LDDR-Niveau II : TRIGONOMETRIE - solutions

EXERCICE 1

degrés	$90°$	$135°$	$30°$	$18°$	22.5	720	$1°$	$x°$
radians	$\pi/2$	$\frac{3\pi}{4}$	$\pi/6$	$\pi/10$	$\pi/8$	4π	$\pi/180$	$x \cdot \frac{\pi}{180}$

degrés → radians

$\alpha° \rightarrow \alpha° \cdot \frac{\pi}{180}$

radians	$\pi/3$	$\pi/20$	$3\pi/8$	$2\pi/3$	$-\pi/2$	6π	1	2.5	x
degrés	$60°$	$9°$	$67.5°$	$120°$	$-90°$	$1080°$	$57.3°$	$143.2°$	$x \cdot \frac{180}{\pi}$

radians → degrés

$x \rightarrow x \frac{180°}{\pi}$

EXERCICE 2

1. $2578° = 7 \cdot 360° + 58° \rightarrow \varphi = 58°$

$5555° = 15 \cdot 360° + 155° \rightarrow \varphi° = 155°$

$-1111° = -3 \cdot 360° - 31° \rightarrow \varphi = 360 - 31 = 329°$

$-9876° = -27 \cdot 360 - 156° \rightarrow \varphi = 360 - 156° = 204°$

2. $\frac{97\pi}{6} = \frac{96\pi}{6} + \frac{\pi}{6} = 16\pi + \frac{\pi}{6} = 8 \cdot 2\pi + \frac{\pi}{6} \rightarrow \varphi = \frac{\pi}{6}$

$\frac{111\pi}{8} = \frac{104\pi}{8} + \frac{7\pi}{8} = 13\pi + \frac{7\pi}{8} = 6 \cdot 2\pi + \pi + \frac{7\pi}{8} \rightarrow \varphi = \pi + \frac{7\pi}{8}$

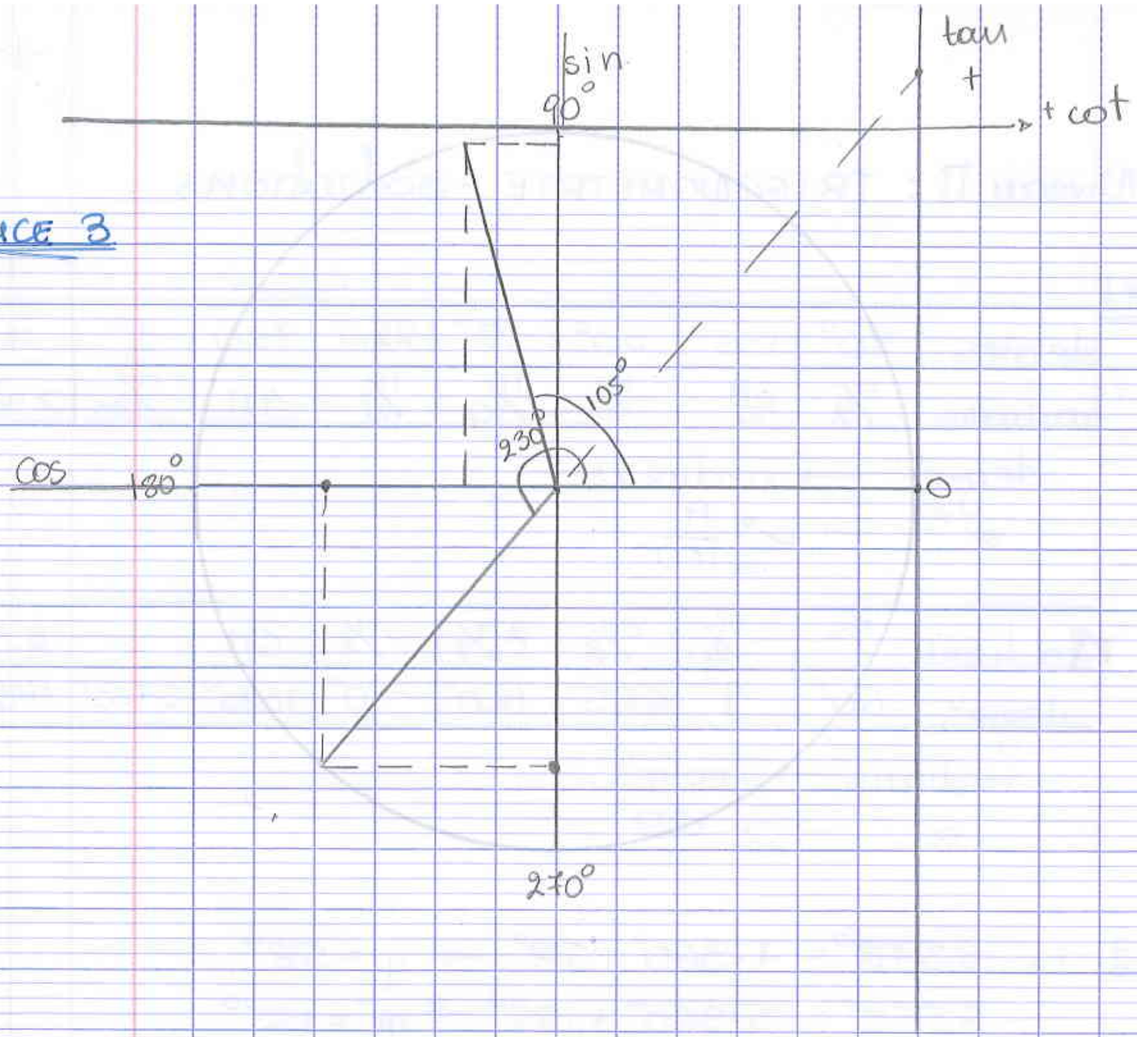
$-\frac{304\pi}{3} = -\frac{303\pi}{3} - \frac{\pi}{3} = -101\pi - \frac{\pi}{3} = -50 \cdot 2\pi - \pi - \frac{\pi}{3} \rightarrow$

$\rightarrow \varphi = 2\pi - \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$-\frac{123\pi}{4} = -\frac{120\pi}{4} - \frac{3\pi}{4} = -30\pi - \frac{3\pi}{4} = -15 \cdot 2\pi - \frac{3\pi}{4} \rightarrow$

$\rightarrow \varphi = 2\pi - \frac{3\pi}{4} = \frac{5\pi}{4}$

Exercice 3



α	$\cos(\alpha)$	$\sin(\alpha)$	$\tan(\alpha)$	$\cot(\alpha)$
230°	-0,625	-0,77	1,146	0,854
105°	-0,25	0,958	-3,832	-0,261

Exercice 4

1. $\cos(\alpha) = 0 \Rightarrow \alpha = 90° + k \cdot 180$
   $\alpha = \pi/2 + k \cdot \pi$
2. $\sin(\alpha) = -1 \Rightarrow \alpha = 270° + k \cdot 360°$
   $\alpha = \frac{3\pi}{2} + 2k\pi$
3. $\cos(\alpha) = -1 \Rightarrow \alpha = 180° + k \cdot 360°$
   $\alpha = \pi + k \cdot 2\pi$
4. $\sin(\alpha) = 0 \Rightarrow \alpha = 0° + k \cdot 180°$
   $= \alpha = 0° \quad k \cdot \pi$
5. $\tan(\alpha) = -1 \Rightarrow \alpha = -45° + k \cdot 180°$
   $\alpha = -\frac{\pi}{4} + k \cdot \pi$
6. $\cot(\alpha) = 0 \Rightarrow \alpha = 90° + k \cdot 180°$
   $\alpha = \frac{\pi}{2} + k \cdot \pi$

7. $\tan(\alpha) = 0 \Rightarrow \alpha = 0° + k \cdot 180°$

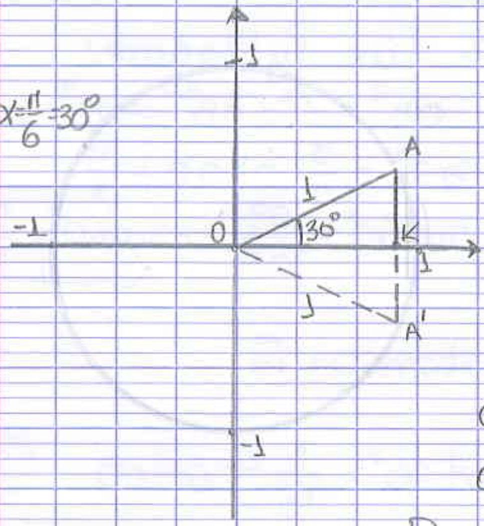
$\alpha = 0 + k \cdot \pi$

8. $\cot(\alpha) = -1 \Rightarrow \alpha = -45° + k \cdot 180°$

$\alpha = -\frac{\pi}{4} + k \cdot \pi$

EXERCICE 5

a. $\alpha = \frac{\pi}{6} = 30°$



$\left.\begin{array}{l} \hat{AOA'} = 60° \\ OA = OA' = 1 \end{array}\right\}$ OAA' est équilatéral $\Rightarrow AK = \frac{1}{2}$

Alors $\sin 30° = \frac{AK}{OA} = \frac{1/2}{1} = \frac{1}{2} = \sin 30°$

Par Pythagore :

$OK^2 = OA^2 - AK^2 = 1^2 - \left(\frac{1}{2}\right)^2 \Rightarrow$

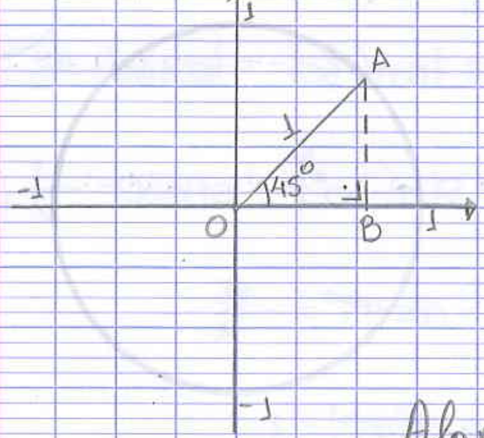
$OK^2 = \frac{3}{4} \Rightarrow OK = \frac{\sqrt{3}}{2}$

Donc $\cos 30 = \frac{OK}{OA} = \frac{\sqrt{3}/2}{1} \Rightarrow \cos 30° = \frac{\sqrt{3}}{2}$

$\tan\frac{\pi}{6} = \frac{\sin(\pi/6)}{\cos(\pi/6)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \Rightarrow \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\cos\left(\frac{\pi}{6}\right) = \frac{1}{\tan\left(\frac{\pi}{6}\right)} = \frac{3}{\sqrt{3}} \Rightarrow \cos\left(\frac{\pi}{6}\right) = \sqrt{3}$

b. $\alpha = \frac{\pi}{4} = 45°$



Le triangle OAB est rectangle isocèle $\Rightarrow OB = AB$

Par Pythagore :

$OB^2 + AB^2 = 1 \Rightarrow 2OB^2 = 1 \Rightarrow$

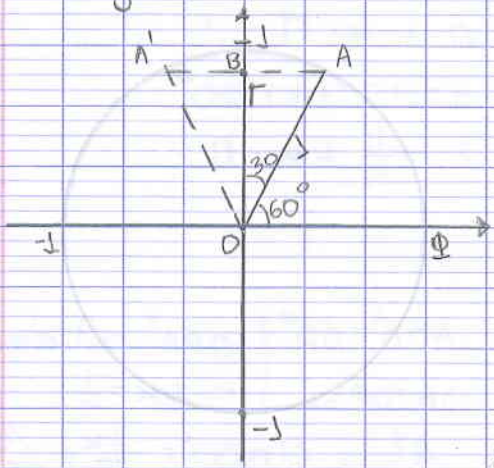
$\Rightarrow OB^2 = \frac{1}{2} \Rightarrow OB = \frac{1}{\sqrt{2}} \Rightarrow OB = \frac{\sqrt{2}}{2}$

Alors $\cos 45 = \frac{OB}{OA} \Rightarrow \cos 45° = \frac{\sqrt{2}}{2} = \sin 45°$

$\tan 45° = \frac{\sin 45}{\cos 45} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1 \Rightarrow \tan 45° = 1$

$\cot 45° = \frac{1}{\tan 45°} = \frac{1}{1} = 1 \Rightarrow \cot 45° = 1$

C. $\alpha = \frac{\pi}{3} = 60^\circ$



Le triangle OAA' est équilatéral

$\Rightarrow AA' = 1 \Rightarrow AB = \frac{1}{2} \Rightarrow$

$\Rightarrow \cos 60^\circ = \frac{AB}{OA} = \frac{1/2}{1} \Rightarrow \cos 60^\circ = \frac{1}{2}$

Par Pythagore:

$OB^2 = 1^2 - AB^2 \Rightarrow OB^2 = 1 - \frac{1}{2^2} \Rightarrow$

$OB^2 = \frac{3}{4} \Rightarrow OB = \frac{\sqrt{3}}{2}$

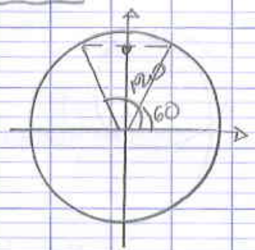
$\sin 60^\circ = \frac{OB}{OA} \Rightarrow \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\tan 60^\circ = \frac{\sin 60}{\cos 60} = \frac{\sqrt{3}/2}{1/2} \Rightarrow \tan 60 = \sqrt{3}$

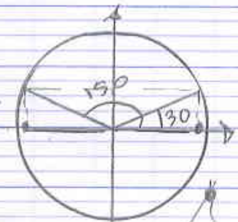
$\cot 60^\circ = \frac{1}{\tan 60^\circ} \Rightarrow \cot 60^\circ = \frac{1}{\sqrt{3}} \Rightarrow \cot 60 = \frac{\sqrt{3}}{3}$

EXERCICE 6.

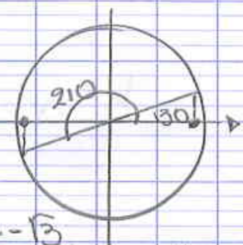
1. $\sin(120^\circ) = \sin(180 - 60^\circ) = \sin 60 = \frac{\sqrt{3}}{2}$



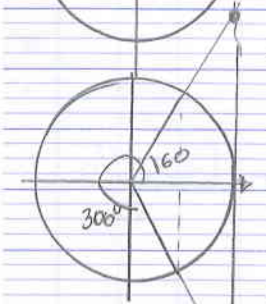
2. $\cos(135) = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$



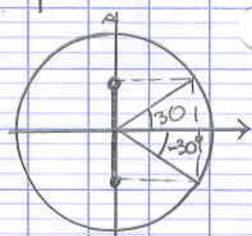
3. $\tan(150^\circ) = \tan(180^\circ - 30^\circ) = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$



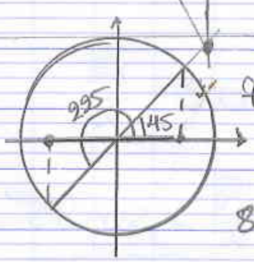
4. $\cos(210^\circ) = \cos(180^\circ + 30^\circ) = -\cos 30 = -\frac{\sqrt{3}}{2}$



5. $\tan(300^\circ) = \tan(360^\circ - 60^\circ) = \tan(-60) = -\tan 60^\circ = -\sqrt{3}$



6. $\sin(330^\circ) = \sin(360 - 30^\circ) = \sin(-30^\circ) = -\sin 30 = -\frac{1}{2}$

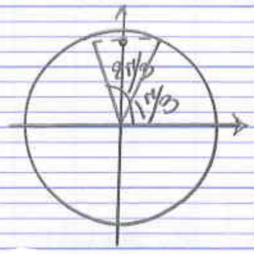


7. $\cos(225^\circ) = \cos(180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

8. $\cot(225^\circ) = \cot(180 + 45^\circ) = \cot(45^\circ) = 1$

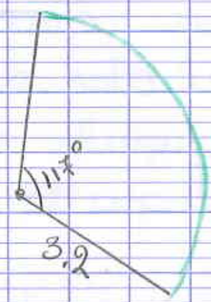
Exercice 7

- $\cos(-945°) = \cos(-2\cdot 360 - 225) = \cos(-225) = \cos(225)$
$= \cos(180+45) = -\cos(45) = -\frac{\sqrt{2}}{2}$



- $\sin\left(-\frac{10\pi}{3}\right) = \sin\left(4\pi - \frac{10\pi}{3}\right) = \sin\left(\frac{12\pi}{3} - \frac{10\pi}{3}\right) =$
$= \sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

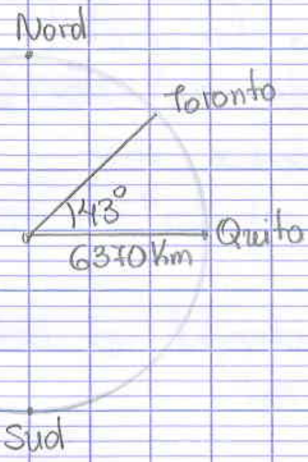
Exercice 8



$L = 2\cdot\pi\cdot r\cdot\frac{\alpha}{360}$

$L = 2\cdot\pi\cdot 3.2\cdot\frac{117°}{360°} \Rightarrow L \approx 6.5\text{ m}$

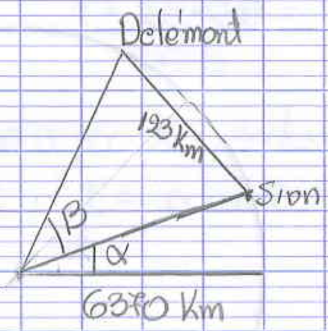
Exercice 9



La longueur de l'arc correspondant à 43° est:

$L = 2\pi\cdot 6370\cdot\frac{43°}{360°} \approx 4780,6\text{ Km}$
= la distance Toronto-Quito

Exercice 10



$\alpha = 46°14' = 46 + \frac{14}{60} = 46.2\bar{3}°$

La longueur de l'arc correspondant à B° est ≈ 123 Km

Donc

$123 = 2\cdot\pi\cdot 6370\cdot\frac{B}{360°} \Rightarrow$

$\Rightarrow B° = 1.106°$

Alors la latitude de Délemont est $\alpha + B = 46.2\bar{3} + 1.106° = 47.34° =$
$= 47° + 0.34\cdot 60 = 47°20' N$

EXERCICE 11 La grande aiguille met 60 min pour faire 360°

360°	60 min
1°	x

$\Rightarrow$ 1° en $0.1\bar{6}$ min

La petite aiguille met 12 h pour faire 360°

360	$12 \cdot 60 = 720$ min
1°	x

$\Rightarrow$ 1° en 2 min

Les 2 aiguilles sont au même endroit à midi. La prochaine fois qu'elles seront au même endroit, la grande aiguille aura fait $360° + \alpha$ et la petite aiguille α degrés

Le temps mis pour la grande aiguille pour effectuer $360 + \alpha$ est $(360+\alpha) \cdot 0.1\bar{6}$ min
Le temps mis pour la petite aiguille pour effectuer α degrés est $\alpha \cdot 2$ min.
Ces temps sont égaux $\Rightarrow$
$(360+\alpha) \cdot 0.1\bar{6} = 2\alpha \Rightarrow 60° = 1.83\alpha \Rightarrow$
$\Rightarrow \underline{\alpha = 32.73°}$
Le temps correspondant à cet angle pour la petite aiguille est

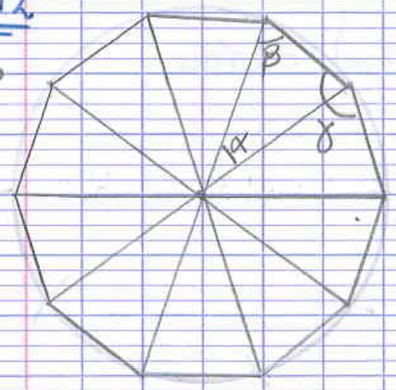
360	720
32.73°	x

$x = \frac{720 \cdot 32.73}{360} \Rightarrow x = 65.46$ min

Donc le temps entre 2 rencontres des aiguilles est 65.46 min = 1 h 5 min 27.6 s

Exercice 12

a.



On a $\alpha = \frac{360}{10} = 36°$

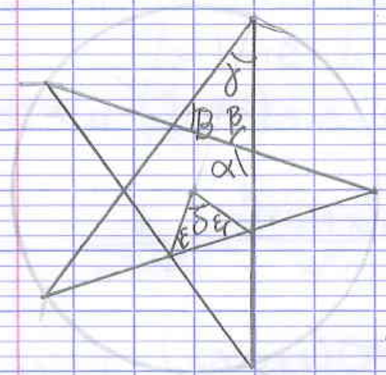
Chaque triangle est isocèle.

Ainsi $2\beta + \alpha = 180° \Leftrightarrow 2\beta = 144° \Rightarrow$
$\Rightarrow \beta = 72°$

Comme $\gamma = 2\beta$, on a $\underline{\gamma = 144°}$

En radians: $144° \longleftrightarrow \frac{\pi}{180} \cdot 144 = \frac{4\pi}{4} = \gamma$

b.



On a $\delta = \frac{360}{5} = 72°$

Ainsi $2\varepsilon = 180 - 72 = 108° \Rightarrow \varepsilon = 54°$

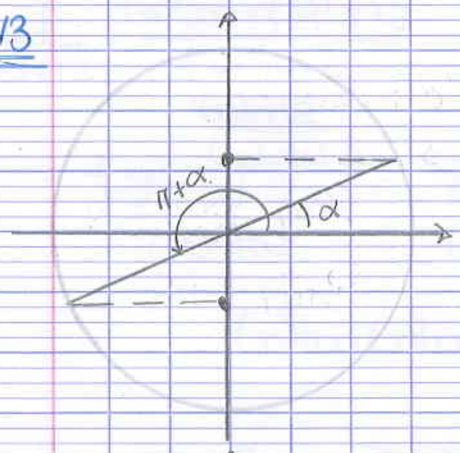
Comme $\alpha = 2\varepsilon$, on a $\alpha = 108°$.

Donc $\beta = 180° - 108° = 72°$

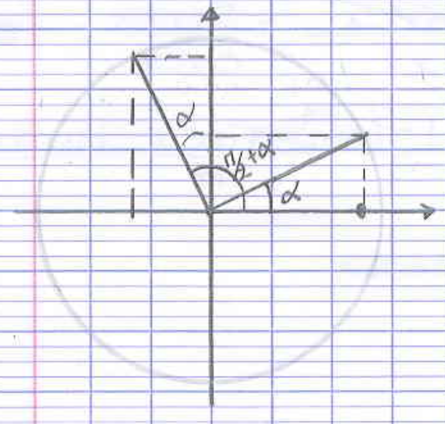
Par concéquent $\gamma = 180° - 2 \cdot 72° = \underline{\underline{36°}}$

En radians: $\gamma = \frac{\pi}{180} \cdot 36 = \underline{\underline{\frac{\pi}{5}}}$

Exercice 13

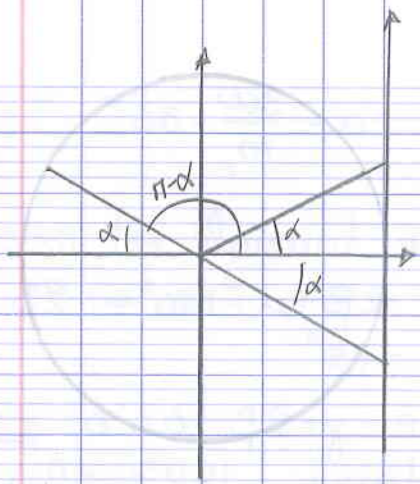


$\sin(\pi + \alpha) = -\sin\alpha$



$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$

car $\cos\left(\frac{\pi}{2} + \alpha\right)$ est négatif et $\sin\alpha$ est positif



$\tan(\pi-\alpha) = -\tan\alpha$

Alors

- $-\cos\alpha = \cos(\pi-\alpha)$
- $-\sin(\alpha) = \sin(\pi+\alpha)$
  $-\sin(\alpha) = \sin(-\alpha)$
- $\cos\alpha = \sin\left(\frac{\pi}{2}-\alpha\right)$
- $\sin(\alpha) = \cos\left(\frac{\pi}{2}-\alpha\right)$
- $-\cos(\alpha) = \sin\left(\alpha-\frac{\pi}{2}\right)$
- $-\sin\alpha = \cos\left(\frac{\pi}{2}+\alpha\right)$

Exercice 14

1. $1+\tan^2(\alpha) = 1+\frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$

2. $\frac{\sin^2(x)}{1-\cos x} = \frac{1-\cos^2 x}{1-\cos x} = \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}} = 1+\cos x.$

3. $1+\cot^2(\alpha) = 1+\frac{\cos^2(\alpha)}{\sin^2(\alpha)} = \frac{\sin^2(\alpha)+\cos^2(\alpha)}{\sin^2(\alpha)} = \frac{1}{\sin^2(\alpha)}$

Exercice 15

1. $\frac{\tan(x)}{1+\tan^2(x)} = \frac{\frac{\sin(x)}{\cos(x)}}{1+\frac{\sin^2(x)}{\cos^2(x)}} = \frac{\frac{\sin(x)}{\cos(x)}}{\frac{\overbrace{\cos^2(x)+\sin^2(x)}^{1}}{\cos^2(x)}} =$

$= \frac{\sin(x)\cdot\cos^{\cancel{2}}(x)}{\cancel{\cos(x)}} = \sin(x)\cos(x)$

2. $\frac{\sin(x)\cos(x)}{1-\sin^2(x)} = \frac{\sin x\cdot\cancel{\cos x}}{\cos^{\cancel{2}} x} = \frac{\sin x}{\cos x} = \tan(x)$

3. $\frac{1-(\sin(x)-\cos(x))^2}{\sin(x)} = \frac{1-(\overbrace{\sin^2 x+\cos^2 x}^{1}-2\sin x\cos x)}{\sin x} =$

$= \frac{\cancel{1}-\cancel{1}+2\cancel{\sin x}\cos x}{\cancel{\sin x}} = 2\cos x$

4. $(\sin x+\cos x)^2+(\sin x-\cos x)^2=$
$=\underbrace{\sin^2 x+\cos^2 x}+2\cancel{\sin x\cos x}+\sin^2 x+\cos^2 x-2\cancel{\sin x\cos x}$
$= \quad 1 \quad + 1 = 2$

EXERCICE 16

1. $\cos(t)=\frac{3}{5} \quad t\in Q_{IV}$
$\sin^2 t=1-\left(\frac{3}{5}\right)^2=1-\frac{9}{25}=\frac{16}{25} \Rightarrow \sin t=-\frac{4}{5}$ car $t\in Q_{IV}$
$\tan t=\frac{\sin t}{\cos t}=\frac{-4/5}{-3/5} \Rightarrow \tan t=\frac{4}{3} \Rightarrow \cot(t)=\frac{3}{4}.$

2. $\sin t=-0.28 \quad t\in Q_{III}$
$\cos^2 t=1-\sin^2 t=1-(-0.28)^2 \Rightarrow \cos^2 t=0.9216 \Rightarrow$
$\Rightarrow \cos t=\pm 0.96$ Comme $t\in Q_{III}$ $\cos t=-0.96.$
$\tan t=\frac{-0.28}{-0.96} \Rightarrow \tan t=0.292 \Rightarrow \cot(t)=3.43.$

3. $\tan(t)=2 \quad t\in Q_{I}$
$\tan(t)=\frac{\sin(t)}{\cos(t)}=2 \Leftrightarrow \sin(t)=2\cos(t)\ (1)$

$\cos^2 t+\sin^2 t=1 \overset{(1)}{\Rightarrow} \cos^2 t+4\cos^2 t=1 \Leftrightarrow$
$5\cos^2 t=1 \Leftrightarrow \cos^2 t=\frac{1}{5} \Rightarrow \cos t=\pm\sqrt{\frac{1}{5}} \xrightarrow{t\in Q_I} \cos t=\sqrt{\frac{1}{5}}$
$\sin t=2\cdot\sqrt{\frac{1}{5}} \Rightarrow \sin t=\frac{2}{\sqrt{5}}$
$\tan(t)=\frac{2/\sqrt{5}}{1/\sqrt{5}} \Rightarrow \tan(t)=2 \Rightarrow \cos(t)=\frac{1}{2}$

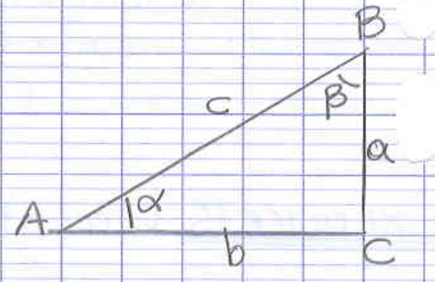
4. $\cot(t)=-\frac{9}{40} \quad t\in Q_{IV}$

$\cot(t)=\frac{\cos t}{\sin t}=-\frac{9}{40} \Leftrightarrow \cos t=-\frac{9}{40}\sin t\ (2)$

$\cos^2 t+\sin^2 t=1 \overset{(2)}{\Leftrightarrow} \frac{81}{1600}\sin^2 t+\sin^2 t=1 \Leftrightarrow$

$\Leftrightarrow \frac{1681}{1600}\sin^2 t=1 \Leftrightarrow \sin^2 t=\frac{1600}{1681} \Rightarrow \sin^2 t=0.95 \Rightarrow$
$\Rightarrow \sin t=\pm 0.976 \xrightarrow{Q_{IV}} \sin t=-0.976$
$(2): \cos(t)=-\frac{9}{40}(-0.976) \Rightarrow \cos(t)=0.22$
$\tan(t)=\frac{-0.976}{0.22}=-4.45$

EXERCICE 17



1. $a = 7.8cm \quad \alpha = 27^\circ$

$\sin\alpha = \frac{a}{c} \Leftrightarrow c = \frac{a}{\sin\alpha} \Rightarrow$

$c = \frac{7.8}{\sin 27} \Rightarrow c = 17.18cm$

$\tan\alpha = \frac{a}{b} \Leftrightarrow b = \frac{a}{\tan\alpha} \Rightarrow b = \frac{7.8}{\tan 27} \Rightarrow b = 15.31cm$

$\beta = 90^\circ - \alpha^\circ \Rightarrow \beta = 63^\circ$

2. $a = 6.3cm, \ c = 9.2cm$

$\sin\alpha = \frac{a}{c} \Rightarrow \sin\alpha = 0.685 \Rightarrow \alpha = 43.22^\circ \Rightarrow$

$\beta = 90 - 43.22 \Rightarrow \beta = 46.78^\circ$

$\sin\beta = \frac{b}{c} \Rightarrow b = c \cdot \sin\beta \Rightarrow b = 9.2 \cdot \sin 46.78 \Rightarrow$

$b = 6.7 cm$

3. $c = 4.8 cm \quad \beta = 10^\circ$

$\alpha = 90 - \beta \Rightarrow \alpha = 80^\circ$

$\sin\alpha = \frac{a}{c} \Rightarrow a = c \cdot \sin\alpha \Rightarrow a = 4.8 \cdot \sin 80 \Rightarrow a = 4.73cm$

$b = \sqrt{c^2 - a^2} = \sqrt{4.8^2 - 4.73^2} \Rightarrow b = 0.83 cm$

4. $a = 13.4cm, \ b = 20cm.$

$\tan\alpha = \frac{a}{b} \Rightarrow \tan\alpha = 0.67 \Rightarrow \alpha = 33.82^\circ$

$\beta = 90 - \alpha \Rightarrow \beta = 56.18^\circ$

$c = \sqrt{a^2 + b^2} \Rightarrow c = \sqrt{13.4^2 + 20^2} \Rightarrow c = 24.cm$

5. $a = 5cm \quad A = 6cm^2$

$A = \frac{b \cdot a}{2} = 6 \Leftrightarrow b \cdot a = 12 \xLeftrightarrow{a=5} b = \frac{12}{5} \Leftrightarrow b = 2.4cm$

$\tan\alpha = \frac{a}{b} \Rightarrow \tan\alpha = 2.08 \Rightarrow \alpha = 64.36^\circ$

$\beta = 90 - \alpha \Rightarrow \beta = 25.64^\circ$

$c = \sqrt{a^2 + b^2} \Rightarrow c = 5.55 cm$

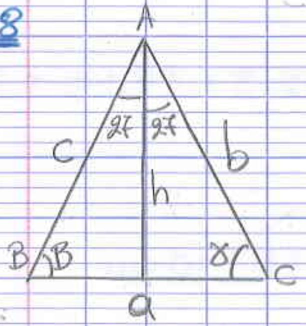
6. $b = 2a$

$\tan\alpha = \frac{a}{b} = \frac{a}{2a} \Rightarrow \tan\alpha = \frac{1}{2} \Rightarrow \alpha = 26.56°$

$\Rightarrow \beta = 90 - \alpha \Rightarrow \beta = 63.43°$

$c = \sqrt{a^2 + b^2} = \sqrt{a^2 + 4a^2} = a\sqrt{5}$

Exercice 18



$\beta = \frac{180-54}{2} = 63° = \gamma$

$A = \frac{BC \cdot h}{2} \Rightarrow h \cdot BC = 240$ (1)

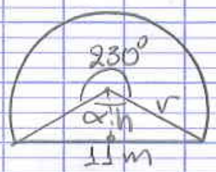
$\tan\gamma = \frac{h}{\frac{BC}{2}} = \frac{2h}{BC} \Rightarrow \tan 63 = \frac{2h}{BC}$ (2)

On résouds le système

$$\left.\begin{array}{l} h \cdot a = 240 \\ 1.96 = \frac{2h}{a} \Leftrightarrow h = 0.98a \end{array}\right\} \Rightarrow 0.98a^2 = 240 \Leftrightarrow a^2 = 244.6 \Leftrightarrow a = 15.64$$

$\cos\gamma = \frac{a/2}{b} \Rightarrow b = \frac{a}{2\cos\gamma} \Rightarrow b = \frac{15.64}{2 \cdot \cos 63} \Rightarrow b = 17.22 = c$

Exercice 19



$\alpha = 360° - 230° \Rightarrow \alpha = 130°$

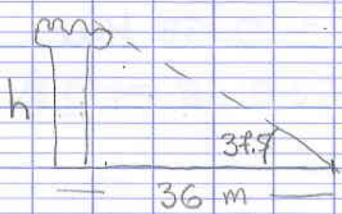
$\sin\frac{\alpha}{2} = \frac{11/2}{r} \Rightarrow \sin 65 = \frac{11}{2r} \Rightarrow r = \frac{11}{2\sin 65} \Rightarrow$

$\Rightarrow r = 6.07\ m$

$h = \sqrt{r^2 - \left(\frac{11}{2}\right)^2} \Rightarrow h = \sqrt{6.07^2 - \left(\frac{11}{2}\right)^2} \Rightarrow h = 2.57\ m$

$\max H = r + h = 8.64\ m$

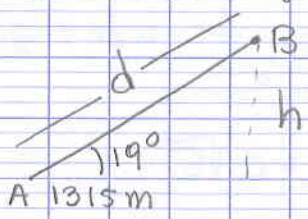
Exercice 20



$\tan 37.5° = \frac{h}{36} \Leftrightarrow h = 36 \cdot \tan 37.5° \Rightarrow$

$h = 27.62\ m$

2. $v = 7\,m/s \quad t = 4\,min\ 20\,sec = 260\,sec$



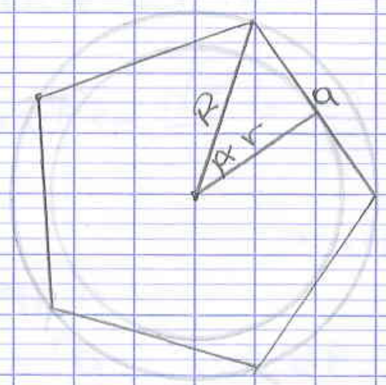
$d = v \cdot t = 7 \cdot 260 \Rightarrow d = 1820\,m$

$\sin 19^\circ = \frac{h}{d} \Rightarrow h = d \cdot \sin 19 \Rightarrow$

$\Rightarrow h = 592.5\,m$

Donc altitude de B: $592.5 + 1315 = 1907.5\,m$

3.



$a = \frac{50}{5} \Rightarrow \underline{a = 10\,cm}$

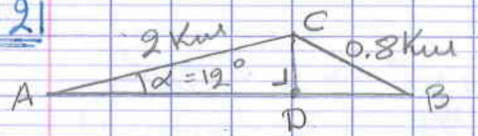
$\alpha = \frac{1}{2}\frac{360}{5} = 36^\circ$

$\sin\alpha = \frac{a/2}{R} \Rightarrow R = \frac{a/2}{\sin\alpha} = \frac{5}{\sin 36} \Rightarrow$

$\Rightarrow \underline{R = 8.506\,cm}$

$\tan 36 = \frac{a/2}{r} \Leftrightarrow r = \frac{a/2}{\tan 36} \Rightarrow r = \frac{5}{\tan 36} \Rightarrow \underline{r = 6.88\,cm}$

EXERCICE 21



Dans le triangle ACD on a: $\cos 12 = \frac{AD}{2} \Rightarrow AD = 2 \cdot \cos 12$

$\Rightarrow AD = 1.96\,Km$.

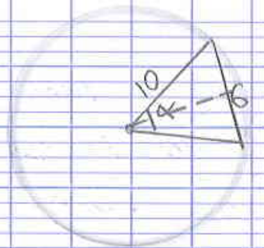
$\sin 12 = \frac{CD}{2} \Rightarrow CD = 2 \cdot \sin 12 \Rightarrow CD = 0.42\,Km$

Dans le triangle DCB on a: $DB = \sqrt{CB^2 - CD^2} \Rightarrow$

$\Rightarrow DB = \sqrt{0.8^2 - 0.42^2} \Rightarrow DB = 0.68\,Km$

Alors $AB = AD + DB = 1.96 + 0.68 \Rightarrow \underline{AB = 2.64\,Km}$

Exercice 22



On cherche l'angle α

$$\sin\frac{\alpha}{2} = \frac{3}{10} \Rightarrow \sin\frac{\alpha}{2} = 0.3 \Rightarrow \frac{\alpha}{2} = 17.46^\circ$$

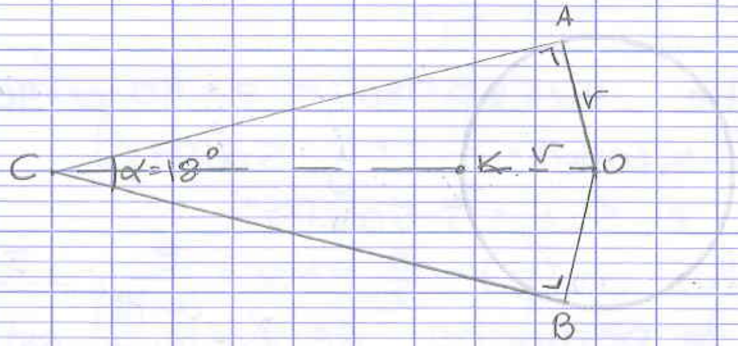
$$\Rightarrow \alpha = 34.9^\circ$$

$$Aire = \pi \cdot r^2 \cdot \frac{\alpha}{360} = \pi \cdot 10^2 \cdot \frac{34.9}{360} \Rightarrow Aire = 30.45\ cm^2$$

$$P = 2\pi \cdot r \cdot \frac{\alpha}{360} + 2r = 2\pi \cdot 10 \, \frac{34.9}{360} + 2 \cdot 10 \Rightarrow$$

$$P = 26.09\ cm$$

Exercice 23



1. On cherche la distance CK.

$$P = 2\pi \cdot r = 50 \Rightarrow r = \frac{50}{2\pi} \Rightarrow r = 7.96\ m$$

Triangle CAO: $\sin 9^\circ = \frac{r}{CO} \Rightarrow CO = \frac{r}{\sin 9} \Rightarrow$

$$\Rightarrow CO = \frac{7.96}{\sin 9} \Rightarrow CO = 50.9\ m$$

Alors $CK = CO - r \Rightarrow CK = 50.9 - 7.96 \Rightarrow$

$$\Rightarrow CK = 42.9\ m$$

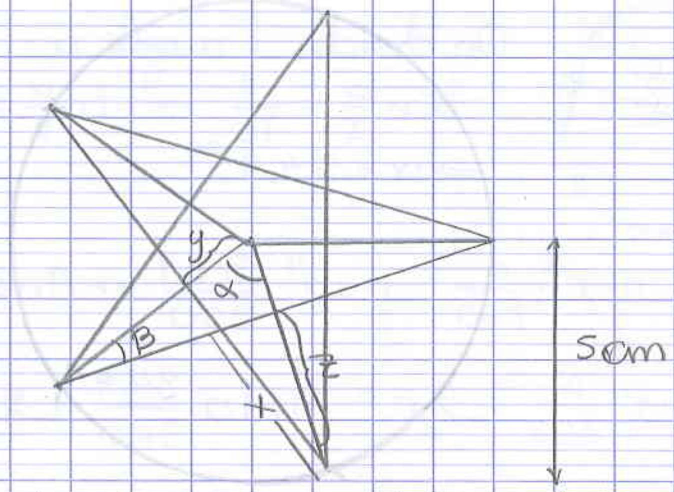
2. On suppose $CK = 100 \Rightarrow CO = 100 + 7.96 \Rightarrow$

$CO = 107.96\ m$.

$$\sin\frac{\alpha}{2} = \frac{r}{CO} \Rightarrow \sin\frac{\alpha}{2} = \frac{7.96}{107.96} \Rightarrow \sin\frac{\alpha}{2} = 0.074 \Rightarrow$$

$$\Rightarrow \frac{\alpha}{2} = 4.23 \Rightarrow \alpha = 8.46^\circ$$

EXERCICE 24



Le périmètre de l'étoile est $10x$

On a $\alpha = \frac{360}{5} = 72°$

En outre $2\alpha + 2\beta = 180° \Rightarrow \beta = 90 - \alpha = 90 - 72 = 18°$

De plus $\cos(\alpha) = \frac{y}{5} \Rightarrow y = 5\cos(72°)$

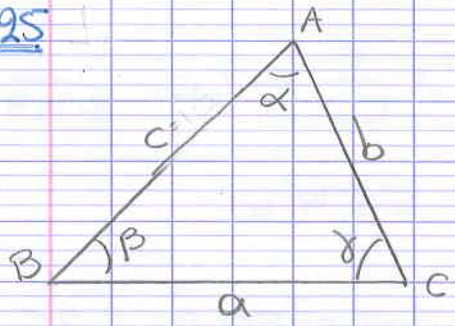
Ainsi $z = 5 - y = 5 - 5\cos(72°)$

De plus $\cos(\beta°) = \frac{z}{x} \Rightarrow x = \frac{z}{\cos(\beta)} = \frac{5 - 5\cos 72}{\cos 18} \approx 3.63$

Ainsi le périmètre de l'étoile est:

$10 \cdot x \cong 36.33\text{cm}$

EXERCICE 25



Par le théorème du cosinus:

$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$b^2 = a^2 + c^2 - 2ac\cos(\beta)$

$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$

Et le théorème du sinus

$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$

1. $c = 10$, $\alpha = 65$ $\gamma = 43° \Rightarrow \beta = 180 - 65 - 43 \Rightarrow \underline{\beta = 72°}$

$\frac{a}{\sin(\alpha)} = \frac{c}{\sin(\gamma)} \Rightarrow a = \frac{c\sin(\alpha)}{\sin\gamma} \Rightarrow \underline{a = 13.289}$

$\frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} \Rightarrow b = \frac{c\sin\beta}{\sin(\gamma)} \Rightarrow \underline{b = 13.945}$

2. $a = 7.32$, $b = 4.65$ $\gamma = 71{,}6°$

$c^2 = a^2 + b^2 - 2ab\cos(\gamma) = 7.32^2 + 4.65^2 - 2 \cdot 7{,}32 \cdot 4{,}65 \cdot \cos(71.6°) \Rightarrow$

$c^2 = 53{,}717 \Rightarrow \underline{c = 7.329}$

$$\frac{a}{\sin(\alpha)} = \frac{c}{\sin(\gamma)} \Rightarrow \sin(\alpha) = \frac{a\sin(\gamma)}{c} \approx 0.948 \Rightarrow$$

$$\Rightarrow \underline{\alpha = 71.386^\circ}$$

$$\beta = 180 - \alpha - \gamma \Rightarrow \underline{\beta = 37.014^\circ}$$

3. $a = 2$, $b = \sqrt{6}$ $c = 1 + \sqrt{3}$.

$$b^2 = a^2 + c^2 - 2ac\cos(\beta) \Rightarrow \cos(\beta) = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow$$

$$\Rightarrow \cos(\beta) = \frac{1}{2} \Rightarrow \underline{\beta = 60^\circ}$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow \sin\alpha = \frac{a\sin\beta}{b} = \frac{2\sin 60}{\sqrt{6}} = \frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow \underline{\alpha = 45^\circ}$$

$$\gamma = 180 - 60^\circ - 45^\circ \Rightarrow \underline{\gamma = 75^\circ}$$

4. On a: $\alpha = 54.08^\circ$, $\beta = 88,94^\circ$ $Aire = 12.52\ m^2$

$$\gamma = 180 - 54.08 - 88.94 \Rightarrow \underline{\gamma = 36.98^\circ}$$

On connait la formule:

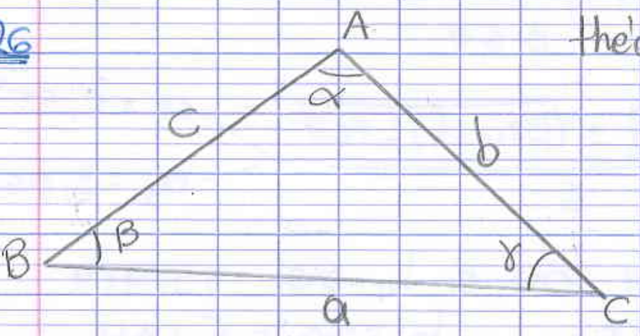
$$aire = \frac{1}{2}bc\sin(\alpha) = \frac{a^2\sin(\beta)\sin(\gamma)}{2\sin(\alpha)}$$

$$\Rightarrow a^2 = \frac{2\sin(\alpha)\,Aire}{\sin(\beta)\sin(\gamma)} \Rightarrow a^2 = 33.717 \Rightarrow a = 5.807\ m$$

$$\frac{b}{\sin(\beta)} = \frac{a}{\sin(\alpha)} \Rightarrow b = \frac{a\sin(\beta)}{\sin(\alpha)} \approx \underline{b = 7.168\ m}$$

$$\frac{c}{\sin(\gamma)} = \frac{b}{\sin(\beta)} \Rightarrow c = \frac{b\sin(\gamma)}{\sin(\beta)} \Rightarrow \underline{c \approx 4.312\ m}$$

EXERCICE 26



théorème du cosinus:

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$
$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$
$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

théorème du sinus:

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

$$Aire = \frac{1}{2}bc\sin(\alpha) = \frac{1}{2}ac\sin(\beta) = \frac{1}{2}ab\sin(\gamma)$$

1. $Aire = 24$, $\sin(\alpha) = \frac{12}{13}$, $b+c = 17$ et $c > b$.

$Aire = 24 \Rightarrow \frac{1}{2} bc\sin(\alpha) = 24 \Rightarrow \frac{1}{2} bc \cdot \frac{12}{13} = 24 \Rightarrow$

$\Rightarrow \frac{12}{26} bc = 24 \Rightarrow 12\, bc = 624 \Rightarrow bc = 52.$

$b + c = 17 \Rightarrow c = 17 - b$

Ainsi $b(17-b) = 52 \Rightarrow 17b - b^2 = 52 \Rightarrow b^2 - 17b + 52 = 0$

$\Delta = 17^2 - 4 \cdot 52 = 81$

$b_{1,2} = \frac{17 \pm 9}{2} = \begin{cases} b_1 = 13 \\ b_2 = 4 \end{cases}$

Avec $b_1 = 13$ on a $c_1 = 17 - b_1 \Rightarrow c_1 = 17 - 13 = 4$

$b_2 = 4$, on a $c_2 = 17 - b_2 = 17 - 4 \Rightarrow c_2 = 13$

On a $c > b$ Donc $\underline{b = 4}$ et $\underline{c = 13}$

Avec $\sin(\alpha) = \frac{12}{13} \Rightarrow \underline{\alpha \approx 67{,}38^\circ}$

De plus, on a $a^2 = b^2 + c^2 - 2bc\cos(\alpha) \Rightarrow a = \sqrt{145} = 12.04$

On a: $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \Rightarrow \sin(\beta) = \frac{b\sin(\alpha)}{a} = 0.307$

$\Rightarrow \underline{\beta = 17.86^\circ}$

$\gamma = 180^\circ - \alpha - \beta \Rightarrow \underline{\gamma = 94.76^\circ}$

2. $c = 8$, $\alpha = \frac{\pi}{9}$, $b = 2a$

$a^2 = b^2 + c^2 - 2bc\cos(\alpha) \Rightarrow a^2 = (2a)^2 + 8^2 - 2 \cdot 2a \cdot 8\cos(\frac{\pi}{9})$

$\Rightarrow a^2 = 4a^2 + 64 - 32a\cos(\frac{\pi}{9})$

$\Rightarrow 3a^2 - 32a\cos(\frac{\pi}{9}) + 64 = 0$

$\Delta = (-32\cos\frac{\pi}{9})^2 - 4 \cdot 3 \cdot 64$

$a_{1,2} = \frac{32\cos\frac{\pi}{9} \pm \sqrt{(32\cos\frac{\pi}{9})^2 - 768}}{6} = \begin{cases} a_1 = 6{,}957 \\ a_2 = 3{,}067 \end{cases}$

Avec $a_1 = 6.957$ on a $b_1 = 2a_1 = 13.914$

$a_2 = 3.067$ $\qquad b_2 = 2a_2 = 6{,}134$

Avec $a_1 = 6.957$, $b_1 = 13,914$ on a :
$$\frac{a_1}{\sin(\alpha)} = \frac{b_1}{\sin(\beta)} \Rightarrow \sin(\beta) = \frac{b_1 \sin(\alpha)}{a_1} \Rightarrow$$
$$\Rightarrow \sin(\beta) = 0.684 \Rightarrow \underline{\beta = 0.753 \text{ rad}}$$

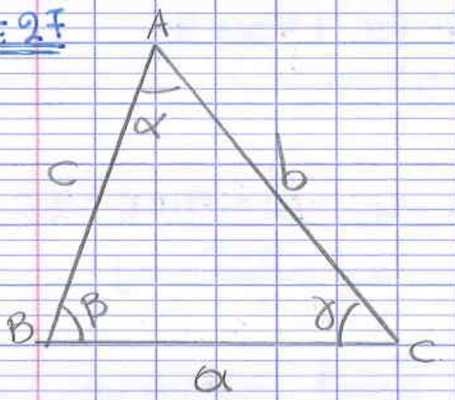
Avec $a_2 = 3.067$ $b_2 = 6,134$ on a :
$$\frac{a_2}{\sin(\alpha)} = \frac{b_2}{\sin(\beta)} \Rightarrow \sin(\beta) = \frac{b_2 \sin(\alpha)}{a_2}$$
$$\Rightarrow \sin(\beta) = 0,684 \Rightarrow \boxed{\beta = 0.753 \text{ rad}}$$

Finalement : $\gamma = \pi - \frac{\pi}{9} - 0.753 \simeq \underline{2,04 \text{ rad} = \gamma}$

On a donc 2 solutions :

1. $a = 6.957$, $b = 13.914$, $c = 8$ $\alpha = \frac{\pi}{9}$, $\beta = 0.753$ $\gamma = 2.04$ rad
2. $a = 3,067$, $b = 6,134$, $c = 8$, $\alpha = \frac{\pi}{9}$, $\beta = 0.753$ rad, $\gamma = 2.04$ rad

Exercice 27



On a :
$$\left.\begin{array}{l} a = x \\ c = x+1 \\ b = x+2 \\ \beta = 2\alpha \end{array}\right\} x \in \mathbb{N}$$

Par le théorème du cosinus et du sinus :

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha) \quad ①$$
$$b^2 = a^2 + c^2 - 2ac\cos(\beta) \quad ②$$
$$c^2 = a^2 + b^2 - 2ab\cos(\gamma) \quad ③$$
$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} \quad ④$$

① s'écrit : $x^2 = (x+2)^2 + (x+1)^2 - 2(x+2)(x+1)\cos(\alpha)$

② s'écrit : $(x+2)^2 = x^2 + (x+1)^2 - 2x(x+1)\cos(\beta)$

③ — " — : $(x+1)^2 = x^2 + (x+2)^2 - 2x(x+2)\cos(\gamma)$

(4) S'écrit : $\frac{x}{\sin(\alpha)} = \frac{x+2}{\sin(\beta)} = \frac{x+1}{\sin(\gamma)}$

On a $\sin(\beta) = \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

On obtient $\frac{x}{\sin(\alpha)} = \frac{x+2}{2\sin(\alpha)\cos(\alpha)} \Rightarrow$

$\Rightarrow x = \frac{x+2}{2\cos(\alpha)} \Rightarrow \cos(\alpha) = \frac{x+2}{2x}$

Par substitution dans (1) :

$x^2 = (x+2)^2 + (x+1)^2 - 2(x+2)(x+1)\frac{x+2}{2x} \Rightarrow$

$\Rightarrow x^2 = (x+2)^2 + (x+1)^2 - \frac{(x+2)^2(x+1)}{x}$

$\Rightarrow x^3 = x(x+2)^2 + x(x+1)^2 - (x+2)^2(x+1)$

$\Rightarrow x^3 = (x+2)^2(x-(x+1)) + x(x+1)^2$

$\Rightarrow x^3 = (x+2)^2(-1) + x(x+1)^2$

$\Rightarrow x^3 = x(x+1)^2 - (x+2)^2$

$\Rightarrow x^3 = x(x^2+2x+1) - (x^2+4x+4)$

$\Rightarrow x^3 = x^3 + 2x^2 + x - x^2 - 4x - 4$

$\Rightarrow 0 = x^2 - 3x - 4$

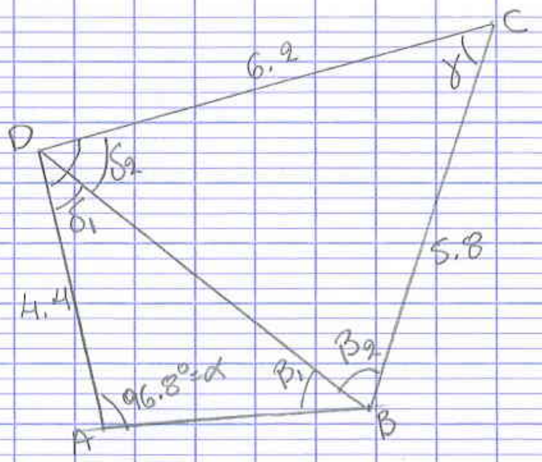
On a : $a = 1$, $b = -3$, $c = -4$. $\Delta = 9 + 16 = 25$

Ainsi $x_1 = \frac{3+5}{2} = 4$ $x_2 = \frac{3-5}{2} = -1$

Comme les longueurs sont positives on accepte $x = 4$.

Les côtés du triangle sont $\underline{a=4}$, $\underline{c=5}$, $\underline{b=6}$

EXERCICE 28



Dans le triangle ABC avec $a = AB$, $b = BD$ et $c = DA$
on a $BD^2 = AB^2 + DA^2 - 2 \cdot AB \cdot AD \cos(\alpha) =$
$= 3.5^2 + 4,4^2 - 2 \cdot 3,5 \cdot 4,4 \cos(96,8^\circ) \Rightarrow BD = \sqrt{35.26} = 5.94$

Encore, $\dfrac{AB}{\sin(\delta_1)} = \dfrac{BD}{\sin(\alpha)} \Rightarrow \sin(\delta_1) = \dfrac{AB \sin(\alpha)}{BD} \approx 0.59$

$\Rightarrow \delta_1 = 35.8^\circ$

Finalement, $\beta_1 = 180^\circ - \alpha - \delta_1 = 180^\circ - 96.8^\circ - 35.8^\circ = 47.4^\circ$

Dans le triangle BCD, on a : $a = BC$, $b = CD$, $c = BD$

Ainsi, $BC^2 = CD^2 + BD^2 - 2 CD \cdot BD \cos(\delta_2)$

$\Leftrightarrow 5.8^2 = 6.2^2 + 5.94^2 - 2 \cdot 6.2 \cdot 5.94 \cdot \cos(\delta_2)$

$\Leftrightarrow 73.656 \cdot \cos(\delta_2) = 40.08 \Leftrightarrow \cos(\delta_2) = 0.544$

$\Rightarrow \delta_2 = 57^\circ$.

De plus, $\dfrac{BC}{\sin(\delta_2)} = \dfrac{CD}{\sin(\beta_2)} \Rightarrow \sin(\beta_2) = \dfrac{CD \sin(\delta_2)}{BC} \Rightarrow$

$\sin \beta_2 = 0.9 \Rightarrow \beta_2 = 63.75^\circ$

Finalement, $\gamma = 180^\circ - \beta_2 - \delta_2 = 180^\circ - 63.75^\circ - 57^\circ \Rightarrow \gamma = 59.25^\circ$

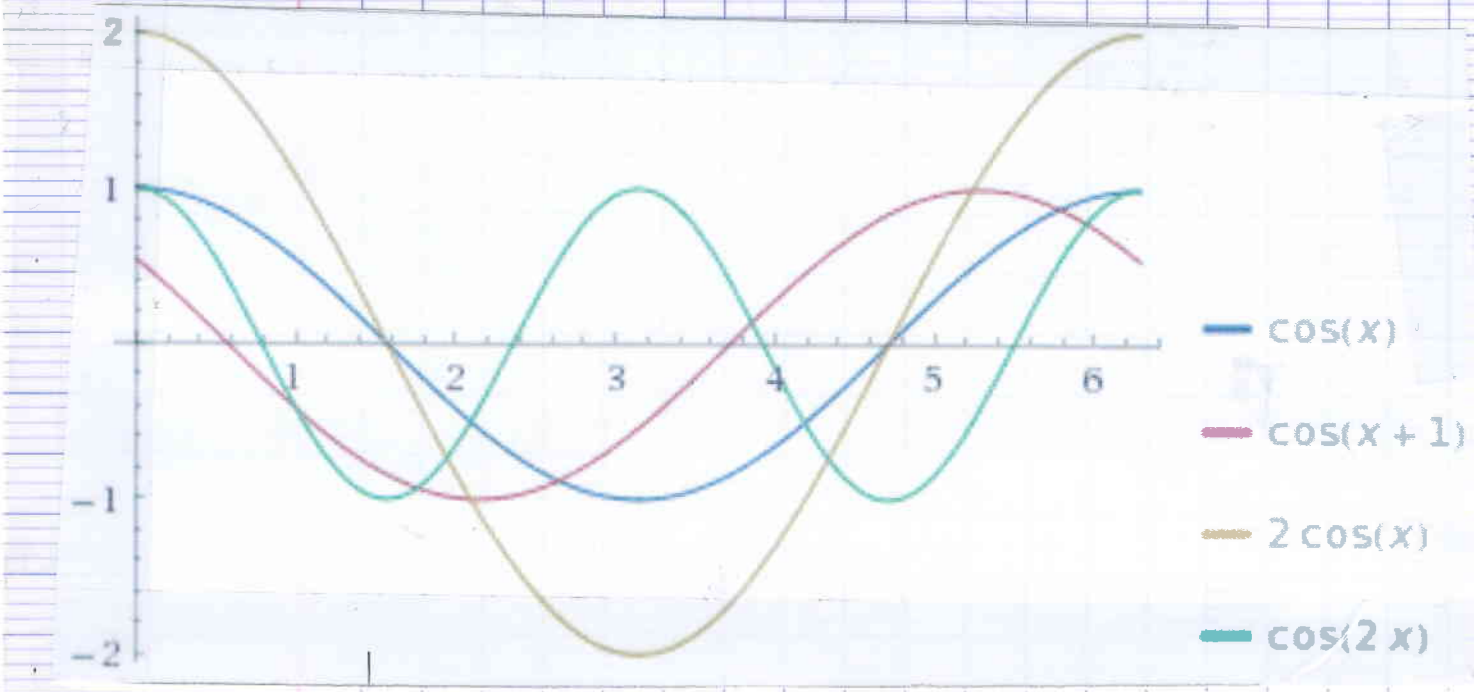
Ainsi les autres angles du quadrilatère sont

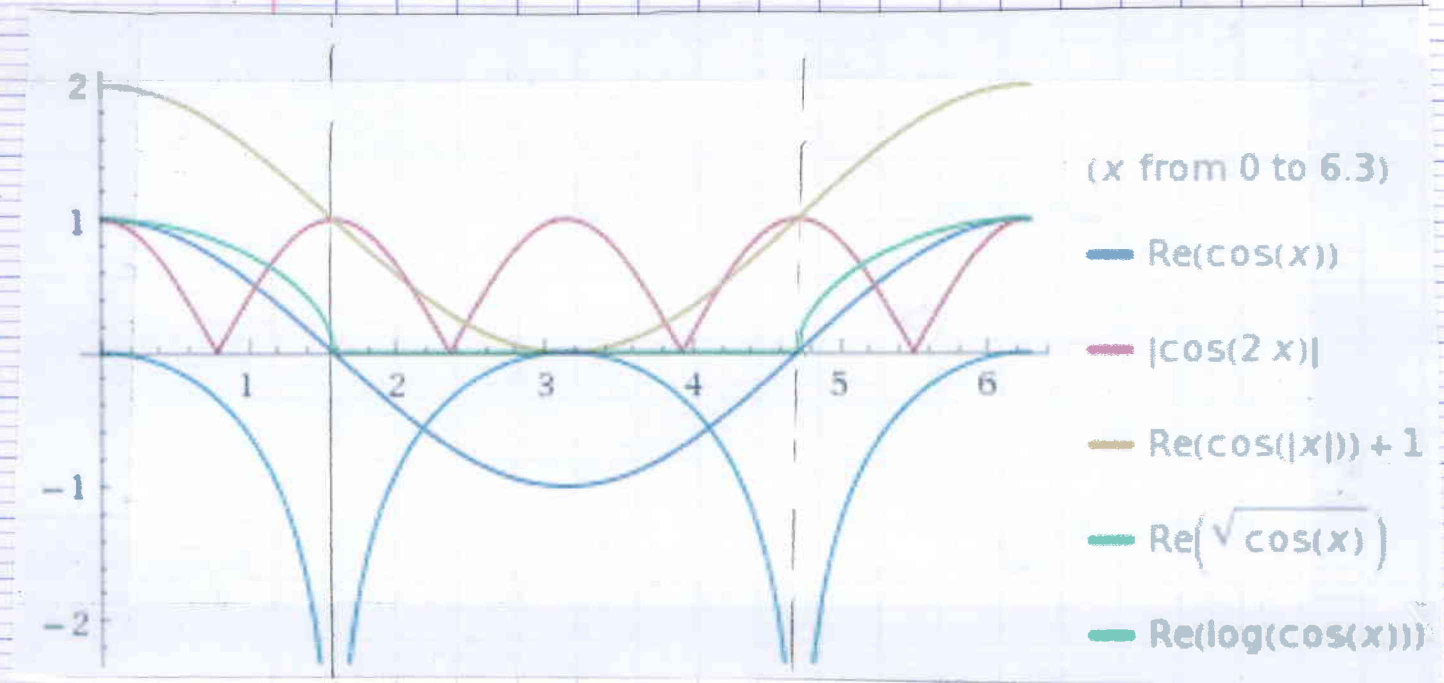
$\beta = \beta_1 + \beta_2 = 47.4^\circ + 63.75^\circ \Rightarrow \underline{\beta = 111,15^\circ}$

$\underline{\gamma = 59.25^\circ}$

$\delta = \delta_1 + \delta_2 = 35.8^\circ + 57^\circ \Rightarrow \underline{\delta = 92.8^\circ}$

EXERCICE 29





$f(x)$: $D=\mathbb{R}$ $f(D)=[-1;1]$ $T=2\pi$ ni paire ni impaire
$g(x)$: $D=\mathbb{R}$, $g(D)=[-2;2]$; $T=2\pi$ paire.
$h(x)$: $D=\mathbb{R}$, $h(D)=[-1;1]$; $T=\pi$ paire
$i(x)$: $D=\mathbb{R}$, $i(D)=[0,1]$; $T=\frac{\pi}{2}$ paire

$j(x)$: $D = \mathbb{R}$, $k(D) = [0;2]$, $T = 2\pi$ paire.
$k(x)$: $D = \ldots \cup [-\frac{\pi}{2};\frac{\pi}{2}] \cup [\frac{3\pi}{2};\frac{5\pi}{2}] \cup \ldots$ $\ell(D) = [0,1]$ $T = 2\pi$, paire
$\ell(x)$: $D = \ldots \cup [-\frac{\pi}{2};\frac{\pi}{2}] \cup [\frac{3\pi}{2};\frac{5\pi}{2}] \cup \ldots$, $\ell(D) =]-\infty;0]$, $T = 2\pi$ paire

Exercice 30

1. $\sin(x) = 0.1 \Leftrightarrow \sin x = \sin(5.7) \Leftrightarrow$

$$\begin{cases} x = 5.7 + k \cdot 360 & (1) \\ x = 180 - 5.7 + k \cdot 360 \Leftrightarrow x = 174.3 + k \cdot 360 & (2) \end{cases}$$

(1) $x = 5.7 + k \cdot 360$

$k = 0$ $\quad \underline{x_1 = 5.7}$

(2) $x = 174.3 + k \cdot 360$.

$k = 0$ $\quad \underline{x_2 = 174.3^\circ}$

2. $\sin x = -0.84 \Leftrightarrow \sin x = \sin(-57.1)$

$$\Leftrightarrow \begin{cases} x = -57.1 + k \cdot 360 & (1) \\ x = 180 + 57.1 + k \cdot 360 \Leftrightarrow x = 237.1 + k\,360 & (2) \end{cases}$$

(1) $k = 1$ $\quad \underline{x_1 = 302.9}$

(2) $k = 0$ $\quad \underline{x_2 = 237.1}$

3. $\sin(x + 15^\circ) = 0.951 \Leftrightarrow \sin(x + 15^\circ) = \sin(72^\circ)$

$$\begin{cases} x + 15 = 72 + k \cdot 360 \Leftrightarrow x = 57 + k \cdot 360 & (1) \\ x + 15 = 180 - 72 + k \cdot 360 \Leftrightarrow x = 93 + k \cdot 360 & (2) \end{cases}$$

(1) $k = 0$ $\quad \underline{x_1 = 57^\circ}$

(2) $k = 0$ $\quad \underline{x_2 = 93^\circ}$

4. $\cos(x) = 0.8 \Leftrightarrow \cos(x) = \cos(36.9)$

$$\begin{cases} x = 36.9 + k \cdot 360 & (1) \\ x = -36.9 + k\,360 & (2) \end{cases}$$

(1) $k = 0$ $\quad \underline{x_1 = 36.9}$

(2) $k = 1$ $\quad \underline{x_2 = 323.1}$

5. $\cos x = -0.84 \Leftrightarrow \cos x = \cos(147.1)$

$$\begin{cases} x = 147.1 + k \cdot 360 & (1) \\ x = -147.1 + k \cdot 360 & (2) \end{cases}$$

(1) $k=0$ $\quad \underline{x_1 = 147.1^\circ}$

(2) $k=1$ $\quad \underline{x_2 = 212.86^\circ}$

6. $\cos(x-20) = \sqrt{\frac{2}{3}} \Leftrightarrow \cos(x-20) = \cos 35.26$

$$\begin{cases} x = 35.26 + k \cdot 360 & (1) \\ x = -35.26 + k \cdot 360 & (2) \end{cases}$$

(1) $k=0$ $\quad x_1 = 35.26$

(2) $k=1$ $\quad x_2 = 324.74$

7. $\tan(x) = 4 \Leftrightarrow \tan x = \tan(75.96)$

$x = 75.96 + k \cdot 180$

$k=0$ $\quad x_1 = 75.96^\circ$

$k=1$ $\quad x_1 = 255.96^\circ$

8. $\tan(x) = -0.32 \Leftrightarrow \tan(x) = \tan(-17.74)$

$x = -17.74 + k \cdot 180$

$k=1$ $\quad x_1 = 162.2$

$k=2$ $\quad x_2 = 342.2$

9. $\tan(x+100^\circ) = 0.11 \Leftrightarrow \tan(x+100) = \tan(6.3)$

$x+100 = 6.3 + k \cdot 180 \Leftrightarrow x = -93.7^\circ + k \cdot 180$

$k=1$ $\quad x_1 = 86.3$

$k=2$ $\quad x_2 = 266.3$

EXERCICE 31

1. $\cos 2x = \frac{1}{3} \Leftrightarrow \cos 2x = \cos(70{,}5)$

$$\begin{cases} 2x = 70{,}5 + k \cdot 360 \Leftrightarrow x = 35{,}3 + k \cdot 180 \quad ① \\ 2x = -70{,}5 + k \cdot 360 \Leftrightarrow x = -35{,}3 + k \cdot 180 \quad ② \end{cases}$$

① $k = 0 \quad x_1 = 35{,}3$

$k = 1 \quad x_2 = 215{,}3$

② $k = 1 \quad x_1 = 144{,}7$

$k = 2 \quad x_2 = 324{,}7.$

2. $\tan(3x) = 2 \Leftrightarrow \tan(3x) = \tan 63{,}4$

$3x = 63{,}4 + k \cdot 180 \Leftrightarrow x = 21{,}1 + k \cdot 60.$

$k = 0 \quad x_1 = 21{,}1 \qquad k = 2 \quad x_3 = 141{,}1$

$k = 1 \quad x_2 = 81{,}1 \qquad k = 3 \quad x_4 = 201{,}1$

$k = 4 \quad x_5 = 261{,}1 \qquad k = 5 \quad x_6 = 321{,}1$

3. $\sin(2x) = -0{,}6 \Leftrightarrow \sin(2x) = \sin(-36{,}9)$

$$\begin{cases} 2x = -36{,}9 + k \cdot 360 \\ 2x = 180 - (-36{,}9) + k \cdot 360 \Leftrightarrow 2x = 216{,}9 + k \cdot 360 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -18{,}45 + k \cdot 180 \quad ① \\ x = 108{,}45 + k \cdot 180 \quad ② \end{cases}$$

① $k = 1 \quad x_1 = 161{,}55$

$k = 2 \quad x_2 = 341{,}55$

② $k = 0 \quad x_1 = 108{,}45$

$k = 1 \quad x_2 = 288{,}45$

4. $\cos(4x + 30°) = -\frac{1}{4} \Leftrightarrow \cos(4x+30) = \cos(104{,}5)$

$$\begin{cases} 4x + 30 = 104{,}5 + k \cdot 360 \\ 4x + 30 = -104{,}5 + k \cdot 360 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x = 18{,}6 + 90 \cdot k \quad ① \\ x = -33{,}6 + k \cdot 90 \quad ② \end{cases}$$

① $k=0$ $x_1 = 18.6$ $k=1$ $x_2 = 108.6$
$k=2$ $x_3 = 198.6$ $k=3$ $x_4 = 288.6$
$k=4$ $x_5 = 378.6$

② $k=1$ $x_1 = 56.4$ $k=2$ $x_2 = 146.4$
$k=3$ $x_3 = 236.4$ $k=4$ $x_4 = 326.4$

5. $\tan(2x-90) = 0.4 \Leftrightarrow \tan(2x-90) = \tan(21.8)$
$2x - 90 = 21.8 + k\cdot 180$
$2x = 111.8 + k\cdot 180 \Leftrightarrow x = 55.9 + k\cdot 90$
$k=0$ $x_1 = 55.9$
$k=1$ $x_2 = 145.9$
$k=2$ $x_3 = 235.9$
$k=3$ $x_4 = 325.9$

6. $\sin(3x-45) = -0.42 \Leftrightarrow \sin(3x-45) = \sin(-24.8)$
$$\begin{cases} 3x - 45 = -24.8 + k\cdot 360 \\ 3x - 45 = 204.8 + k\cdot 360 \end{cases} \Leftrightarrow$$
$$\Leftrightarrow \begin{cases} x = 6.7 + k\cdot 120 & ① \\ x = 83.3 + k\cdot 120 & ② \end{cases}$$
① $k=0$ $x_1 = 6.7$ $k=1$ $x_2 = 126.7$
$k=2$ $x_3 = 246.7$
② $k=0$ $x_1 = 83.3$ $k=1$ $x_2 = 203.3$
$k=2$ $x_3 = 323.3$

7. $\cot(x) = \frac{\sqrt{3}}{3} \Leftrightarrow \cot(x) = \cot(60°)$

$\Leftrightarrow x = 60 + k\cdot 180$
$k=0$ $x_1 = 60$ $k=1$ $x_2 = 240$

8. $\cot(x-20^\circ)=\sqrt{3} \Leftrightarrow \cot(x-20)=\cot(30) \Leftrightarrow$

$x-20=30+k\cdot180^\circ \Leftrightarrow x=50+k\cdot180^\circ$

$k=0 \quad x_1=50 \quad k=1 \quad x_2=230^\circ$

9. $\cot(2x)=2.8 \Leftrightarrow \cot(2x)=\cot(19.6)$

$2x=19.6+k\cdot180 \Leftrightarrow x=9.8+k\cdot90$

$k=0 \quad x_1=9.8 \quad k=1 \quad x_2=99.8$

$k=2 \quad x_3=189.8 \quad k=3 \quad x_4=279.8$

EXERCICE 32. $\sin(x)=-0.7 \Leftrightarrow \sin(x)=\sin(0.77)$

$\Leftrightarrow \begin{cases} x=-0.77+2k\pi & ① \\ x=\pi+0.7+2k\pi & ② \end{cases}$

② $k=1 \quad x=\pi+0.7+2\pi=10.125$

Donc $\underline{x=10.125}$

EXERCICE 33

1. $\sin(5x)=\sin(7x)$

$\begin{cases} 5x=7x+k\cdot360 \Leftrightarrow -2x=k\cdot360 \Leftrightarrow x=k\cdot180 \; ① \\ 5x=180-7x+k\cdot360 \Leftrightarrow 12x=180+k\cdot360 \Leftrightarrow \end{cases}$

$x=15+k\cdot30 \quad ②$

2. $\cos(2x)=\cos\left(\frac{2\pi}{3}\right) \Leftrightarrow$

$\begin{cases} 2x=\frac{2\pi}{3}+2k\pi \Leftrightarrow x=\frac{\pi}{3}+k\pi \\ 2x=-\frac{2\pi}{3}+2k\pi \Leftrightarrow x=-\frac{\pi}{3}+k\pi \end{cases}$

3. $\sin(4x)+\sin(x)=0 \Leftrightarrow \sin(4x)=-\sin(x) \Leftrightarrow$

$\Leftrightarrow \sin(4x)=\sin(-x) \Leftrightarrow$

$\begin{cases} 4x=-x+2k\pi \Leftrightarrow 5x=2k\pi \Leftrightarrow x=\frac{2k\pi}{5} \\ 4x=\pi+x+2k\pi \Leftrightarrow 3x=\pi+2k\pi \Leftrightarrow \end{cases}$

$x=\frac{\pi}{3}+\frac{2k\pi}{3}$

4. $\tan\left(x+\frac{\pi}{6}\right) = \tan(3x) \Leftrightarrow$

$x + \frac{\pi}{6} = 3x + k\pi \Leftrightarrow$

$-2x = -\frac{\pi}{6} + k\pi \Leftrightarrow x = \frac{\pi}{12} - \frac{k\pi}{2}$

EXERCICE 34

1. $2\sin x + \cos x = 0 \Leftrightarrow 2\sin x = -\cos x \quad \left(x \neq \frac{\pi}{2} + k\pi\right)$

$\Leftrightarrow \frac{\sin x}{\cos x} = -\frac{1}{2} \Leftrightarrow \tan x = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \tan x = \tan(-26{,}6) \Leftrightarrow$

$\Leftrightarrow x = -26{,}6 + k \cdot 180$

2. $4\cos^2(x) - 4\cos(x) - 3 = 0$

On pose $t = \cos(x)$

$4t^2 - 4t - 3 = 0 \quad \Delta = 64 \quad t_{1,2} = \frac{4 \pm 8}{8} = \begin{cases} t_1 = \frac{3}{2} \\ t_2 = -\frac{1}{2} \end{cases}$

$t_1 = \frac{3}{2} = \cos x$ impossible

$t_2 = -\frac{1}{2} = \cos x \Leftrightarrow \cos x = \cos\frac{2\pi}{3} \Leftrightarrow$

$\begin{cases} x = \frac{2\pi}{3} + 2k\pi \\ x = -\frac{2\pi}{3} + 2k\pi \end{cases}$

3. $2\sin^2(x) - 3\sin(x) + 1 = 0$

$\sin x = t$

$2t^2 - 3t + 1 = 0 \quad \Delta = 1 \quad t_{1,2} = \frac{3 \pm 1}{4} \begin{cases} t_1 = 1 \\ t_2 = 1/2 \end{cases}$

$\sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + 2k\pi$

$\sin x = \frac{1}{2} \Leftrightarrow \sin x = \sin\frac{\pi}{6} \Leftrightarrow \begin{cases} x = \frac{\pi}{6} + 2k\pi \\ x = \pi - \frac{\pi}{6} + 2k\pi \end{cases}$

4. $\sqrt{3}\sin(2x)-\cos(2x)=0$ $\left(2x \neq \frac{\pi}{2}+k\pi \Leftrightarrow x \neq \frac{\pi}{4}+\frac{k\pi}{2}\right)$

$\Leftrightarrow \frac{\sin(2x)}{\cos(2x)} = \frac{\sqrt{3}}{3} \Leftrightarrow$

$\Leftrightarrow \tan(2x) = \tan\left(\frac{\pi}{6}\right) \Leftrightarrow$

$2x = \frac{\pi}{6} + k\cdot\pi \Leftrightarrow x = \frac{\pi}{12} + \frac{k\pi}{2}$

5. $2\cos^2(x)+\sin(x)-1=0$

$\Leftrightarrow 2(1-\sin^2(x))+\sin x-1=0 \Leftrightarrow$

$\Leftrightarrow -2\sin^2 x+\sin x+1=0$

$\Delta = 1+8=9$ $\sin x = \frac{-1\pm 3}{-4} = \begin{cases} 1 \\ -1/2 \end{cases}$

- $\sin x = 1 \Leftrightarrow x = \frac{\pi}{2}+2k\pi$
- $\sin x = -\frac{1}{2} \Leftrightarrow \sin x = \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow \begin{cases} x = -\frac{\pi}{6}+2k\pi \\ x = \pi+\frac{\pi}{6}+2k\pi \end{cases}$

6. $\sqrt{3}\cos x-\sin x=1 \Leftrightarrow$

$\Leftrightarrow \tan\frac{\pi}{3}\cos x-\sin x=1 \Leftrightarrow$

$\Leftrightarrow \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}\cos x-\sin x=1 \Leftrightarrow$

$\Leftrightarrow \sin\frac{\pi}{3}\cos x-\cos\frac{\pi}{3}\sin x = \cos\frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow \sin\left(\frac{\pi}{3}-x\right) = \cos\frac{\pi}{3} \Rightarrow$

$\sin\left(\frac{\pi}{3}-x\right) = \sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right) \Leftrightarrow$

$\sin\left(\frac{\pi}{3}-x\right) = \sin\left(\frac{\pi}{6}\right)$

$\begin{cases} \frac{\pi}{3}-x = \frac{\pi}{6}+2k\pi \Leftrightarrow -x = -\frac{\pi}{6}+2k\pi \Leftrightarrow x = \frac{\pi}{6}+2k\pi \\ \frac{\pi}{3}-x = \pi-\frac{\pi}{6}+2k\pi \Leftrightarrow -x = \frac{\pi}{2}+2k\pi \Leftrightarrow x = -\frac{\pi}{2}+2k\pi \end{cases}$

EXERCICE 35 1. $\tan(\arctan(x)) = x$

2. $\sin(\arccos(x)) \overset{y=\arccos x}{=\!=} \sin y = \sqrt{1-\cos^2 y} =$

$= \sqrt{1-(\cos(\arccos x))^2} = \sqrt{1-x^2}$

3. $\tan(\arccos(x)) \overset{y=\arccos(x)}{=} \tan(y) =$

$= \sqrt{\tan^2 y} = \sqrt{\frac{1}{\cos^2 y} - 1} = \sqrt{\frac{1}{(\cos(\arccos x))^2} - 1} =$

$= \sqrt{\frac{1}{x^2} - 1}$

4. $\sin(2\arcsin x) \overset{y=\arcsin x}{=} \sin(2y) = 2\sin y \cos y =$

$= 2\sin(\arcsin x)\cos(\arcsin x) =$
$= 2x\sqrt{1-x^2}$

EXERCICE 36 1. $\cos(2x) = \cos(x+x) = \cos x \cos x - \sin x \sin x =$
$= \cos^2 x - \sin^2 x.$
$\sin(2x) = \sin(x+x) = \sin x \cos x + \sin x \cos x =$
$= 2\sin x \cos x.$
$\tan 2x = \tan(x+x) = \frac{\tan x + \tan x}{1 - \tan x \tan x} =$

$= \frac{2\tan x}{1-\tan^2 x}$

2. $\cos(3x) = \cos(2x+x) = \cos 2x \cdot \cos x - \sin 2x \cdot \sin x =$
$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x =$
$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x =$
$= 2\cos^3 x - \cos x - 2(1-\cos^2 x)\cos x =$
$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x =$
$= 4\cos^3 x - 3\cos x.$

$= \cos x\,(4\cos^2 x - 3) = \cos x\,(4 - 4\sin^2 x - 3) =$
$= \cos x\,(1 - 4\sin^2 x)$

$\sin(3x) = \sin(2x + x) = \sin 2x \cos x + \cos 2x \cdot \sin x$
$= 2\sin x \cos^2 x + (1 - 2\sin^2 x)\sin x =$
$= 2\sin x(1 - \sin^2 x) + (1 - 2\sin^2 x)\sin x =$
$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x =$
$= 3\sin x - 4\sin^3 x =$
$= \sin x\,(3 - 4\sin^2 x) =$
$= \sin x\,(3 - 4 + 4\cos^2 x) =$
$= \sin x\,(4\cos^2 x - 1)$

EXERCICE 37

- $\cos(15°) = \cos(45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30 =$
$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

- $\sin(15) = \sin(45 - 30) = \sin 45 \cos 30 - \sin 30 \cos 45 =$
$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

- $\sin(105°) = \sin(60 + 45) = \sin 60 \cos 45 + \cos 60 \sin 45$
$= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{1}{2} \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

- $\cos(105°) = \cos(60 + 45) = \cos 60 \cos 45 - \sin 60 \sin 45 =$
$= \frac{1}{2} \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

- $\cos(255) = \cos(270 - 15) = \cos 270 \cos 15 + \sin 270 \sin 15 =$
$= 0 - 1 \cdot \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$

EXERCICE 38

1. $\cos x + \cos(x + 120) + \cos(x + 240) =$
$= \cos x + \cos x \cos 120 - \sin x \sin 120 + \cos x \cos 240 - \sin x \sin 240$
$= \cos x + \cos x\left(-\frac{1}{2}\right) - \sin x \cdot \frac{\sqrt{3}}{2} + \cos x \cdot \left(-\frac{1}{2}\right) - \sin x\left(-\frac{\sqrt{3}}{2}\right) =$

$= \cos x - \frac{1}{2}\cos x - \frac{\sqrt{3}}{\cancel{2}}\sin x - \frac{1}{2}\cos x + \frac{\sqrt{3}}{\cancel{2}}\sin x =$

$= \cos x - \cos x = 0$

2. $\sin(x) + \sin(x+120) + \sin(x+240) =$
$= \sin x + \sin x \cos 120 + \sin 120 \cos x + \sin x \cos 240 + \cos x \sin 240 =$
$= \sin x - \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cancel{\cos x} - \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cancel{\cos x} =$
$= \sin x - \sin x = 0$

EXERCICE 39 $\sin x = \frac{2}{3}$ $x \in Q_{II}$ $\cos y = -\frac{1}{4}$ $y \in Q_{III}$

- $\cos^2 x = 1 - \left(\frac{2}{3}\right)^2 \Rightarrow \cos^2 x = 1 - \frac{4}{9} \Rightarrow \cos^2 x = \frac{5}{9} \xrightarrow{x \in Q_{II}} \cos x = -\frac{\sqrt{5}}{3}$
  $\tan x = \frac{2/3}{-\frac{\sqrt{5}}{3}} \Rightarrow \tan x = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

- $\sin^2 y = 1 - \left(-\frac{1}{4}\right)^2 \Rightarrow \sin^2 y = 1 - \frac{1}{16} = \frac{15}{16} \xrightarrow{y \in Q_{III}} \sin y = -\frac{\sqrt{15}}{4}$
  $\tan y = \frac{-\frac{\sqrt{15}}{4}}{-1/4} = \frac{\sqrt{15}}{1}$

- $\sin(x+y) = \sin x \cos y + \cos x \sin y =$
  $= \frac{2}{3}\left(-\frac{1}{4}\right) + \left(-\frac{\sqrt{5}}{3}\right)\cdot\left(-\frac{\sqrt{15}}{4}\right) =$

  $= -\frac{1}{6} + \frac{5\sqrt{3}}{12} = \frac{-2+5\sqrt{3}}{12}$

- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} = \frac{-\frac{2\sqrt{5}}{5} - \sqrt{15}}{1 - \frac{2\sqrt{5}}{5}\cdot\sqrt{15}} =$

  $= \frac{-\frac{2\sqrt{5}}{5} - \frac{5\sqrt{15}}{5}}{1 - \frac{2\cancel{5}\sqrt{3}}{\cancel{5}}} = \frac{\frac{-2\sqrt{5}-5\sqrt{15}}{5}}{1-2\sqrt{3}} = \frac{-2\sqrt{5}-5\sqrt{15}}{5(1-2\sqrt{3})}$

EXERCICE 40 $\sin\alpha = \frac{3}{5}$ $\cos\beta = \frac{24}{25}$ $\alpha, \beta \in Q_I$
$\cos^2\alpha = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow \cos\alpha = \frac{4}{5}$
$\tan\alpha = \frac{3/5}{4/5} \Rightarrow \tan\alpha = 3/4$

2. $\tan\left(\frac{\pi}{4}+t\right)-\tan\left(\frac{\pi}{4}-t\right)=$

$$\frac{\tan\frac{\pi}{4}+\tan t}{1-\tan\frac{\pi}{4}\cdot\tan t}-\frac{\tan\frac{\pi}{4}-\tan t}{1+\tan\frac{\pi}{4}\tan t}=$$

$$=\frac{1+\tan t}{1-\tan t}-\frac{1-\tan t}{1+\tan t}=$$

$$=\frac{(1+\tan t)^2-(1-\tan t)^2}{1-\tan^2 t}=\frac{(\cancel{1}+\tan t-\cancel{1}+\tan t)(1+\cancel{\tan t}+1-\cancel{\tan t})}{1-\tan^2 t}$$

$$=\frac{2\tan t\cdot 2}{1-\tan^2 t}=\frac{4\tan t}{1-\tan^2 t}$$

EXERCICE 1.42

1. $\cos 2x = 1-\sin^2(x)$

donc $\cos x = 1-2\sin^2\frac{x}{2} \Rightarrow \cos x = 1-2\left(1-\cos^2\frac{x}{2}\right) \Leftrightarrow$

$\Leftrightarrow \cos x = 1-2+2\cos^2\frac{x}{2} \Leftrightarrow 2\cos^2\frac{x}{2} = 1-\cos x \Leftrightarrow$

$\Leftrightarrow \cos^2\frac{x}{2} = \frac{1-\cos x}{2}$

2. $\sin^2(22{,}5°) = \sin^2\left(\frac{45}{2}\right) = \frac{1-\cos 45}{2} \Rightarrow$

$\Rightarrow \sin^2(22{,}5) = \frac{1-\frac{\sqrt{2}}{2}}{2} \Rightarrow \sin^2(22{,}5) = \frac{2-\sqrt{2}}{4} \Rightarrow$

$\Rightarrow \sin 22{,}5 = \frac{\sqrt{2-\sqrt{2}}}{2}$

EXERCICE 1.43

1. $A\left(6;\frac{7\pi}{4}\right)$ $\begin{cases} x = 6\cdot\cos\frac{7\pi}{4} \\ y = 6\cdot\sin\frac{7\pi}{4}\end{cases} \Rightarrow \begin{cases} x = 6\cdot\cos\left(2\pi-\frac{\pi}{4}\right) \\ y = 6\cdot\sin\left(2\pi-\frac{\pi}{4}\right)\end{cases} \Rightarrow$

$\begin{cases} x = 6\cdot\frac{\sqrt{2}}{2} \\ y = 6\cdot\left(-\frac{\sqrt{2}}{2}\right)\end{cases} \Rightarrow \begin{cases} x = 3\sqrt{2} \\ y = -3\sqrt{2}\end{cases}$

2. $B\left(4;\frac{5\pi}{6}\right)$ $\begin{cases} x = 4\cos\frac{5\pi}{6} \\ y = 4\sin\frac{5\pi}{6}\end{cases} \Rightarrow \begin{cases} x = 4\cdot\cos\left(\pi-\frac{\pi}{6}\right) \\ y = 4\sin\left(\pi-\frac{\pi}{6}\right)\end{cases} \Rightarrow$

$\begin{cases} x = 4\cdot\left(-\frac{\sqrt{3}}{2}\right) \\ y = 4\cdot\frac{1}{2}\end{cases} \Rightarrow \begin{cases} x = -2\sqrt{3} \\ y = 2\end{cases}$

3. $C\left(2;\frac{3\pi}{2}\right)$ $\begin{cases} x = 2\cos\frac{3\pi}{2} \\ y = 2\cdot\sin\frac{3\pi}{2} \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = -2 \end{cases}$

4. $D\left(\frac{1}{2};60°\right)$ $\begin{cases} x = \frac{1}{2}\cos 60 \\ y = \frac{1}{2}\sin 60 \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{2}\cdot\frac{1}{2} \\ y = \frac{1}{2}\cdot\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \begin{cases} x = \frac{1}{4} \\ y = \frac{\sqrt{3}}{4} \end{cases}$

5. $E(3;-45°)$
$\begin{cases} x = 3\cos(-45°) \\ y = 3\sin(-45°) \end{cases} \Rightarrow \begin{cases} x = \frac{3\sqrt{2}}{2} \\ y = -\frac{3\sqrt{2}}{2} \end{cases}$

6. $F(5;253°)$ $\begin{cases} x = 5\cos 253° = -1.46 \\ y = 5\sin 253 = -4.78 \end{cases}$

Exercice 1.44

1. $A(-9;12)$ $r = \sqrt{(-9)^2+12^2} = \sqrt{225} = 15$
$\varphi = \tan^{-1}\left(\frac{12}{-9}\right) \Rightarrow \varphi = 126.87°$

2. $B(2;-1)$ $r = \sqrt{4+1} = \sqrt{5}$
$\varphi = \tan^{-1}\left(-\frac{1}{2}\right) \Rightarrow \varphi = 333.43°$

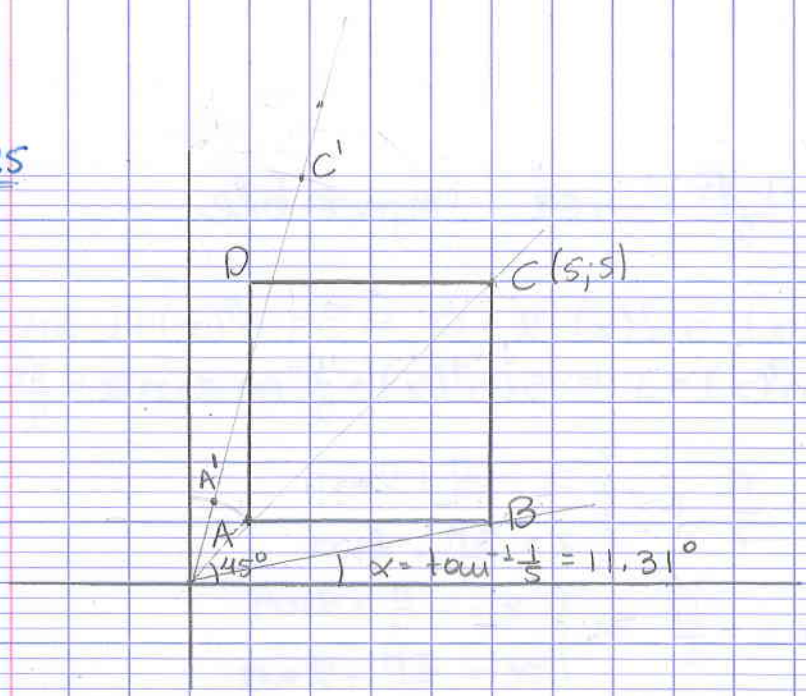
3. $C(-2\sqrt{3};-6)$ $r = \sqrt{3\cdot4+36} = \sqrt{48}$
$\varphi = \tan^{-1}\left(\frac{-6}{-2\sqrt{3}}\right) = \tan^{-1}(\sqrt{3}) = 240°$

4. $D(-5;5)$ $r = \sqrt{25+25} = 5\sqrt{2}$
$\varphi = \tan^{-1}(-1) = 180-45 = 135°$

5. $E(-2;0)$ $r = \sqrt{4+0} = 2$
$\varphi = \tan^{-1} 0 = 180°$

6. $F\left(\frac{x\sqrt{3}}{2};-\frac{x\sqrt{3}}{2}\right)$ $x>0$ $r = \sqrt{\frac{x^2\cdot3}{4}\cdot2} = x\cdot\sqrt{\frac{3}{2}} = x\cdot1.225$
$\varphi = \tan^{-1}(-1) = 315°$

Exercice 1.45



A(1;1) B(5;1) C(5;5) D(1;5)

On va utiliser les coordonnées polaires

$\underline{A'}$. $r = \sqrt{1+1} = \sqrt{2}$ $\quad \varphi = 45+30 = 75$ $\quad \begin{cases} x = r\cdot\cos\varphi \\ y = r\cdot\sin\varphi \end{cases} \Rightarrow \begin{cases} x = \sqrt{2}\cos 75 \\ y = \sqrt{2}\sin 75 \end{cases} \Rightarrow \begin{cases} x = 0.37 \\ y = 1.37° \end{cases}$

$\underline{B'}$: $r = \sqrt{25+1} = \sqrt{26}$ $\quad \varphi = 11.31 + 30 = 41.31°$ $\quad \begin{cases} x = \sqrt{26}\cdot\cos 41,31 \\ y = \sqrt{26}\cdot\sin 41.31 \end{cases} \Rightarrow \begin{cases} x = 3,83 \\ y = 3.37 \end{cases}$

$\underline{C'}$: $r = \sqrt{25+25} = 5\sqrt{2}$ $\quad \varphi = 75$ $\quad \begin{cases} x = 5\sqrt{2}\cos 75 \\ y = 5\sqrt{2}\sin 75 \end{cases} \Rightarrow \begin{cases} x = 1,83 \\ y = 6.83 \end{cases}$

$\underline{D'}$: $r = \sqrt{1+5^2} = \sqrt{26}$ $\quad \varphi = \tan^{-1}\frac{5}{1} + 30 = 108.7°$ $\quad \begin{cases} x = \sqrt{26}\cos 108.7° \\ y = \sqrt{26}\sin 108.7 \end{cases} \Rightarrow \begin{cases} x = -1,63 \\ y = 4,83 \end{cases}$

Exercice 1.46

1. $\cos x = \tan x \qquad x \neq \frac{\pi}{2} + k\pi$

$\Leftrightarrow \cos x = \frac{\sin x}{\cos x} \Leftrightarrow \cos^2 x - \sin x = 0 \Leftrightarrow 1 - \sin^2 x - \sin x = 0$

$\Leftrightarrow \sin^2 x + \sin x - 1 = 0 \qquad \Delta = 1 + 4 = 5$

$\sin x = \frac{-1 \pm \sqrt{5}}{2} \begin{cases} \frac{-1+\sqrt{5}}{2} \\ \frac{-1-\sqrt{5}}{2} \end{cases}$

$\sin x = \frac{-1+\sqrt{5}}{2} \Leftrightarrow \sin x = \sin 38.2$

$\begin{cases} x = 38,2 + k\cdot 360 \\ x = 141,8 + k\cdot 360 \end{cases}$

- $\sin x = \frac{-1-\sqrt{5}}{2} = -1,62$ impossible.

2. $3\sin^2(x) + \cos^2(x) - 2 = 0 \Leftrightarrow 3\sin^2(x) + 1 - \sin^2(x) - 2 = 0$
$\Leftrightarrow 2\sin^2(x) = 1 \Leftrightarrow \sin^2(x) = \frac{1}{2} \Leftrightarrow \sin x = \frac{\sqrt{2}}{2}$ ou $\sin x = -\frac{\sqrt{2}}{2}$

- $\sin x = \frac{\sqrt{2}}{2} \Leftrightarrow \begin{cases} x = \frac{\pi}{4} + 2k\pi \\ x = \frac{3\pi}{4} + 2k\pi \end{cases}$
- $\sin x = -\frac{\sqrt{2}}{2} \Leftrightarrow \begin{cases} x = -\frac{\pi}{4} + 2k\pi \\ x = \frac{5\pi}{4} + 2k\pi \end{cases}$

3. $\sin(2x) + 3\cos(2x) = 2 \Leftrightarrow \sin(2x) + \tan 71.6 \cos 2x = 2$
$\Leftrightarrow \sin 2x \cdot \cos 71.6 + \sin 71.6 \cdot \cos 2x = 2 \cdot \cos 71.6 \Leftrightarrow$
$\Leftrightarrow \sin(2x + 71.6) = 0.63 \Leftrightarrow$
$\Leftrightarrow \sin(2x + 71.6) = \sin 39.23^\circ \Leftrightarrow$
$\begin{cases} 2x + 71.6 = 39.23 + k \cdot 360 \\ 2x + 71.6 = 140.77 + k \cdot 360 \end{cases} \Leftrightarrow$
$\begin{cases} 2x = -32.37 + k \cdot 360 \\ 2x = 69.17 + k \cdot 360 \end{cases} \Leftrightarrow \begin{cases} x = -16.2^\circ + 180 \cdot k \\ x = 34.6^\circ + 180 \cdot k \end{cases}$

4. $\tan\left(x + \frac{\pi}{6}\right) = \cot(3x)$

$x + \frac{\pi}{6} \neq \frac{\pi}{2} + k\pi \Rightarrow$
$x \neq \frac{\pi}{3} + k\pi$
$3x \neq k\pi \Rightarrow x \neq \frac{k\pi}{3}$

$\Leftrightarrow \tan\left(x + \frac{\pi}{6}\right) = \tan\left(\frac{\pi}{2} - 3x\right)$

$\Leftrightarrow x + \frac{\pi}{6} = \frac{\pi}{2} - 3x + k\pi \Leftrightarrow$

$\Leftrightarrow 4x = \frac{\pi}{3} + k\pi \Leftrightarrow x = \frac{\pi}{12} + \frac{k\pi}{4}$

5. $\sin x + 3\cos x = 3 \Leftrightarrow \sin x + \tan 71.6 \cos x = 3 \Leftrightarrow$
$\Leftrightarrow \sin x \cdot \cos 71.6 + \sin 71.6 \cdot \cos x = 3 \cdot \cos 71.6 \Leftrightarrow$
$\Leftrightarrow \sin(x + 71.6) = 0.95 \Leftrightarrow \sin(x + 71.6) = \sin(71.6)$
$\Leftrightarrow \begin{cases} x + 71.6 = 71.6 + k \cdot 360 \\ x + 71.6 = 108.4 + k \cdot 360 \end{cases} \Leftrightarrow \begin{cases} x = k \cdot 360 \\ x = 36.83 + k \cdot 360 \end{cases}$

6. $\tan^4(x) - 4\tan^2(x) + 3 = 0$ $\quad\quad$ $x \neq \frac{\pi}{2} + k\pi$

$t = \tan^2 x$

$t^2 - 4t + 3 = 0$

$\Delta = 16 - 12 = 4 \quad t_{1,2} = \frac{4 \pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$

• $\tan^2 x = 3 \Leftrightarrow \tan x = \pm\sqrt{3}$

$\rightarrow \tan x = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3} + k\pi$

$\rightarrow \tan x = -\sqrt{3} \Rightarrow x = \frac{2\pi}{3} + k\pi$

• $\tan^2 x = 1 \Leftrightarrow \tan x = \pm 1$

$\rightarrow \tan x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi$

$\rightarrow \tan x = -1 \Leftrightarrow x = \frac{3\pi}{4} + k\pi$

7. $2\tan x + 2\cot x = 5 \Leftrightarrow$ $\quad\quad$ $x \neq \frac{\pi}{2} + k\pi$, $x \neq k\pi$

$\Leftrightarrow 2\tan x + 2\frac{1}{\tan x} = 5 \Leftrightarrow$

$\Leftrightarrow 2\tan^2 x + 2 = 5\tan x \Leftrightarrow$

$\Leftrightarrow 2\tan^2 x - 5\tan x + 2 = 0$

$\Delta = 25 - 16 = 9 \quad \tan x = \frac{5 \pm 3}{4} = \begin{cases} 2 \\ 1/2 \end{cases}$

• $\tan x = 2 \Rightarrow x = 63 + k \cdot 180$

• $\tan x = \frac{1}{2} \Rightarrow x = 26{,}6 + k \cdot 180$

8. $\sin^4(x) + \cos^4(x) = \frac{2}{3} \Leftrightarrow \sin^4(x) + (1 - \sin^2 x)^2 = \frac{2}{3} \Leftrightarrow$

$\Leftrightarrow \sin^4(x) + 1 - 2\sin^2 x + \sin^4 x = \frac{2}{3} \Leftrightarrow$

$\Leftrightarrow 2\sin^4(x) - 2\sin^2 x + \frac{1}{3} = 0 \Leftrightarrow 6\sin^4 x - 6\sin^2 x + 1 = 0$

$\Delta = 36 - 24 = 12 \quad \sin^2 x = \frac{6 \pm \sqrt{12}}{12} = \begin{cases} 0.79 \\ 0.21 \end{cases}$

• $\sin^2 x = 0.79 \Rightarrow \sin x = 0.89$ ou $\sin x = -0.89$

$\rightarrow \sin x = 0.89 \Leftrightarrow \sin x = \sin 62.72 \Rightarrow$

$$\begin{cases} x = 62.72 + k\cdot 360 \\ x = 117.27 + k\cdot 360 \end{cases}$$

$\rightarrow \sin x = -0.89 \Leftrightarrow \sin x = \sin(-62.87^\circ) = \sin 117.13^\circ$

$$\Leftrightarrow \begin{cases} x = 117.13 + k\cdot 360 \\ x = 62.87^\circ + k\cdot 360 \end{cases}$$

$\cdot \sin^2 x = 0.21 \Leftrightarrow \sin x = \pm 0.46$

$\rightarrow \sin x = 0.46 \Leftrightarrow \sin x = \sin 27.3 \Leftrightarrow$

$$\begin{cases} x = 27.3 + k\cdot 360 \\ x = 152.7 + k\cdot 360 \end{cases}$$

$\rightarrow \sin x = -0.46 \Leftrightarrow \sin x = \sin(152.6) \Leftrightarrow$

$$\begin{cases} x = 152.6 + k\cdot 360 \\ x = 27.4 + k\cdot 360 \end{cases}$$

EXERCICE 1.47

1. $\sin x = \sin\left(\frac{\pi}{4} - x\right) \Leftrightarrow \begin{cases} x = \frac{\pi}{4} - x + k\cdot 360 \\ x = \frac{\pi}{2} - \frac{\pi}{4} + x + k\cdot 360 \quad \text{imp} \end{cases} \Leftrightarrow \left\{ 2x = \frac{\pi}{4} + k\cdot 360 \right.$

$\Leftrightarrow x = \frac{\pi}{8} + k\cdot 180$

2. $\cos(2x) = \cos x \Leftrightarrow \begin{cases} 2x = x + 2k\pi \\ 2x = -x + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = 2k\pi \\ 3x = 2k\pi \end{cases} \Leftrightarrow$

$$\begin{cases} x = 2k\pi \\ x = \frac{2k\pi}{3} \end{cases}$$

3. $\sin\left(\frac{5x}{3}\right) + \cos\left(\frac{x}{2}\right) = 0 \Leftrightarrow \sin\left(\frac{5x}{3}\right) = -\cos\left(\frac{x}{2}\right) \Leftrightarrow$

$\Leftrightarrow \sin\left(\frac{5x}{3}\right) = \cos\left(\pi - \frac{x}{2}\right) \Leftrightarrow \sin\left(\frac{5x}{3}\right) = \sin\left(\frac{\pi}{2} - \pi + \frac{x}{2}\right) \Leftrightarrow$

$\Leftrightarrow \sin \frac{5x}{3} = \sin\left(-\frac{\pi}{2} + \frac{x}{2}\right) \Leftrightarrow$

$$\begin{cases} \frac{5x}{3} = -\frac{\pi}{2} + \frac{x}{2} + 2k\pi \\ \frac{5x}{3} = \pi + \frac{\pi}{2} - \frac{x}{2} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} \frac{7x}{6} = -\frac{\pi}{2} + 2k\pi \\ \frac{13}{6}x = \frac{3\pi}{2} + 2k\pi \end{cases} \Leftrightarrow$$

$$\begin{cases} x = -\frac{3\pi}{7} + \frac{6k\pi}{5} \\ x = \frac{9\pi}{13} + \frac{12k\pi}{13} \end{cases}$$

4. $\sin^2(2x) = \sin^2(x+\frac{\pi}{4}) \Leftrightarrow \begin{cases} \sin 2x = \sin(x+\frac{\pi}{4}) & ① \\ \sin 2x = -\sin(x+\frac{\pi}{4}) & ② \end{cases}$

① $\sin 2x = \sin(x+\frac{\pi}{4}) \Leftrightarrow$

$\begin{cases} 2x = x + \frac{\pi}{4} + 2k\pi \\ 2x = \pi - x - \frac{\pi}{4} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{4} + 2k\pi \\ 3x = \frac{3\pi}{4} + 2k\pi \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x = \frac{\pi}{4} + 2k\pi \\ x = \frac{\pi}{4} + \frac{2k\pi}{3} \end{cases}$

② $\sin 2x = \sin(-x-\frac{\pi}{4}) \Leftrightarrow$

$\Leftrightarrow \begin{cases} 2x = -x - \frac{\pi}{4} + 2k\pi \\ 2x = \pi + x + \frac{\pi}{4} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} 3x = -\frac{\pi}{4} + 2k\pi \\ x = \frac{5\pi}{4} + 2k\pi \end{cases} \Leftrightarrow$

$\begin{cases} x = -\frac{\pi}{12} + \frac{2}{3}k\pi \\ x = \frac{5\pi}{4} + 2k\pi \end{cases}$

5. $6\cos^2(x) = 5\sin x \Leftrightarrow 6(1-\sin^2 x) - 5\sin x = 0$

$-6\sin^2 x - 5\sin x + 6 = 0 \Leftrightarrow$

$\Leftrightarrow 6\sin^2 x + 5\sin x - 6 = 0 \qquad \Delta = 169$

$\sin x = \frac{-5 \pm 13}{12} = \begin{cases} \frac{8}{12} = \frac{2}{3} \\ -\frac{18}{12} = -\frac{3}{4} \end{cases}$

- $\sin x = \frac{2}{3} \Leftrightarrow \sin x = \sin 41.8 \Leftrightarrow$

$\begin{cases} x = 41.8 + k \cdot 360 \\ x = 138.2 + k \cdot 360 \end{cases}$

- $\sin x = -\frac{3}{4} \Leftrightarrow \sin x = \sin(-48.6)$

$\begin{cases} x = -48.6 + k \cdot 360 \\ x = 228.6 + k \cdot 360 \end{cases}$

6. $3\sin x + 6\cos^2(x) = 5 \Leftrightarrow$

$3\sin x + 6(1-\sin^2(x)) = 5 \Leftrightarrow$

$3\sin x + 6 - 6\sin^2 x - 5 = 0 \Leftrightarrow$

$6\sin^2 x - 3\sin x - 1 = 0$

$\Delta = 9 + 24 = 33 \qquad \sin x = \frac{3 \pm \sqrt{33}}{12} \begin{cases} 0.73 \\ -0.23 \end{cases}$

- $\sin x = 0.73 \Leftrightarrow \sin x = \sin(46.9) \Rightarrow$
$$\begin{cases} x = 46.9 + k\cdot 360 \\ x = 133.1 + k\cdot 360 \end{cases}$$
- $\sin x = -0.23 \Leftrightarrow \sin x = \sin(-13.3) \Leftrightarrow$
$$\begin{cases} x = -13.3 + k\cdot 360 \\ x = 193.3 + k\cdot 360 \end{cases}$$

7. $\cos^2(x) + 2\sin x + 1 = 0 \Leftrightarrow 1 - \sin^2 x + 2\sin x + 1 = 0$
$\Leftrightarrow \sin^2 x - 2\sin x - 2 = 0$
$\Delta = 4 + 8 = 12 \qquad \sin x = \dfrac{2 \pm \sqrt{12}}{2} = \begin{cases} 2.73 & \underline{\text{imp}} \\ -0.73 \end{cases}$

- $\sin x = -0.73 \Leftrightarrow \sin x = \sin(-47) \Leftrightarrow$
$$\begin{cases} x = -47 + k\cdot 360 \\ x = 227 + k\cdot 360 \end{cases}$$

8. $\tan^4(x) - 15\tan^2(x) + 26 = 0$
$\tan^2(x) = t$
$t^2 - 15t + 26 = 0 \qquad \Delta = 121$
$t_{1,2} = \dfrac{15 \pm 11}{2} = \begin{cases} t_1 = 2 \\ t_2 = 13 \end{cases}$

- $\tan^2 x = 2 \Leftrightarrow \tan x = \pm\sqrt{2}$
  - $\rightarrow \tan x = \sqrt{2} \Leftrightarrow \tan x = \tan(54.7) \Leftrightarrow$
  $x = 54.7 + k\cdot 180$
  - $\rightarrow \tan x = -\sqrt{2} \Leftrightarrow \tan x = \tan(-54.7) \Rightarrow$
  $x = -54.7 + k\cdot 180$
- $\tan^2 x = 13 \Leftrightarrow \tan x = \pm\sqrt{13}$
  - $\rightarrow \tan x = \sqrt{13} \Rightarrow \tan x = \tan(74.5)$
  $x = 74.5 + k\cdot 180$
  - $\rightarrow \tan x = -\sqrt{13} \Rightarrow \tan x = \tan(-74.5)$
  $\Leftrightarrow x = -74.5 + k\cdot 180$

9. $2\cos(3x)+2\sin(3x)=1+\sqrt{3} \Leftrightarrow$
$\Leftrightarrow \cos(3x)+\sin(3x)=\frac{1+\sqrt{3}}{2} \Leftrightarrow$
$\Leftrightarrow \cos(3x)+\tan 45\sin(3x)=\frac{1+\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos(3x)\cos 45+\sin(45)\sin 3x=\cos 45\cdot\frac{1+\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos(3x-45)=0{,}96 \Leftrightarrow \cos(3x-45)=\cos 15$
$\Leftrightarrow \begin{cases} 3x-45=15+k\cdot 360 \\ 3x-45=-15+k\cdot 360 \end{cases} \Leftrightarrow \begin{cases} 3x=60+k\cdot 360 \\ 3x=30+k\cdot 360 \end{cases} \Leftrightarrow$
$\Leftrightarrow \begin{cases} x=20+k\cdot 120 \\ x=10+k\cdot 120 \end{cases}$

10. $3\sin x+4\cos x=5 \Leftrightarrow \sin x+\frac{4}{3}\cos x=\frac{5}{3} \Leftrightarrow$
$\Leftrightarrow \sin x+\tan 53\cos x=\frac{5}{3} \Leftrightarrow$
$\Leftrightarrow \cos 53\sin x+\sin 53\cos x=\frac{5}{3}\cos 53 \Leftrightarrow$
$\Leftrightarrow \sin(53+x)=1 \Leftrightarrow$
$\begin{cases} 53+x=k\cdot 360 \Leftrightarrow x=-53+k\cdot 360 \end{cases}$

11. $\cos x-2\sin x=2 \Leftrightarrow \cos x-\tan 63.43\sin x=2 \Leftrightarrow$
$\Leftrightarrow \cos x\cdot\cos 63.43-\sin 63.43\sin x=2\cos 63.43 \Leftrightarrow$
$\Leftrightarrow \cos(x-63.43)=0{,}8944 \Leftrightarrow \cos(x-63{,}43)=\cos 26.6$
$\begin{cases} x-63.43=26{,}6+k\cdot 360 \\ x-63{,}43=-26{,}6+k\cdot 360 \end{cases} \Leftrightarrow$
$\begin{cases} x=90+k\cdot 360 \\ x=36{,}86+k\cdot 360 \end{cases}$

12. $4\sin^2\left(\frac{x}{2}\right)=1 \Leftrightarrow \sin^2\left(\frac{x}{2}\right)=\frac{1}{4} \Leftrightarrow \sin\left(\frac{x}{2}\right)=\pm\frac{1}{2}$
• $\sin\left(\frac{x}{2}\right)=\frac{1}{2} \Leftrightarrow \sin\left(\frac{x}{2}\right)=\sin\frac{\pi}{6} \Leftrightarrow$
$\begin{cases} \frac{x}{2}=\frac{\pi}{6}+2k\pi \\ \frac{x}{2}=\pi-\frac{\pi}{6}+2k\pi \end{cases} \Leftrightarrow \begin{cases} \frac{x}{2}=\frac{\pi}{6}+2k\pi \\ \frac{x}{2}=\frac{5\pi}{6}+2k\pi \end{cases} \Leftrightarrow \begin{cases} x=\frac{\pi}{3}+k4\pi \\ x=\frac{5\pi}{3}+4k\pi \end{cases}$
• $\sin\left(\frac{x}{2}\right)=-\frac{1}{2} \Leftrightarrow \sin\left(\frac{x}{2}\right)=\sin\left(-\frac{\pi}{2}\right) \Leftrightarrow$
$\begin{cases} \frac{x}{2}=-\frac{\pi}{2}+2k\pi \\ \frac{x}{2}=\pi+\frac{\pi}{2}+2k\pi \end{cases} \Leftrightarrow \begin{cases} \frac{x}{2}=-\frac{\pi}{2}+2k\pi \\ \frac{x}{2}=\frac{3\pi}{2}+2k\pi \end{cases} \Leftrightarrow \begin{cases} x=-\pi+4k\pi \\ x=3\pi+4k\pi \end{cases}$

13. $\tan(x)+3\cot(x)=4 \Leftrightarrow$ $\quad \left\{\begin{array}{l} x \neq \frac{\pi}{2}+k\pi \\ x \neq k\pi \end{array}\right.$

$\Leftrightarrow \tan x + \dfrac{3}{\tan x} = 4 \Leftrightarrow$

$\Leftrightarrow \tan^2 x + 3 - 4\tan x = 0 \Leftrightarrow$

$\Leftrightarrow \tan^2 x - 4\tan x + 3 = 0.$

$\Delta = 16 - 12 = 4 \qquad \tan x = \dfrac{4 \pm 2}{2} \left\langle \begin{array}{l} 3 \\ 1 \end{array} \right.$

• $\tan x = 3 \Leftrightarrow \tan x = \tan(71.57)$

$x = 71.57 + k \cdot 180.$

• $\tan x = 1 \Leftrightarrow \tan x = \tan(45) \Leftrightarrow$

$x = 45 + k \cdot 180$

14. $\arccos(2x) = \arcsin(x) \Leftrightarrow$ $\quad$ $\underline{ex\ 1{,}35}$

$\Leftrightarrow \cos(\arccos(2x)) = \cos(\arcsin(x)) \Leftrightarrow$

$2x = \sqrt{1-x^2} \Leftrightarrow 4x^2 = 1 - x^2 \Leftrightarrow 5x^2 = 1 \Leftrightarrow$

$\Leftrightarrow x^2 = \frac{1}{5} \Leftrightarrow x = \pm \frac{\sqrt{5}}{5}.$

• <u>Vérification</u>

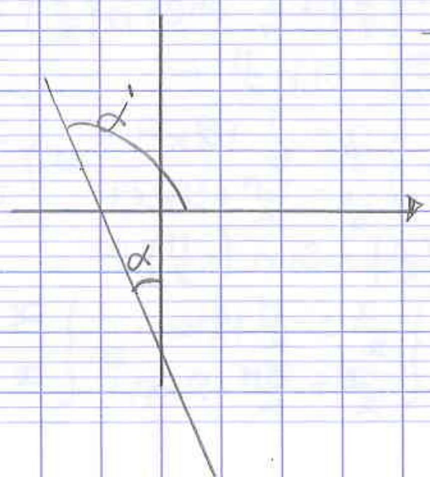
• $x = \frac{\sqrt{5}}{5}.$ $\quad 2 \cdot \frac{\sqrt{5}}{5} = \sqrt{1 - \frac{5}{25}} = \sqrt{1 - \frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5}$ ✓

• $x = -\frac{\sqrt{5}}{5}$ $\quad$ impossible

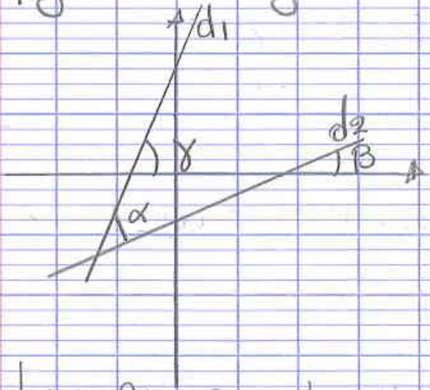
<u>Exercice 1.48</u>

1. $y = 3x + 1 \quad y = 0 \quad \tan\alpha = 3 \Leftrightarrow \alpha = 71{,}565°$
2. $y = -0{,}5x - 7 \quad x = 0 \quad \tan\alpha' = -0{,}5 \Leftrightarrow \alpha' = 153.43°$

$\rightarrow \alpha = 153{,}43° - 90 = 63.43$



3. $d_1\ y = 2x + 9 \quad y = 7x - 5 ; d_2$

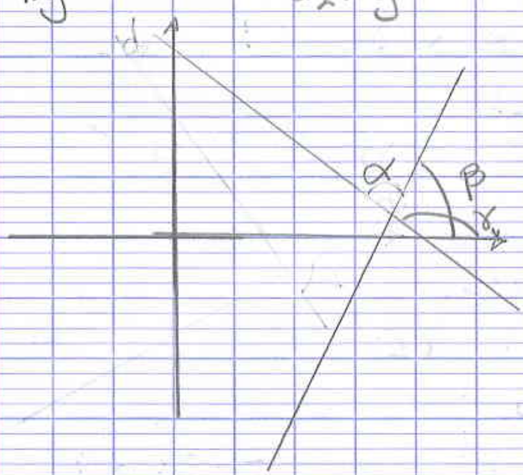


$\beta = \tan^{-1}(7) = 81.87°$

$\gamma = \tan^{-1}(2) = 63.43$

$\alpha = \gamma - \beta = 18.43°$

4. $d_1 : y = 2x - 8 \quad d_2 : y = -0.3x + 9$



$\beta = \tan^{-1}(2) = 63.43$

$\gamma = \tan^{-1}(-0.3) = 163.3$

$\alpha = 163.3 - 63.43 \Rightarrow$

$\alpha = 99.8°$

$\Rightarrow \alpha'$ aigu $= 180 - 99.8 = 80.13°$

5. $y = 2x \quad y = -3x + 9$

$\left.\begin{array}{l}\beta = \tan^{-1}(2) = 63.43 \\ \gamma = \tan^{-1}(-3) = 108.4\end{array}\right\} \alpha = 108.4 - 63.43 \Rightarrow \alpha = 45°$

Exercice 1.49 $\quad a = \tan 40 = 0.84 \Rightarrow y = 0.84x + h$

$(3;-6) : -6 = 0.84 \cdot 3 + h \Rightarrow h = -8.52$

$y = 0.84x - 8.52$

ou $\quad a = \tan(-40) = -0.84 \quad y = -0.84x + h$

$(3;-6) : -6 = -0.84 \cdot 3 + h \Rightarrow h = -3.48$

$y = -0.84x - 3.48$

Exercice 1.50 $\quad f(x) = A \sin(wx)$

La fonction atteint son max pour $\sin(wx) = 1$

$\Rightarrow wx = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{2w} = 7 \Rightarrow \frac{\pi}{14} = w$

et le max $= 7 = A \cdot 1 \Rightarrow \underline{A = 7}$