ $\quad x=y=z=0$ $\quad T_1(0,0,0) \equiv T_3$

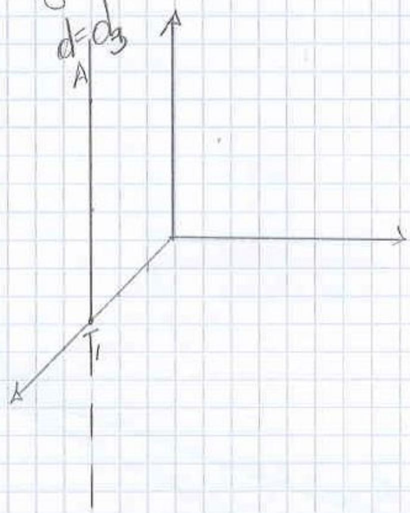


g) 1. d: A(3;0;4) $\vec{d} // Oz \Rightarrow \vec{d} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ d: $\begin{cases} x = 3 \\ y = 0 \\ z = 4 + t \end{cases}$

2. T_1: $z = 0 \Rightarrow t = -4$ $T_1(3;0;0)$

T_2: $x = 0$ Impossible $\rightarrow$ d // mur

T_3: $y = 0$ $\forall t \in \mathbb{R}$ $\rightarrow$ $T_3(3;0;4+t)$ $t \in \mathbb{R}$



d_1: le point T_1

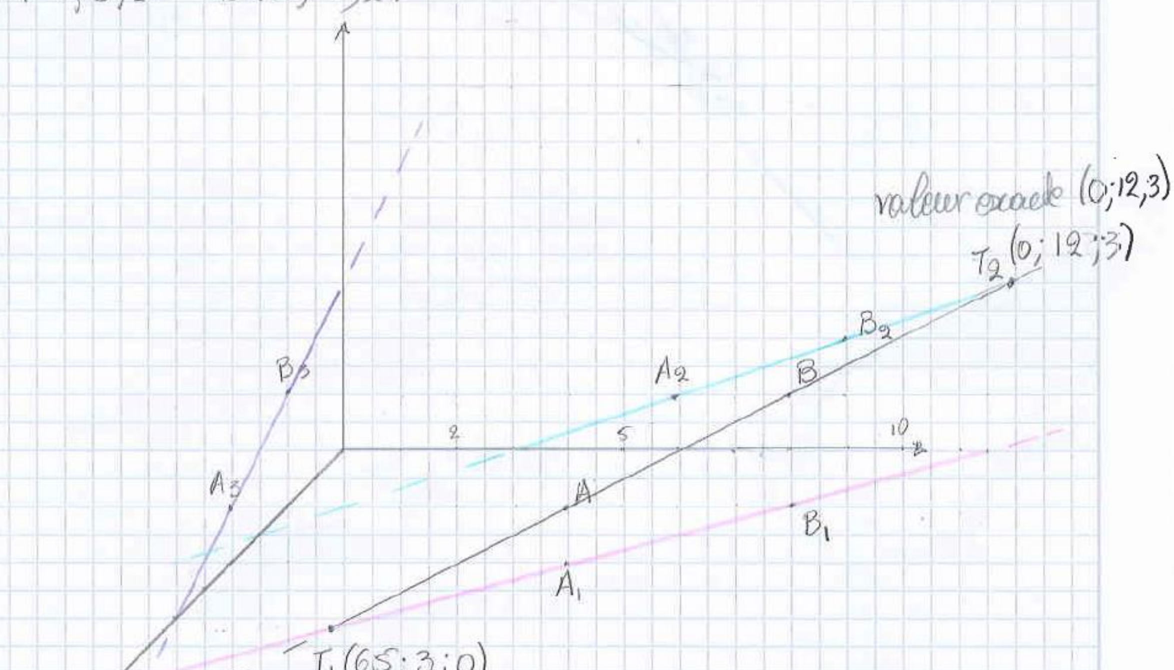
d_2: = l'axe Oz

d_3: $\equiv$ d

EXERCICE 7.

d: A(4;6;1) B(2;9;2)

1.



EXERCICE 8

d_1 : $A(3;5;0)$ $B(1;8;0)$
d_2 : $C(0;2;1)$ $D(0;8;3)$