LDDR-Niveau II:Trigonometrie

1.17 Exercises

1. 1 : Complete the following tables (answer with a multiple of π for the angles in radians).

degrees	$90°$	$135°$	$30°$	$18°$	$22,5°$	$720°$	$270°$	$1°$	$x°$
radians									

radians	$\frac{\pi}{3}$	$\frac{\pi}{20}$	$\frac{3\pi}{8}$	$\frac{2\pi}{3}$	$-\frac{\pi}{2}$	6π	1	$2,5$	x
degrees									

1. 2 :

1) Find the angles between $0°$ and $360°$ which are equivalent, on the trigonometric circle, to $2578°$, $5555°$, $-1111°$ and $-9876°$.

2) Find the angles between 0 and 2π which are equivalent, on the trigonometric circle, to $\frac{97\pi}{6}$, $\frac{111\pi}{8}$, $\frac{-304\pi}{3}$ and $-\frac{123\pi}{4}$.

1. 3 : Use a drawing to fill the table below with approximations

α	$\cos(\alpha)$	$\sin(\alpha)$	$\tan(\alpha)$	$\cot(\alpha)$
$230°$				
$105°$				

1. 4 : In each case, find all the values of α (in both units) that satisfy the equalities.

1) $\cos(\alpha) = 0$ 3) $\cos(\alpha) = -1$ 5) $\tan(\alpha) = -1$ 7) $\tan(\alpha) = 0$

2) $\sin(\alpha) = -1$ 4) $\sin(\alpha) = 0$ 6) $\cot(\alpha) = 0$ 8) $\cot(\alpha) = -1$

1. 5 : Use a drawing and special triangles to find the exact values of $\cos(\alpha)$, $\sin(\alpha)$, $\tan(\alpha)$ and $\cot(\alpha)$ for 1) $\alpha = 30° = \frac{\pi}{6}$ rad, 2) $\alpha = 45° = \frac{\pi}{4}$ rad, and 3) $\alpha = 60° = \frac{\pi}{3}$ rad

1. 6 : **Without your calculator** find the exact value of :

1) $\sin(120°)$ 4) $\cos(210°)$ 7) $\cos(225°)$

2) $\cos(135°)$ 5) $\tan(300°)$

3) $\tan(150°)$ 6) $\sin(330°)$ 8) $\cot(225°)$

1. 7 : Determine the exact value of $\cos(-945°)$ and $\sin\left(\frac{-10\pi}{3}\right)$.

1. 8 : Given a circle with radius 3.2m. We consider, on that circle, an arc corresponding to an angle of $117°$. Calculate the length of that arc.

1. 9 : Toronto and Quito are on the same meridian. Given that the radius of the earth is about 6370km and that the difference between their latitude is $43°$, calculate the distance between these two towns.

1.10 : Sion and Delémont are on the same meridian. The straight line distance between them is 123km. Given that the latitude of Sion is 46°14′N and that the radius of the earth is about 6370km, calculate the one of Delémont.

Indication : Sexagesimal is a numeral system with sixty as its base used to express geographic coordinates. The practical unit of angular measure is the degree, of which there are 360 in a circle. There are 60 minutes of arc in a degree (60 minutes = 1 degree), and 60 arcseconds in a minute (60 seconds = 1 minute).

1.11 : The hands of a clock rotates at constant speed, so they overlap at regular intervals. Determine the time between two meetings.

1.12 : Calculate, in degrees and in radians, the following angles :

1) Interior angle at a vertex of a regular decagon.

2) Angle at the tip of a regular five branch star.

1.13 : Using the trigonometric circle :

1) Express $\sin(\pi+\alpha)$, $\cos\left(\frac{\pi}{2}+\alpha\right)$ and $\tan(\pi-\alpha)$ with $\cos(\alpha)$, $\sin(\alpha)$ and/or $\tan(\alpha)$.

2) Complete the following equalities :

- □ $-\cos(\alpha)=\cos(...)$ □ $-\sin(\alpha)=\sin(...)$ □ $\cos(\alpha)=\sin(...)$
- □ $\sin(\alpha)=\cos(...)$ □ $-\cos(\alpha)=\sin(...)$ □ $-\sin(\alpha)=\cos(...)$

1.14 : Show that :

1) $1+\tan^2(\alpha)=\dfrac{1}{\cos^2(\alpha)}$ 2) $\dfrac{\sin^2(x)}{1-\cos(x)}=1+\cos(x)$ 3) $1+\cot^2(\alpha)=\dfrac{1}{\sin^2(\alpha)}$

1.15 : Simplify :

1) $\dfrac{\tan(x)}{1+\tan^2(x)}$

3) $\dfrac{1-(\sin(x)-\cos(x))^2}{\sin(x)}$

2) $\dfrac{\sin(x)\cos(x)}{1-\sin^2(x)}$

4) $(\sin(x)+\cos(x))^2+(\sin(x)-\cos(x))^2$

1.16 : In each case, find the exact value of $\sin(t)$, $\cos(t)$, $\tan(t)$ and $\cot(t)$ without looking for the value of the angle t.

1) $\cos(t)=\frac{3}{5}$ with $t\in Q_{IV}$

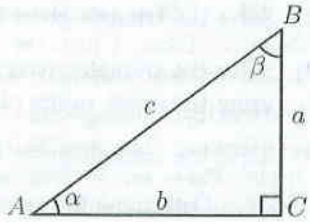
3) $\tan(t)=2$ with $t\in Q_I$

2) $\sin(t)=-0.28$ with $t\in Q_{III}$

4) $\cot(t)=-\frac{9}{40}$ with $t\in Q_{IV}$

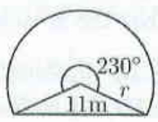
1.17 : The triangle ABC is represented opposite. In each case, find the unknown angles and sides.

1) $a = 7.8\text{cm}$, $\alpha = 27°$
2) $a = 6.3\text{cm}$, $c = 9.2\text{cm}$
3) $c = 4.8\text{cm}$, $\beta = 10°$
4) $a = 13.4\text{cm}$, $b = 20\text{cm}$
5) $a = 5\text{cm}$, Area $=6\text{cm}^2$
6) $b = 2a$



1. 18 : The area of an isosceles triangle is 120m^2, the angle at its vertex is 54°. Calculate its basis, its height and the length of its sides.

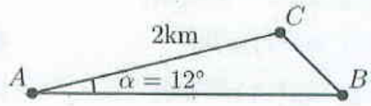
1. 19 : The archway of a road tunnel is an arc of circle whose angle at the center mesure 230°. Calculate the radius r of this arc knowing that the width of the road is 11m. Then, calculate the maximum height of the archway.



1. 20 :

1) Calculate the height of a tower whose shadow on the ground measures 36 meters when the sun is 37.5° over the horizon ?
2) The average speed of a cable car is 7m/s and the run lasts 4 minutes and 20 seconds. The start is at the altitude of 1315 meters and the cables make a 19° angle with the ground. What is the altitude of the arrival of the cable car ?
3) The perimeter of a regular pentagon is 50 centimeters. Calculate the radius of the incircle and the one of the circumcircle of the pentagon.

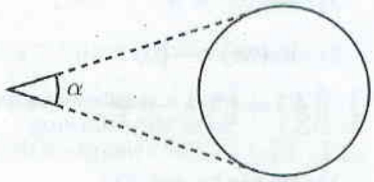
1. 21 : In the fog, a man wants to walk from a point A to a point B. He starts its walk at point A but its trajectory makes an angle of 12° with the right way. After 2km he arrives at point C and modifies its course. He reaches the point B after 0.8 km. What is the distance between A and B ?



1. 22 : In a circle with radius 10cm, we consider a sector corresponding to a chord of 6cm. Calculate the perimeter and the area of that sector.

1. 23 : A man watches a circular tower whose circumference is 50m. He looks at the tower with an angle α of 18°.

1) What is the shortest distance from the man to the tower ?
2) What would the angle be if that distance was 100m ?



1. 24 : A regular five branch star is inscribed in a circle with diameter 10cm. Calculate its perimeter.

1. 25 :

1) Solve the triangle given that $c = 10$, $\alpha = 65°$ and $\gamma = 43°$

2) Solve the triangle given that $a = 7.32$, $b = 4.65$ and $\gamma = 71.6°$

3) Solve the triangle given that $a = 2$, $b = \sqrt{6}$ and $c = 1 + \sqrt{3}$. Start by calculating β by using the exact values of the sides.

4) Solve the triangle given that the area is 12.52m^2, $\alpha = 54.08°$ and $\beta = 88.94°$

1. 26 : Determine the sides and the angles of the triangles in which :

1) Area $= 24$, $\sin(\alpha) = \frac{12}{13}$, $b + c = 17$ and $c > b$.
2) $c = 8$, $\alpha = 20°$ and $b = 2a$

1. 27 : Determine the measures of the sides of a triangle given that they are three consecutive integers and that the greatest angle is twice the smaller.

1. 28 : In a convex quadrilateral $ABCD$, we have the angle $\alpha = 96.8°$ (at A) and the lengths of the sides $AB = 3.5$, $BC = 5.8$, $CD = 6.2$ and $DA = 4.4$. Calculate the other angles of the quadrilateral $ABCD$.

1. 29 : After having sketched the graph of $x \longmapsto \cos(x)$, $x \in [-2\pi; 2\pi]$, sketch, in the same set of axes, the graph of the functions below. Also determine their domain, range, period and parity.

1) $f(x) = \cos(x + 1)$
2) $g(x) = 2\cos(x)$
3) $h(x) = \cos(2x)$
4) $i(x) = |\cos(2x)|$
5) $j(x) = \cos(|x|) + 1$
6) $k(x) = \sqrt{\cos(x)}$
7) $l(x) = \log(\cos(x))$

1. 30 : Find the positive solutions (0< x <360°) of the following trigonometric equations (give one digit)

1) $\sin(x) = 0.1$
2) $\sin(x) = -0.84$
3) $\sin(x + 15°) = 0.951$
4) $\cos(x) = 0.8$
5) $\cos(x) = -0.84$
6) $\cos(x - 20°) = \sqrt{\frac{2}{3}}$
7) $\tan(x) = 4$
8) $\tan(x) = -0.32$
9) $\tan(x + 100°) = 0.11$

1. 31 : Determine all the solutions of the following equations in the interval $0° < x \leq 360°$.

1) $\cos(2x) = \frac{1}{3}$
2) $\tan(3x) = 2$
3) $\sin(2x) = -0.6$
4) $\cos(4x + 30°) = -\frac{1}{4}$
5) $\tan(2x - 90°) = 0,4$
6) $\sin(3x - 45°) = -0.42$
7) $\cot(x) = \frac{\sqrt{3}}{3}$
8) $\cot(x - 20°) = \sqrt{3}$
9) $\cot(2x) = 2.8$

1. 32 : Find a number between 10 and 11 whose sine is equal to -0.7.

1. 33 : Solve the following :

1) $\sin(5x) = \sin(7x)$
2) $\cos(2x) = \cos\left(\frac{2\pi}{3}\right)$
3) $\sin(4x) + \sin(x) = 0$
4) $\tan\left(x + \frac{\pi}{6}\right) = \tan(3x)$

1. 34 : Solve the following :

1) $2\sin(x) + \cos(x) = 0$
2) $4\cos^2(x) - 4\cos(x) - 3 = 0$
3) $2\sin^2(x) - 3\sin(x) + 1 = 0$
4) $\sqrt{3}\sin(2x) - \cos(2x) = 0$
5) $2\cos^2(x) + \sin(x) - 1 = 0$
6) $\sqrt{3}\cos(x) - \sin(x) = 1$

1. 35 : Rewrite the following expressions without using the trigonometric functions but by using the following example :

$$\cos(\arcsin(x)) \overset{y=\arcsin(x)}{=} \cos(y) = \sqrt{1-\sin^2(y)} = \sqrt{1-(\sin(\arcsin(x)))^2} = \sqrt{1-x^2}$$

1) $\tan(\arctan(x))$ 2) $\sin(\arccos(x))$ 3) $\tan(\arccos(x))$ 4) $\sin(2\arcsin(x))$

1. 36 :

1) Using the addition formulae for sine and cosine establish the double angle formulae :

- $\cos(2x) = ...$
- $\sin(2x) = ...$
- $\tan(2x) = ...$

2) Prove the following :

- $\cos(3x) = 4\cos^3(x) - 3\cos(x) = \cos(x)\cdot(1 - 4\sin^2(x))$
- $\sin(3x) = 3\sin(x) - 4\sin^3(x) = \sin(x)\cdot(4\cos^2(x) - 1)$

1. 37 : Find the exact values of $\cos(15°)$, $\sin(15°)$, $\sin(105°)$, $\cos(105°)$ and of $\cos(255°)$.

1. 38 : Simplify :

1) $\cos(x)+\cos(x+120°)+\cos(x+240°) = ...$ 2) $\sin(x)+\sin(x+120°)+\sin(x+240°) = ...$

1. 39 : Given that $\sin(x) = \frac{2}{3}$, $(x \in Q_{II})$ and $\cos(y) = -\frac{1}{4}$ $(y \in Q_{III})$, determine, without calculator, the exact value of $\sin(x+y)$ and $\tan(x-y)$.

1. 40 : If $\sin(\alpha) = \frac{3}{5}$ and $\cos(\beta) = \frac{24}{25}$ and if α, β are acute, calculate the exact value of $\tan(\alpha)$ and $\sin(\beta)$ and thoses of $\cos(\alpha-\beta)$ and $\tan(\alpha+\beta)$.

1. 41 :

1) Determine, without calculator, the exact value of $\tan(2\theta)$ given that $\sin(\theta) = \frac{3}{5}$ and that θ is in the second quadrant.

2) Simplify, as most as possible, the expression : $\tan\left(\frac{\pi}{4}+t\right) - \tan\left(\frac{\pi}{4}-t\right)$.

1. 42 :

1) Prove that $\cos^2\left(\frac{x}{2}\right) = \dfrac{1+\cos(x)}{2}$ by using the calculation opposite.

2) Determine, without calculator, the exact value of $\sin(22,5°)$

$$\begin{aligned} \cos(2x) &= 1 - 2\sin^2(x) \\ \sin^2(x) &= \frac{1-\cos(2x)}{2} \\ \sin^2\left(\frac{x}{2}\right) &= \frac{1-\cos(x)}{2} \end{aligned}$$

1. 43 : Calculate the Cartesian coordinates of the followings. Use exact values where possible.

1) $A\left(6; \frac{7\pi}{4}\right)$
2) $B\left(4; \frac{5\pi}{6}\right)$
3) $C\left(2; \frac{3\pi}{2}\right)$
4) $D\left(\frac{1}{2}; 60°\right)$
5) $E(3; -45°)$
6) $F(5; 253°)$

1. 44 : Calculate the polar coordinates of the followings. Give your answers in degrees.

1) $A(-9;12)$
2) $B(2;-1)$
3) $C(-2\sqrt{3};-6)$
4) $D(-5;5)$
5) $E(-2;0)$
6) $F(\frac{x\sqrt{3}}{2};-\frac{x\sqrt{3}}{2})$, $x>0$

1.45 : Determine the coordinates of the vertices of the square $A'B'C'D'$, obtained by a rotation of 30° around the origin of the square $ABCD$ whose vertices $A(1;1)$ and $C(5;5)$ are given.

1.46 : Solve the following (answers in degrees) :

1) $\cos(x)=\tan(x)$
2) $3\sin^2(x)+\cos^2(x)-2=0$
3) $\sin(2x)+3\cos(2x)=2$
4) $\tan\left(x+\frac{\pi}{6}\right)=\cot(3x)$
5) $\sin(x)+3\cos(x)=3$
6) $\tan^4(x)-4\tan^2(x)+3=0$
7) $2\tan(x)+2\cot(x)=5$
8) $\sin^4(x)+\cos^4(x)=\frac{2}{3}$

1.47 : Solve the following :

1) $\sin(x)=\sin\left(\frac{\pi}{4}-x\right)$
2) $\cos(2x)=\cos(x)$
3) $\sin\left(\frac{5x}{3}\right)+\cos\left(\frac{x}{2}\right)=0$
4) $\sin^2(2x)=\sin^2\left(x+\frac{\pi}{4}\right)$
5) $6\cos^2(x)=5\sin(x)$
6) $3\sin(x)+6\cos^2(x)=5$
7) $\cos^2(x)+2\sin(x)+1=0$
8) $\tan^4(x)-15\tan^2(x)+26=0$
9) $2\cos(3x)+2\sin(3x)=1+\sqrt{3}$
10) $3\sin(x)+4\cos(x)=5$
11) $\cos(x)-2\sin(x)=2$
12) $4\sin^2\left(\frac{x}{2}\right)=1$
13) $\tan(x)+3\cot(x)=4$
14) $\arccos(2x)=\arcsin(x)$

1.48 : Determine the acute angle between

1) the line $y=3x+1$ and the x-axis.
2) the line $y=-0.5x-7$ and the y-axis
3) the lines $y=2x+9$ and $y=7x-1$
4) the line $y=2x-8$ and $y=-0.3x+9$
5) the line $y=2x$ and $y=-3x+9$

1.49 : Give the equation of a line that forms an angle of 40° with the x-axis and that passes through the point $(3;-6)$. Give all the possible answers.

1.50 : The graph of $f(x)=A\sin(\omega x)$ is plotted. The point $M(7;4)$ is a maximum of the function. Determine the values of A and ω and find the coordinates of P, point on the x-axis. Determine the distance between the origin and the point N.

1.51 : The graph of $f(x)=A\cos(\omega x)$ is plotted. The point $M(4;-5)$ is a minimum of the function. Determine the values of A and ω and find the coordinates of N.

1.52 : The graph of $f(x)=A\cos(\omega x)$ is plotted. The point $M(7;-2)$ is a minimum of the function. Determine the values of A and ω. Find the exact values of a and b.

1.53 :

1) Find the coordinates of a minimum of the graph of $f(x) = \cos(x) + 2\sin(x)$
2) Solve the equation $6\cos(2x) - 8\sin(2x) = 5$, using the Physicist's formula.
3) Determine the range of the graph of $f(x) = 2\cos(3x) - \sin(3x + \frac{\pi}{4}) + 2$.