LDDR - Niveau 2 : TE 8 Calcul Integral Solutions

EXERCICE 1

a) $f(x) = \dfrac{2}{\sqrt{4-5x^2}}$ On pose $x = \dfrac{2}{\sqrt5}\sin t \to dx = \dfrac{2}{\sqrt5}\cos t\,dt$

$$\int f(x)\,dx = 2\int \frac{1}{\sqrt{4-5\cdot\frac{4}{5}\sin^2 t}}\,\frac{2}{\sqrt5}\cos t\,dt = \frac{4}{\sqrt5}\int\frac{\cos t}{\sqrt{4-4\sin^2 t}}\,dt =$$

$$= \frac{4}{2\sqrt5}\int\frac{\cos t}{\sqrt{1-\sin^2 t}}\,dt = \frac{2}{\sqrt5}\int\frac{\cos t}{\cos t}\,dt = \frac{2}{\sqrt5}\int dt = \frac{2}{\sqrt5}\,t = \frac{2}{\sqrt5}\,\mathrm{Arcsin}\left(\frac{\sqrt5}{2}x\right)+c$$

b) $f(x) = \dfrac{3x}{x^2+1}$ On pose $x^2+1 = t \to 2x\,dx = dt \Leftrightarrow dx = \dfrac{dt}{2x}$

$$\int f(x)\,dx = 3\int\frac{x}{t}\,\frac{dt}{2x} = \frac{3}{2}\int\frac{1}{t}\,dt = \frac{3}{2}\ln|t| = \frac{3}{2}\ln(x^2+1)+c.$$

c) $f(x) = \arcsin(x)$ $t = \arcsin(x) \Rightarrow x = \sin(t) \Rightarrow dx = \cos(t)\,dt.$

$$\int f(x)\,dx = \int t\cos(t)\,dt = \int t(\sin t)'\,dt = t\sin t - \int \sin t\,dt =$$

$$= t\sin t + \cos t + c$$

EXERCICE 2

a) $\displaystyle\int_{-2}^{2}\frac{dx}{3x^2+4} = \frac{1}{3}\int_{-2}^{2}\frac{dx}{x^2+\frac{4}{3}} =$ $x = \sqrt{\frac{4}{3}}\,t \Rightarrow dx = \frac{2}{\sqrt3}\,dt$

$x = -2$ $t = -\sqrt3$

$x = 2$ $t = \sqrt3$

$$= \frac{2}{3\sqrt3}\int_{-\sqrt3}^{\sqrt3}\frac{dt}{\frac{4}{3}t^2+\frac{4}{3}} =$$

$$= \frac{2}{3\sqrt3}\,\frac{3}{4}\int_{-\sqrt3}^{\sqrt3}\frac{dt}{t^2+1} = \frac{1}{2\sqrt3}\,\mathrm{Arctan}(t)\Big]_{-\sqrt3}^{\sqrt3} = \frac{1}{2\sqrt3}\left(\frac{\pi}{3}-\left(-\frac{\pi}{3}\right)\right) = \frac{1}{2\sqrt3}\,\frac{2\pi}{3} =$$

$$= \frac{\pi}{3\sqrt3} \simeq 0.60460$$

b) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}}\sin^3(x)\,dx =$ $t = \cos x \Rightarrow dt = -\sin x\,dx$

$x = -\frac{\pi}{2} \Rightarrow t = 0$ $x = \frac{\pi}{4} \Rightarrow t = \frac{\sqrt2}{2}$

$$= -\int_0^{\sqrt2/2}\sin^2(x)\sin x\,\frac{dt}{\sin x} = -\int_0^{\sqrt2/2}(1-\cos^2 x)\,dt = \int_0^{\sqrt2/2}(t^2-1)\,dt =$$

$$= \frac{1}{3}t^3 - t\Big]_0^{\sqrt2/2} = \frac{1}{3}\left(\frac{\sqrt2}{2}\right)^3 - \frac{\sqrt2}{2} = \frac{1}{3}\,\frac{\cancel{2}\sqrt2}{\cancel{8}4} - \frac{\sqrt2}{2} = \frac{\sqrt2}{12} - \frac{\sqrt2}{2} = \frac{\sqrt2}{12} - \frac{6\sqrt2}{12} =$$

$$= -\frac{5\sqrt2}{12}$$

EXERCICE 3

$$f(x) = \frac{1}{x[\ln^2(x)+2\ln(x)+2]}$$

$\left.\begin{array}{l}\cdot\ x \neq 0 \quad x > 0 \\ \cdot\ \ln^2 x + 2\ln(x)+2 \neq 0 \quad \Delta < 0\end{array}\right\} \Rightarrow D_f = \,]0, +\infty[$

$f(x) > 0$ $\forall x \in D_f$

b) $\lim\limits_{x\to 0} f(x) = +\infty \qquad \lim\limits_{x\to +\infty} f(x) = 0$

c) $u = \ln(x) \Rightarrow du = \frac{1}{x}dx$

$\int f(x)dx = \int \frac{\cancel{x}\,du}{\cancel{x}(u^2+2u+2)} = \int \frac{du}{(u+1)^2+1} \qquad u+1 = t \Rightarrow du = dt$

$= \int \frac{dt}{t^2+1} = \text{Arctan}\,t = \text{Arctan}(u+1) = \text{Arctan}(\ln x+1) + C$

d) $I = \int_0^\infty f(x)dx = \left[\text{Arctan}(\ln x+1)\right]_0^{+\infty} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

EXERCICE 4

$f(x) = \sqrt{x}\,(x^2-4)$

a) • $x \geq 0 \qquad D_f = [0, +\infty[$

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x	0		2	+∞
f	0	−	0	+

• $\lim\limits_{x\to 0} f(x) = 0 \qquad \lim\limits_{x\to +\infty} f(x) = +\infty \qquad \varnothing$ AV, AH

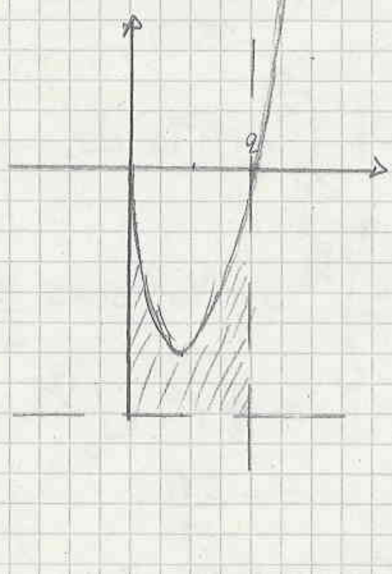
• $f'(x) = \frac{1}{2\sqrt{x}}(x^2-4) + \sqrt{x}\,(2x) = \frac{x^2-4+4x\cdot x}{2\sqrt{x}} = \frac{5x^2-4}{2x} = 0$

$\Leftrightarrow 5x^2 - 4 = 0 \Leftrightarrow x^2 = \frac{4}{5} \Rightarrow x = \pm\frac{2}{\sqrt{5}}$

•

	0		$2/\sqrt{5}$		+∞
f'	‖	−	0	+	
f		↘	Min	↗	

Min $f\left(\frac{2}{\sqrt{5}}\right) \simeq -3.03$



b) $V = \pi\int_0^2 (-2-f(x))^2 dx = \pi\int_0^2 (4+4f(x)+f^2(x))dx$

$= \pi\int_0^2 \left(4 + 4\sqrt{x}(x^2-4) + x(x^2-4)^2\right)dx =$

$= \pi \left[\frac{8x^{7/2}}{7} - \frac{32x^{3/2}}{3} - \frac{x^6}{6} - 2x^4 + 8x^2 + 4x\right]_0^2 =$

$= \frac{8}{21}\left(49 - 32\sqrt{2}\right)\pi \simeq 4.4822$