1.31. EXERCICES

1.1. 1) $2-5\cdot 4+18:6 = 2-20+3 = -15$

2) $2\cdot(5\cdot 9)+(-4)-18:2 = 90-4-9 = 77$

3) $(\sqrt{25}-3)^0-28:4+4(2+3\cdot(-4))^2 = 1-7+4(2-12)^2 =$

$= -6+4\cdot 100 = 400-6 = 392$

1.2 1) $A = \frac{5}{6}+\frac{6}{7}-\frac{7}{8} = \frac{140}{168}+\frac{144}{168}-\frac{147}{168} = \frac{137}{168}$

2) $B = \frac{13}{\cancel{24}_{2}}\cdot\frac{\cancel{36}^{3}}{7} = \frac{13\cdot 3}{2\cdot 7} = \frac{39}{14}$

3) $C = \frac{85}{49} : \frac{34}{21} = \frac{85}{\cancel{49}_{7}}\cdot\frac{\cancel{21}^{3}}{34} = \frac{255}{238} = \frac{15}{14}$

4) $D = \frac{2}{3-\frac{5}{4}} = \frac{2}{\frac{12-5}{4}} = \frac{2\cdot 4}{7} = \frac{8}{7}$

1.3 1) $A = \frac{1}{4}+\left[\frac{1}{3}+\left[\frac{2}{3}-\left[\frac{1}{6}+\frac{1}{2}\right]\right]\right] = \frac{1}{4}+\left[\frac{1}{3}+\left[\frac{2}{3}-\left[\frac{1}{6}+\frac{3}{6}\right]\right]\right]$

$= \frac{1}{4}+\left[\frac{1}{3}+\left[\frac{2}{3}-\frac{2}{3}\right]\right] = \frac{1}{4}+\frac{1}{3} = \frac{3+4}{12} = \frac{7}{12}$

2) $B = \frac{1}{4}-\left[\frac{1}{3}-\left[\frac{2}{3}-\left[\frac{1}{6}-\frac{1}{2}\right]\right]\right] = \frac{1}{4}-\left[\frac{1}{3}-\left[\frac{2}{3}-\left(-\frac{1}{3}\right)\right]\right] =$

$= \frac{1}{4}-\left[\frac{1}{3}-\left[\frac{2}{3}+\frac{1}{3}\right]\right] = \frac{1}{4}-\left[\frac{1}{3}-\frac{3}{3}\right] = \frac{1}{4}-\left(-\frac{2}{3}\right) = \frac{1}{4}+\frac{2}{3} =$

$= \frac{3+8}{12} = \frac{11}{12}$

3) $C = \frac{1}{2}-\left[\frac{2}{3}-\left[\frac{3}{4}+\left[\frac{4}{5}-\frac{5}{6}\right]\right]\right] = \frac{1}{2}-\left[\frac{2}{3}-\left[\frac{3}{4}+\left[\frac{24-25}{30}\right]\right]\right]$

$= \frac{1}{2}-\left[\frac{2}{3}-\left[\frac{3}{4}-\frac{1}{30}\right]\right] = \frac{1}{2}-\left[\frac{2}{3}-\left[\frac{45-2}{60}\right]\right] = \frac{1}{2}-\left[\frac{2}{3}-\frac{43}{60}\right] =$

$= \frac{1}{2}-\left[\frac{40-43}{60}\right] = \frac{1}{2}+\frac{3}{60} = \frac{1}{2}+\frac{1}{20} = \frac{10+1}{20} = \frac{11}{20}$

4) $D = \frac{1}{2}-\left[\frac{2}{3}+\left[\frac{3}{4}-\left[\frac{4}{5}-\frac{5}{6}\right]\right]\right] = \frac{1}{2}-\left[\frac{2}{3}+\left[\frac{3}{4}+\frac{1}{30}\right]\right] =$

$= \frac{1}{2}-\left[\frac{2}{3}+\frac{45+2}{60}\right] = \frac{1}{2}-\left[\frac{2}{3}+\frac{47}{60}\right] = \frac{1}{2}-\frac{40+47}{60} = \frac{1}{2}-\frac{87}{60}$

$= \frac{30-87}{60} = \frac{-57}{60} = -\frac{19}{20}$

1.4 1) $A(2) = ([(2+5)\cdot 2+3]\cdot 2-8)\cdot 2+3 = ([7\cdot 2+3]\cdot 2-8)\cdot 2+3$

$= 26\cdot 2+3 = 55$

$A(-3) = ([(-3+5)(-3)+3]\cdot(-3)-8)(-3)+3 = ([-6+3](-3)-8)(-3)+3$
$= (9-8)(-3)+3 = -3+3 = 0$

2) $B(2) = [(2+5)\cdot 2+3](2-8)\cdot 2+3 = 17(-6)\cdot 2+3 = -201$
$B(-3) = [(-3+5)(-3)+3](-3-8)(-3)+3 = (-3)(-11)(-3)+3 = -96$

3) $C(2) = 2(1-2[1-2(1-2)]) = 2(1-2\cdot 3) = 2(-5) = -10$
$C(-3) = (-3)(1+3(1+3(1+3)]) = (-3)(1+3\cdot 13) = (-3)\cdot 40 = -120$

4) $D(2) = (40-[30-(20+2)\cdot 2]\cdot 2)\cdot 2 = (40-(-14)\cdot 2)\cdot 2 = (40+28)\cdot 2$
$= 136$
$D(-3) = (40-[30-(20-3)(-3)](-3))(-3) = (40-81(-3))\cdot(-3) =$
$= (40+243)(-3) = -849$

1.5 $S = \{x \mid x^2-12x+35=0\}$ $\Delta = 4$ $x_{1,2} = \frac{12\pm 2}{2} = \begin{cases} x_1 = 7 \\ x_2 = 5 \end{cases}$

$S = \{5; 7\} \Rightarrow 0, 0.2, \frac{1}{7} \notin S$

$T = \{x \mid 7x = 1\} = \{\frac{1}{7}\} \Rightarrow 0, 0.2, 5, 7 \notin T$
$U = \{x \mid x^2-2x=0\}$ $x^2-2x = x(x-2) = 0 \Leftrightarrow \begin{matrix} x_1 = 0 \\ x_2 = 2 \end{matrix}$
$U = \{0; 2\} \Rightarrow 0.2, 7, 5, \frac{1}{7} \notin U$

1.6 $A = \{x \in \mathbb{N}^* \mid x < 11\}$ $B = \{1; 2; 3; 4; 5\}$
$C = \{x \in \mathbb{R} \mid x = 2n+1, n \in \mathbb{N}^*, n < 5\}$
$\Rightarrow A = \{1; 2; 3; \dots; 10\}$ $C = \{3; 5; 7; 9\}$

1) $D = A \setminus (B \cup C)$ $B \cup C = \{1; 2; 3; 4; 5; 7; 9\}$
$D = \{6; 8; 10\}$
2) $A \cup C = A$ $A \cup B = B$ $E = A \setminus A = \emptyset$
3) $A \setminus C = \{1; 2; 4; 6; 8; 10\}$ $A \setminus B = \{6; 7; 8; 9; 10\}$
$F = \{6; 8; 10\}$

1.7 1) $A =]-3; 5[$ 2) $B =]-\infty; 1[$ 3) $C =]-\infty, -1] \cup]3; +\infty[$
4) $D =]-\infty, +\infty[$ 5) $E = [-8; 8]$
6) $1 < x^2 \Leftrightarrow x^2 - 1 > 0$

	-1		1	
+	0	−	0	+

$x^2 < 5 \Leftrightarrow x^2 - 5 < 0$

	$-\sqrt{5}$		$\sqrt{5}$	
+	0	−	0	+

$F =]-\sqrt{5}; -1[\cup]1, +\sqrt{5}[$

7) $G = [10, +\infty[$ 8) $H =]-\infty, -4[\cup]4, +\infty[$

1.8 $x = 0.\overline{18} \Rightarrow x = 0.181818\ldots \xRightarrow{\cdot 100} 100x = 18.1818\ldots$

$100x = 18.1818\ldots$

$(-) \quad x = 0.1818\ldots$

$99x = 18 \Rightarrow x = \frac{18}{99} \Rightarrow x = \frac{2}{11}$

1.9 $a = 0.0434343\ldots \Rightarrow 1000a = 43.4343\ldots$

$- \quad 10a = 0.4343\ldots$

$990a = 43 \Rightarrow a = \frac{43}{990}$

$b = 0.120234234234\ldots \Rightarrow$

$\Rightarrow 10^6 b = 120234.234234\ldots$

$(-) \quad 10^3 b = 120.234234\ldots$

$999 \cdot 10^3 b = 120114 \Leftrightarrow b = \frac{120114}{999000} = \frac{6673}{55500}$

1.10 1) $A = 1 - \left(\frac{7}{5} - \frac{3}{4}\right) : \left(\frac{2}{3} + \frac{1}{5}\right) = 1 - \left(\frac{13}{20}\right) : \frac{13}{15} = 1 - \frac{\cancel{13}}{\cancel{20}_4} \cdot \frac{\cancel{15}^3}{\cancel{13}}$

$= 1 - \frac{3}{4} = \frac{1}{4}$

2) $B = 3 - \left(\frac{13}{2} - \frac{5}{3}\right) : \left(\frac{7}{3} + \frac{5}{6}\right) = 3 - \left(\frac{29}{6}\right) : \frac{19}{6} = 3 - \frac{29}{\cancel{6}} \cdot \frac{\cancel{6}}{19} = 3 - \frac{29}{19}$

$= \frac{57 - 29}{19} = \frac{28}{19}$

3) $C = \left(2 + \frac{1}{a}\right) : \left(1 + \frac{2}{a}\right) = \left(\frac{2a+1}{a}\right) : \left(\frac{a+2}{a}\right) = \frac{2a+1}{\cancel{a}} \cdot \frac{\cancel{a}}{a+2} = \frac{2a+1}{a+2}$

4) $D = \dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}} = \dfrac{\frac{b+a}{\cancel{ab}}}{\frac{b-a}{\cancel{ab}}} = \frac{a+b}{b-a}$

5) $E = \dfrac{\frac{1}{ab} + \frac{1}{ac}}{\frac{1}{ab} + \frac{1}{bc}} = \dfrac{\frac{c+b}{\cancel{abc}}}{\frac{c+a}{\cancel{abc}}} = \frac{c+b}{c+a}$

6) $F = \frac{2t}{4 - t^2} + \frac{1}{2-t} = \frac{2t}{(2-t)(2+t)} + \frac{1}{2-t} = \frac{2t}{(2-t)(2+t)} + \frac{2+t}{(2-t)(2+t)}$

$= \frac{2t + 2 + t}{(2-t)(2+t)} = \frac{3t+2}{(2-t)(2+t)}$

7) $G = \frac{1}{b} - \frac{1}{bc} = \frac{c-1}{bc}$

8) $H = \frac{x-y}{2x} - (1-y) = \frac{x-y}{2x} - \frac{(1-y)2x}{2x} = \frac{x - y - 2x + 2xy}{2x}$

$= \frac{-x - y + 2xy}{2x}$

9) $I = \frac{1}{x-1} - \frac{2}{x(x-1)} = \frac{x-2}{x(x-1)}$

10) $J = \frac{1}{2-x} \cdot \frac{x-2}{4} = -\frac{1}{\cancel{x-2}} \cdot \frac{\cancel{x-2}}{4} = -\frac{1}{4}$

11) $K = \frac{m^4-1}{(m+1)^2} = \frac{(m^2-1)(m^2+1)}{(m+1)^2} = \frac{(m-1)\cancel{(m+1)}(m^2+1)}{(m+1)^{\cancel{2}}} =$

$= \frac{(m-1)(m^2+1)}{m+1}$

12) $L = \frac{\frac{3}{x} - \frac{8}{x}}{\frac{x}{2} + \frac{2}{x}} = \frac{-\frac{5}{x}}{\frac{x^2+4}{2x}} = \frac{-5 \cdot 2\cancel{x}}{\cancel{x}(x^2+4)} = \frac{-10}{x^2+4}$

1.11 1) $A = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

2) $B = \sqrt{72} = \sqrt{4 \cdot 18} = \sqrt{4 \cdot 9 \cdot 2} = 2 \cdot 3 \cdot \sqrt{2} = 6\sqrt{2}$

3) $C = \sqrt{216} = \sqrt{6 \cdot 36} = 6\sqrt{6}$

1.12 1) $A = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

2) $B = \frac{\sqrt{27}}{\sqrt{15}} = \sqrt{\frac{\cancel{3} \cdot 9}{\cancel{3} \cdot 5}} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

3) $C = \frac{12}{\sqrt{3}} - \sqrt{27} = \frac{12 \cdot \sqrt{3}}{3} - 3\sqrt{3} = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$

4) $D = \sqrt{63} - \sqrt{28} = \sqrt{7 \cdot 9} - \sqrt{4 \cdot 7} = 3\sqrt{7} - 2\sqrt{7} = \sqrt{7}$

5) $E = \frac{3\sqrt{20} - 5\sqrt{15}}{\sqrt{5}} = \frac{(3 \cdot 2\sqrt{5} - 5\sqrt{15})\sqrt{5}}{5} = \frac{6 \cdot 5 - 5 \cdot 5\sqrt{3}}{5} =$

$= \frac{30 - 25\sqrt{3}}{5} = 6 - 5\sqrt{5}$

6) $F = \frac{\sqrt{12} - \sqrt{3}}{\sqrt{12}} = \frac{(\sqrt{12} - \sqrt{3})\sqrt{12}}{12} = \frac{12 - \sqrt{36}}{12} = \frac{12-6}{12} = \frac{6}{12} = \frac{1}{2}$

7) $G = \frac{\sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{2}(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \frac{\sqrt{2}(\sqrt{5}+\sqrt{2})}{5-2} = \frac{\sqrt{2}(\sqrt{5}+\sqrt{2})}{3} =$

$= \frac{\sqrt{10}+2}{3}$

8) $H = \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2} = \sqrt{18} - \sqrt{12} = 3\sqrt{2} - 2\sqrt{3}$

9) $I = \frac{7\sqrt{5} - 5\sqrt{7}}{\sqrt{7}+\sqrt{5}} = \frac{(7\sqrt{5} - 5\sqrt{7})(\sqrt{7}-\sqrt{5})}{7-5} = \frac{7\sqrt{35} - 5 \cdot 7 - 7 \cdot 5 + 5\sqrt{35}}{2} =$

$= \frac{12\sqrt{35} - 70}{2} = 6\sqrt{35} - 35$

10) $J = \frac{7+2\sqrt{10}}{\sqrt{2}+\sqrt{5}} - \frac{7-2\sqrt{10}}{\sqrt{2}-\sqrt{5}} = \frac{(7+2\sqrt{10})(\sqrt{2}-\sqrt{5}) - (7-2\sqrt{10})(\sqrt{2}+\sqrt{5})}{2-5} =$

$= \frac{\cancel{7\sqrt{2}} + 2\sqrt{20} - 7\sqrt{5} - \cancel{2\sqrt{50}} - \cancel{7\sqrt{2}} + 2\sqrt{20} - 7\sqrt{5} + \cancel{2\sqrt{50}}}{-3} =$

$= \frac{4\sqrt{20} - 14\sqrt{5}}{-3} = \frac{8\sqrt{5} - 14\sqrt{5}}{-3} = \frac{-6\sqrt{5}}{-3} = 2\sqrt{5}$

11) $K = \frac{\sqrt{5}+\sqrt{3}}{1-\sqrt{5}} - \frac{\sqrt{5}-\sqrt{3}}{1+\sqrt{5}} = \frac{(\sqrt{5}+\sqrt{3})(1+\sqrt{5}) - (\sqrt{5}-\sqrt{3})(1-\sqrt{5})}{1-5} =$

$= \frac{\cancel{\sqrt{5}}+\sqrt{3}+5+\cancel{\sqrt{15}}-\cancel{\sqrt{5}}+\sqrt{3}+5-\cancel{\sqrt{15}}}{-4} = \frac{10+2\sqrt{3}}{-4} =$

$= -\frac{5+\sqrt{3}}{2}$

1.13 1) $A = \sqrt{5-2\sqrt{3}} \cdot \sqrt{5+2\sqrt{3}} = \sqrt{25-4\cdot 3} = \sqrt{13}$

2) $B = \sqrt{5+\sqrt{21}} + \sqrt{5-\sqrt{21}} \Leftrightarrow B^2 = 5+\cancel{\sqrt{21}}+5-\cancel{\sqrt{21}}+2\sqrt{(5+\sqrt{21})(5-\sqrt{21})} =$

$= 10+2\sqrt{25-21} \Leftrightarrow B^2 = 10+2\cdot 2 \Leftrightarrow B^2 = 14 \Rightarrow$

$B = \sqrt{14}$ (car $B>0$)

3) $C = \sqrt{6-2\sqrt{5}} + \sqrt{6+2\sqrt{5}} \Leftrightarrow C^2 = 6-\cancel{2\sqrt{5}}+6+\cancel{2\sqrt{5}}+2\sqrt{36-4\cdot 5}$

$\Rightarrow C^2 = 12+8 \Rightarrow C^2 = 20 \Rightarrow C = \sqrt{20}$

1.14 $\sqrt{6}-\sqrt{2} = 2\sqrt{2-\sqrt{3}} \Leftrightarrow (\sqrt{6}-\sqrt{2})^2 = 4\cdot(2-\sqrt{3}) \Leftrightarrow$

$\Leftrightarrow 6+2-2\sqrt{12} = \cancel{8}-4\sqrt{3} \Leftrightarrow -2\cdot 2\sqrt{3} = -4\sqrt{3} \Leftrightarrow$

$4\sqrt{3} = 4\sqrt{3}$ ✓

1.15. 1) $A = (2e^2 f^{-3})^4 = 2^4 e^8 f^{-12} = 16 e^8 f^{-12}$

2) $B = \left(\sqrt[5]{\frac{P^2}{w^{-3}}}\right)^0 = 1$

3) $C = (100 r^3 s^{-1}) : (4 r^{-2} s^5 t) = \frac{100 r^3 s^{-1}}{4 r^{-2} s^5 t} =$

$= 25 r^5 s^{-6} t^{-1} = \frac{25 r^5}{s^6 t}$

4) $D = \frac{\sqrt{x\sqrt{y}}}{\sqrt{y\sqrt{x}}} = \frac{(x y^{1/2})^{1/2}}{(y x^{1/2})^{1/2}} = \frac{x^{1/2} y^{1/4}}{y^{1/2} x^{1/4}} = x^{\frac{1}{2}-\frac{1}{4}} y^{\frac{1}{4}-\frac{1}{2}} = x^{\frac{1}{4}} y^{-1/4}$

5) $E = \sqrt[4]{c^3} \cdot c = c^{3/4} \cdot c = c^{7/4} = \sqrt[4]{c^7}$

6) $F = (125^{2/3} x^{-1} y^2)^{-1/2} = (5^3)^{-\frac{1}{3}} x^{1/2} y^{-1} = 5^{-1} x^{1/2} y^{-1}$

7) $G = (a^{-3} b^7)^{1/2} : (a^{1/4} b^{3/2})^{-2} = \frac{a^{-3/2} b^{7/2}}{a^{-1/2} b^{-3}} = a^{-\frac{2}{2}} b^{13/2} = a^{-1} b^{13/2}$

8) $H = \sqrt[3]{\frac{\sqrt{2}}{64}} = \frac{(2^{\frac{1}{2}})^{1/3}}{(64)^{1/3}} = \frac{2^{\frac{1}{6}}}{(2^6)^{1/3}} = \frac{2^{1/6}}{2^2} = 2^{\frac{1}{6}-2} = 2^{-11/6}$

9) $I = \sqrt{\frac{a^{2/3}}{\sqrt[3]{a^{1/2}}}} \cdot ((a^{2/3})^2)^{1/3} = \frac{(a^{2/3})^{1/2}}{((a^{1/2})^{1/3})^{1/2}} \cdot (a^{2/3})^{2/3} = \frac{a^{1/3}}{a^{1/12}} \cdot a^{4/9} =$

$= a^{1/3 - \frac{1}{12} + \frac{4}{9}} = a^{\frac{25}{36}}$

1.16 1) $A = a^4 \cdot a^{28} = a^{32}$ 2) $B = x^{-25} \cdot x^{14} = x^{-11}$

3) $C = u^{x-4} \cdot u^{6-x} = u^2$ 4) $D = \frac{n^0}{n^5} = n^5$ 5) $E = \frac{h^4}{h^2} = h^2$

6) $F = b^{4n-7} \cdot b^{-n+7} = b^{3n}$ 7) $G = \frac{z^5}{z^{3-6n}} = z^{2+6n}$

8) $H = \frac{e^x}{e^{-x}} = e^{2x}$ 9) $I = (-a)^5 = -a^5$ 10) $J = (-x^6)^4 = x^{24}$

11) $K = (a^5 b^{-8} x)^4 = a^{20} b^{-32} x^4$ 12) $L = (-3x^4)^3 = -27x^{12}$

13) $M = (x^{-3} y^5)^3 = x^{-9} y^{15}$ 14) $N = (a b^0 c^{-3})^{-n} = a^{-n} c^{+3n}$

15) $O = \frac{(-a)^6}{(-a)^3} = \frac{a^6}{-a^3} = -a^3$ 16) $P = \frac{(-a)^{4n+3}}{(-a^n)^4} = -a^3$

1.17 1) $A = \left(\frac{a^5}{b^6}\right)^{-4} \left(\frac{a^4 b^{-4}}{c^7}\right)^3 = \frac{a^{-20}}{b^{-24}} \cdot \frac{a^{12} b^{-12}}{c^{21}} = \frac{a^{-8} b^{12}}{c^{21}} = \frac{b^{12}}{a^8 \cdot c^{21}}$

2) $B = \left(\frac{x^3}{y^4}\right)^6 \left(\frac{x^2 z^{-2}}{y^{-1}}\right)^{-3} = \frac{x^{18}}{y^{24}} \frac{x^{-6} z^6}{y^3} = \frac{x^{12} z^6}{y^{27}}$

3) $C = \left(81^{-1/2} (3x)^3\right)^2 \cdot (y^{-5} x y)^2 = \left(\frac{1}{9} \cdot 27x^3\right)^2 \cdot y^{-10} x^2 y^2 =$

$= \frac{1}{81} 729 x^6 \cdot y^{-8} x^2 = 9 \frac{x^8}{y^8}$

4) $D = \frac{(x^4 y^3)^n}{(x^2 y^4)^{2n-1}} = \frac{x^{4n} y^{3n}}{x^{4n-2} y^{8n-4}} = x^2 y^{-5n+4}$

5) $E = \frac{(a^{n-1} b^n)^2}{(a^2 b^2)^{n-1}} = \frac{\cancel{a^{2n-2}} b^{2n}}{\cancel{a^{2n-2}} b^{2n-2}} = b^2$

6) $F = \frac{a^{-2} b^{-10}}{a^7 b^{-8}} \cdot \frac{a^4 b^{-3}}{(a^{-3} b)^2} = a^{-9} b^{-2} \cdot \frac{a^4 b^{-3}}{a^{-6} b^2} = a\, b^{-7}$

7) $G = \frac{(16 x^4 y^2)^n}{(4x^2 y)^{2n-1}} = \frac{16^n x^{4n} y^{2n}}{4^{2n-1} x^{4n-2} y^{2n-1}} = 4^{-2n+3} \cdot x^2 y$

8) $H = \frac{\sqrt{(2^6 \cdot x^{-12} y^{24})^{1/3}}}{(x^{-2} \cdot y)^{-4}} = \frac{(2^2 x^{-4} y^8)^{1/2}}{x^8 y^{-4}} = \frac{2 x^{-2} y^4}{x^8 y^{-4}} =$

$= 2 x^{-10} y^8$

1.18 1) $A = \frac{3a}{4b} - \frac{4b}{3a} = \frac{9a^2 - 16b^2}{12ab}$

2) $B = \frac{x - \frac{4}{x}}{1 - \frac{2}{x}} = \frac{\frac{x^2 - 4}{\cancel{x}}}{\frac{x-2}{\cancel{x}}} = \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = x + 2$

3) $C = a\sqrt{a\sqrt{a\sqrt{a}}} = a\sqrt{a\sqrt{a^{3/2}}} = a\sqrt{a \cdot a^{3/4}} = a\sqrt{a^{7/4}} = a \cdot a^{7/8}$

$= a^{15/8}$

4) $D = \frac{a+2}{2a-6} - \frac{a-2}{2a+6} = \frac{a+2}{2(a-3)} - \frac{a-2}{2(a+3)} = \frac{(a+2)(a+3)-(a-2)(a-3)}{2(a-3)(a+3)}$

$= \frac{\cancel{a^2}+2a+3a+\cancel{6}-\cancel{a^2}+2a+3a-\cancel{6}}{2(a-3)(a+3)} = \frac{10a}{2(a^2-9)} = \frac{5a}{a^2-9}$

5) $E = \frac{(a^2)^{3n}}{\sqrt[3]{a^{6n-3}}} = \frac{a^{6n}}{a^{2n-1}} = a^{4n+1}$

6) $F = \left(\frac{4a^{-2}}{9x^2}\right)^{-1/2} = \left(\frac{9x^2}{4a^{-2}}\right)^{1/2} = \frac{3x}{2a^{-1}} = \frac{3}{2}xa$

7) $G = \frac{(a^{-2})^3}{\sqrt{b}\cdot a^{2/3}} \cdot \frac{\sqrt{b^{-5}}}{\sqrt[3]{a}} = \frac{a^{-6}}{b^{1/2}\cdot a^{1/3}} \; \frac{b^{-5/2}}{a^{1/3}} = a^{-7}b^{-3}$

1.19 1) $A = (3a+2b)(2a+3b) = 6a^2+4ab+9ab+6b^2 =$
$= 6a^2+13ab+6b^2$

2) $B = (5x+4)(9-11x) = 45x+36-55x^2-44x =$
$= -55x^2+x+36$

3) $C = (a+2b+3c)(3a+2b+c) =$
$= 3a^2+2ab+ac+6ab+4b^2+2bc+9ac+6cb+3c^2$
$= 3a^2+4b^2+3c^2+8ab+10ac+8bc$

4) $D = (x-5y+2)(2x+y-2) =$
$= 2x^2+xy-2x-10xy-5y^2+10y+4x+2y-4 =$
$= 2x^2-5y^2+2x+12y-9xy-4$

1.20 1) $A = (3x+2y)^2 = 9x^2+4y^2+12xy$

2) $B = [3(x+y)]^2 = 9x^2+18xy+9y^2$

3) $C = 3(x+y)^2 = 3x^2+6xy+3y^2$

4) $D = (a+b+c)(a+b-c) = (a+b)^2-c^2 = a^2+b^2-c^2+2ab$

5) $E = (4x-\sqrt{6}y)^2 = 16x^2-8\sqrt{6}xy+6y^2$

6) $F = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right) = x^2-\frac{1}{x^2}$

7) $G = (\sqrt{2}-\sqrt{5})^2 = 2+5-2\sqrt{10} = 7-2\sqrt{10}$

8) $H = (x-3y)^3 = x^3-9x^2y+27xy^2-27y^3$

9) $I = (3x-y)(3x+y) = 9x^2-y^2$

10) $J = (2a+3b)^4 = 16a^4+96a^3b+216a^2b^2+216ab^3+81b^4$

1.21 1) $A = 4x^2+20x+25 = (2x+5)^2$

2) $B = x^2+14x+49 + (x+7)^2$

3) $C = x^2-20x+100 = (x-10)^2$

4) $D = (3x+5)^2 = 9x^2+30x+25$

5) $E = 64+48x^2+9x^4 = (8+3x^2)^2$

6) $F = 9x^2-12x+4 = (3x-2)^2$

7) $G = x^2-16 = (x+4)(x-4)$

8) $H = 4x^2-24x+36 = (2x-6)^2$

1.22 $(x+1)^2-(x-1)^2 = \cancel{x^2}+2x+1-\cancel{x^2}+2x-1 = 4x$

$10001^2 - 9999^2 = (10000+1)^2-(10000-1) = 40000$

1.23 $(x^2-5) = 0 \Leftrightarrow x^2 = 5 \Leftrightarrow x = \pm\sqrt{5}$

$x^2+3x = 0 \Leftrightarrow x(x+3) = 0 \Leftrightarrow x = 0,\ x = -3$

$(x-3)^2-3 = 0 \Leftrightarrow (x-3)^2 = 3 \Leftrightarrow x-3 = \sqrt{3} \Leftrightarrow x = 3+\sqrt{3}$

$x-3 = -\sqrt{3} \Leftrightarrow x = 3-\sqrt{3}$

1.24 1) $x^2+3x-4 = 0 \Leftrightarrow \left(x+\frac{3}{2}\right)^2 - \frac{9}{4} - 4 = 0 \Leftrightarrow$

$\Leftrightarrow \left(x+\frac{3}{2}\right)^2 = \frac{25}{4} \Leftrightarrow \begin{cases} x+\frac{3}{2} = \frac{5}{2} \Leftrightarrow x = 1 \\ x+\frac{3}{2} = -\frac{5}{2} \Leftrightarrow x = -4 \end{cases}$

2) $x^2+3x+4 = 0 \Leftrightarrow \left(x+\frac{3}{2}\right)^2 - \frac{9}{4}+4 = 0 \Leftrightarrow \left(x+\frac{3}{2}\right)^2 = -\frac{7}{4}$ imp

3) $x^2+4x+4 = 0 \Leftrightarrow (x+2)^2 = 0 \Leftrightarrow x = -2$

4) $2x^2-5x+2 = 0 \Leftrightarrow x^2-\frac{5}{2}x+1 = 0 \Leftrightarrow \left(x-\frac{5}{4}\right)^2 - \frac{25}{16}+1 = 0$

$\Leftrightarrow \left(x-\frac{5}{4}\right)^2 = \frac{9}{16} \Leftrightarrow \begin{cases} x-\frac{5}{4} = \frac{3}{4} \Leftrightarrow x_1 = 2 \\ x-\frac{5}{4} = -\frac{3}{4} \Leftrightarrow x_2 = \frac{1}{2} \end{cases}$

5) $ax^2+bx+c = 0$

- $a = 0$ $\quad bx+c = 0$
  - $b = 0$
    - $c = 0$ $\quad x \in \mathbb{R}$
    - $c \neq 0$ $\quad$ impossible
  - $b \neq 0$ $\quad x = -\frac{c}{b}$

- $a \neq 0$ $\quad x^2+\frac{b}{a}x+\frac{c}{a} = 0 \Leftrightarrow \left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}+\frac{c}{a} = 0$

$\Leftrightarrow \left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$\rightarrow b^2-4ac < 0$ $\quad$ Impossible

$\rightarrow b^2-4ac \geq 0$ $\quad x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

1.25 1) $A(x) = x^2+6x+9 = (x+3)^2$ $\qquad x = -3$

2) $B(x) = 4x^2-4x+1 = (2x-1)^2$ $\qquad x = 1/2$

3) $C(x) = x^2-121 = (x-11)(x+11)$ $\qquad x = \pm 11$

4) $D(x) = x^2-10x+25 = (x-5)^2$ $\qquad x = 5$

5) $E(x) = 16x^4-1 = (4x^2-1)(4x^2+1)$

$= (2x-1)(2x+1)(4x^2+1)$ $\qquad x = \pm 1/2$

6) $F(x) = 9x^2+12x+4 = (3x+2)^2$ $\quad x = -\frac{2}{3}$

7) $G(x) = 2x^3-18x = 2x(x^2-9)$
$= 2x(x-3)(x+3)$ $\quad x=0 ;\ x=3 ,\ x=-3$

8) $H(x) = (2x+1)(x-2)+x(2x+1) =$
$= (2x+1)(x-2+x) = (2x+1)(2x-2) =$ $\quad x = -1/2$
$= 2(2x+1)(x-1)$ $\quad x = 1$

9) $I(x) = (x^2+1)(x-5)+2x(x-5) =$
$= (x-5)(x^2+2x+1) = (x-5)(x+1)^2$ $\quad x = 5$ $\quad x = -1$

10) $J(x) = (x^2-5)(x^2-1)-4(x^2-1) =$
$= (x^2-1)(x^2-5-4) = (x-1)(x+1)(x-3)(x+3)$ $\quad x = \pm 1$ $\quad x = \pm 3$

1.26 1) $A(x) = 4x^2+4xy+y^2 = (2x+y)^2$

2) $B(x) = x^2+8x+16 = (x+4)^2$

3) $C(x) = x^2-4y^2 = (x-2y)(x+2y)$

4) $D(x) = 5xy^3-10xy^2+5xy = 5xy(y^2-2y+1) =$
$= 5xy(y-1)^2$

5) $E(x) = (x^2-1)(3x^2-4) = (x-1)(x+1)(\sqrt{3}x-2)(\sqrt{3}x+2)$

1.27 Si $\Delta \geq 0$ $\quad x_1 = \frac{-b+\sqrt{\Delta}}{2a}$ $\quad x_2 = \frac{-b-\sqrt{\Delta}}{2a}$

$\cdot\ x_1+x_2 = \frac{-b+\sqrt{\Delta}}{2a} + \frac{-b-\sqrt{\Delta}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$

$\cdot\ x_1 \cdot x_2 = \frac{(-b+\sqrt{\Delta})(-b-\sqrt{\Delta})}{4a^2} = \frac{(-b)^2-\sqrt{\Delta}^2}{4a^2} = \frac{b^2-\Delta}{4a^2} =$

$= \frac{\cancel{b^2}-\cancel{b^2}+4ac}{4a^2} = \frac{c}{a}$

1.28 1) $25x^2+10x+2 = 1 \Leftrightarrow 25x^2+10x+1 = 0$
$\Leftrightarrow (5x+1)^2 = 0$ $\quad x = -1/5$

2) $2x^2+5x-3=0$ $\quad \Delta = 25+24 = 49$ $\quad x_{1,2} = \frac{-5\pm 7}{4}$ $\quad x_1 = -3$ $\quad x_2 = 1/2$

3) $x^2+kx-k^2=0$ $\quad \Delta = k^2+4k^2 = 5k^2$
$x_{1,2} = \frac{-k \pm k\sqrt{5}}{2}$

4) $x^2-2\sqrt{3}x+1=0$ $\quad \Delta = 4 \cdot 3-4 = 8$
$x_{1,2} = \frac{2\sqrt{3}\pm\sqrt{8}}{2} = \frac{2\sqrt{3}\pm 2\sqrt{2}}{2} = \sqrt{3}\pm\sqrt{2}$

5) $ax^2+(1-a^2)x-a=0$

$\cdot\ a \neq 0$ $\quad \Delta = (1-a^2)^2+4a^2 = 1-2a^2+a^4+4a^2 = a^4+2a^2+1$
$= (a^2+1)^2$

$x_{1,2} = \frac{a^2-1\pm(a^2+1)}{2a}$ $\quad x_1 = \frac{a^2-\cancel{1}+a^2+\cancel{1}}{2a} = a$ $\quad x_2 = \frac{a^2-1-a^2-1}{2a} = -\frac{1}{a}$

$\cdot\, a=0 \qquad x=0$

6) $x^4-16x^3=0 \Leftrightarrow x^3(x-16)=0 \Leftrightarrow x=0 \quad x=16$

1.29 1) $3x^2+dx+d=0$

$\Delta=0 \Leftrightarrow d^2-12d=0 \Leftrightarrow d(d-12)=0 \Leftrightarrow d=0,\ d=12.$

2) $2x^2+x+2=dx \Leftrightarrow 2x^2+(1-d)x+2=0$

$\Delta=0 \Leftrightarrow (1-d)^2-16=0 \Leftrightarrow 1-d=4 \Leftrightarrow d=-3$
$1-d=-4 \Leftrightarrow d=5$

3) $x^2+2x+d=3 \Leftrightarrow x^2+2x+d-3=0$

$\Delta=0 \Leftrightarrow 4-4(d-3)=0 \Leftrightarrow 1-d+3=0 \Leftrightarrow d=4$

4) $\frac{1}{4}(x+1)^2+2=dx \Leftrightarrow (x+1)^2+8=4dx \Leftrightarrow$
$\Leftrightarrow x^2+2x+1+8-4dx \Leftrightarrow x^2+2(1-2d)x+9=0$

$\Delta=0 \Leftrightarrow 4(1-2d)^2-4\cdot 9=0 \Leftrightarrow (1-2d)^2=9 \Leftrightarrow$

$\Leftrightarrow \begin{cases} 1-2d=3 \Leftrightarrow -2=2d \Leftrightarrow d=-1 \\ 1-2d=-3 \Leftrightarrow 4=2d \Leftrightarrow d=2 \end{cases}$

1.30 1) $\dfrac{2x^2-x-3}{2x^2-7x+6}=\dfrac{\cancel{2}\cancel{(x-\frac{3}{2})}(x+1)}{\cancel{2}(x-2)\cancel{(x-\frac{3}{2})}}=\dfrac{x+1}{x-2}$

2) $\dfrac{x^2+x-2}{x^2-x-2}=\dfrac{(x+2)(x-1)}{(x-2)(x+1)}$

3) $\dfrac{(3x^2+10x-8)(5x^2-9x-2)}{(5x^2+16x+3)(3x^2-8x+4)}=\dfrac{\cancel{3}(x+4)\cancel{(x-\frac{2}{3})}\cdot\cancel{5}\cancel{(x-2)}\cancel{(x+\frac{1}{5})}}{\cancel{5}(x+3)\cancel{(x+\frac{1}{5})}\cdot\cancel{3}\cancel{(x-2)}\cancel{(x-\frac{2}{3})}}$

$=\dfrac{x+4}{x+3}$

Exercice 1.31

$P(x) = x^3 - 2x^2 - 5x + 6.$

$\pm 1, \pm 2, \pm 3, \pm 6$

$P(1) = 1 - 2 - 5 + 6 = 0$ ✓

$P(-1) = -1 - 2 + 5 + 6 \neq 0.$

$P(2) = 8 - 8 - 10 + 6 = -4 \neq 0$

$P(-2) = -8 - 8 + 10 + 6 = 0$ ✓

$P(3) = 27 - 18 - 15 + 6 = 0$ ✓

$P(-3) = -27 - 18 + 15 + 6 \neq 0$

$P(6) = 216 - 72 - 30 + 6 = 120 \neq 0$

$P(-6) = -216 - 72 + 30 + 6 \neq 0$

$x = 1 \quad x = -2 \quad x = -3$

$Q(x) = x^3 + x^2 - 3x + 9$

$\pm 1, \pm 3, \pm 9$

$Q(1) = 2 - 3 + 9 \neq 0$

$Q(-1) = -1 + 1 + 3 + 9 \neq 0$

$Q(3) = 27 + 9 - 9 + 9 \neq 0$

$Q(-3) = -27 + 9 + 9 + 9 = 0$ ✓ $\qquad \underline{x = -3}$

$Q(9) = 729 + 81 - 27 + 9 \neq 0$

$Q(-9) = -729 + 81 + 27 + 9 \neq 0$

Exercice 1.32

1) $A = \dfrac{x^4 + 2x^3 - 1}{x^2 - 1}$

$$
\begin{array}{rrrrr|l}
\cancel{x^4} & +2x^3 & & & -1 & x^2-1 \\
-\cancel{x^4} & & +x^2 & & & \hline x^2+2x+1 \\
\hline
 & 2x^3 & +x^2 & & -1 & \\
 & -\cancel{2x^3} & & +2x & & \\
\hline
 & & \cancel{x^2} & +2x & -1 & \\
 & & -x^2 & & +1 & \\
\hline
 & & & 2x & &
\end{array}
$$

$$\underbrace{x^4 + 2x^3 - 1}_{P} = \underbrace{(x^2+2x+1)}_{Q}(x^2-1) + \underbrace{2x}_{R}$$

2)
$$
\begin{array}{l|l}
7x^4 - 3x^2 + 2x - 4 & x^2 - 4 \\
\underline{-7x^4 + 28x^2} & 7x^2 + 25 \\
\quad 25x^2 + 2x - 4 & \\
\quad \underline{-25x^2 \qquad + 100} & \\
\qquad 2x + 96 &
\end{array}
$$

$$7x^4 - 3x^2 + 2x - 4 = (x^2 - 4)\underbrace{(7x^2 + 25)}_{Q} + \underbrace{2x + 96}_{R}$$

3)
$$
\begin{array}{l|l}
x^6 - 3x^5 \qquad\qquad + 2 & x^3 + 3x \\
\underline{-x^6 \qquad -3x^4} & x^3 - 3x^2 - 3x + 9 \\
\quad -3x^5 - 3x^4 \quad + 2 & \\
\quad \underline{+3x^5 - \quad + 9x^3} & \\
\qquad -3x^4 + 9x^3 + 2 & \\
\qquad \underline{+3x^4 \quad + 9x^2} & \\
\qquad\quad 9x^3 + 9x^2 + 2 & \\
\qquad\quad \underline{-9x^3 - 27x} & \\
\qquad\quad 9x^2 - 27x + 2 &
\end{array}
$$

$$x^6 - 3x^5 = (x^3 + 3x)(x^3 - 3x^2 - 3x + 9) + 9x^2 - 27x + 2$$

4)
$$
\begin{array}{l|l}
2x^3 - 11x^2 + 23x - 26 & 2x - 5 \\
\underline{-2x^3 + 5x^2} & x^2 - 3x + 4 \\
\quad -6x^2 + 23x - 26 & \\
\quad \underline{+6x^2 - 15x} & \\
\qquad 8x - 26 & \\
\qquad \underline{-8x + 20} & \\
\qquad\quad -6 &
\end{array}
$$

$$2x^3 - 11x^2 + 23x - 26 = (2x - 5)(x^2 - 3x + 4) - 6$$

EXERCICE 1.33

$P(x) = 4x^2 + 3x - 1$

$\frac{1}{4}, -\frac{1}{4}, \pm\frac{1}{2}, \pm 1$

$P\left(\frac{1}{4}\right) = 4 \cdot \frac{1}{16} + 3 \cdot \frac{1}{4} - 1 = \frac{1}{4} + \frac{3}{4} - 1 = 0$ ✓ $\boxed{x = \frac{1}{4}}$

$P\left(-\frac{1}{4}\right) = 4\frac{1}{16} - 3\frac{1}{4} - 1 \neq 0$

$P\left(\frac{1}{2}\right) = 4\frac{1}{4} + 3\frac{1}{2} - 1 = \frac{3}{2} \neq 0$

$P\left(-\frac{1}{2}\right) = 4\frac{1}{4} - \frac{3}{2} - 1 \neq 0$

$P(1) = 4 + 3 - 1 \neq 0$

$P(-1) = 4 - 3 - 1 = 0$ ✓ $\boxed{x = -1}$

$Q(x) = 6x^2 - x - 2$

$\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm\frac{2}{3}$

$Q\left(\frac{1}{6}\right) \neq 0$

$Q\left(-\frac{1}{2}\right) = 0$ ✓

$Q\left(\frac{2}{3}\right) = 0$ ✓

Ex 1.34

$P(x) = x^3 + 2x^2 - 2x - 12$ $x-3,\ x+1,\ x-2$

$P(3) = 27 + 18 - 6 - 12 \neq 0 \Rightarrow (x-3) \nmid P(x)$

$P(-1) = -1 + 2 + 2 - 12 \neq 0 \Rightarrow (x+1) \nmid P(x)$

$P(2) = 8 + 8 - 4 - 12 = 0$ ✓ $\rightarrow (x-2) \mid P(x)$

EXERCICE 1.35

1)

$$\begin{array}{l|l} x^2-2x+1 & x-1 \\ \cline{2-2} -\cancel{x^2}+x & x-1 \\ \hline \quad -x+1 & \\ \quad \underline{x-1} & \\ \qquad 0 & \end{array}$$

$$x^2-2x+1=\underbrace{(x-1)}_{}\underbrace{(x-1)}_{=Q}+\underbrace{0}_{=R}$$

$$\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ \cline{4-4} \downarrow & 1 & -1 & \\ 1 & -1 & 0 & \end{array}$$

2)

$$\begin{array}{l|l} 2x^2+3x+4 & x+1 \\ \cline{2-2} -\cancel{2x^2}-2x & 2x+1 \\ \hline \quad \cancel{x}+4 & \\ \quad \underline{-\cancel{x}-1} & \\ \qquad 3 & \end{array}$$

$$2x^2+3x+4=(x+1)\underbrace{(2x+1)}_{=Q}+\underbrace{3}_{=R}$$

3)

$$\begin{array}{l|l} 3x^2+2x+5 & x+3 \\ \cline{2-2} -\cancel{3x^2}-9x & 3x-7 \\ \hline \quad -\cancel{7x}+5 & \\ \quad \underline{+\cancel{7x}+21} & \\ \qquad 26 & \end{array}$$

$Q=3x-7$

$R=26$

4)

$$\begin{array}{l|l} \cancel{x^3}+2\cancel{x^2}+3x+4 & x+2 \\ \cline{2-2} -\cancel{x^3}-2x^2 & x^2+3 \\ \hline \quad \cancel{3x}+4 & \\ \quad \underline{-\cancel{3x}-6} & \\ \qquad -2 & \end{array}$$

$Q=x^2+3$

$R=-2$

EXERCICE 1.36

$P(x) = 2(x+1)^3(3x-5)^2(2x+3)(x^2-3x+5)(2x^2+2x+1)$

x^2-3x+5 $\qquad$ $2x^2+2x+1$

$\Delta = 9-20<0$ $\qquad$ $\Delta = 4-8<0$

Roots: $-1 \quad m=3$

$\frac{5}{3} \quad m=2$

$-\frac{3}{2} \quad m=1$

EXERCICE 1.37

1) $P(x) = 4x^3+32x^2-9x-72$

4	32	-9	-72	-8
	-32	0	72	
4	0	-9	0	

$P(x) = (x+8)(4x^2-9) = (x+8)(2x-3)(2x+3)$

$P(x)=0 \Leftrightarrow x=-8, \; x=\frac{3}{2}, \; x=-\frac{3}{2}$

2) $P(x) = -9x^3-9x^2+4x = -x(9x^2+9x-4)$

$\Delta = 225 = 15^2 \qquad x_{1,2} = \frac{-9\pm15}{18} = \frac{-24}{18} = -\frac{4}{3}, \; \frac{6}{18} = \frac{1}{3}$

$P(x) = -x \cdot 9 \cdot (x-\frac{1}{3})(x+\frac{4}{3}) =$

$= -x(3x-1)(3x+4)$

3) $P(x) = (x-5)(x-3)(x+13)$

No. $Q(x) = 2(x-5)(x-3)(x+13)$

But always $Q(x)$ is an multiple of $P(x)$

4) $P(0)=14 \quad P(1)=0 \quad P(-2)=0 \quad deg(P(x))=3$

$P(x)=(x-2)Q(x)-12$

$P(x)=(x-1)(x+2)(ax+b)$

. $P(0)=14 \Rightarrow -1\cdot 2\cdot b=14 \Leftrightarrow b=-7$

. $P(2)=-12 \Leftrightarrow 1\cdot 4\cdot(2a-7)=-12 \Leftrightarrow$

$\Leftrightarrow 2a-7=\frac{-12}{4} \Leftrightarrow 2a=-3+7 \Leftrightarrow$

$\Leftrightarrow 2a=4 \Leftrightarrow a=2$

So $P(x)=(x-1)(x+2)(2x-7)=$

$=(x^2-x+2x-2)(2x-7)=$

$=(x^2+x-2)(2x-7)=$

$=2x^3+2x^2-4x-7x^2-7x+14=$

$=2x^3-5x^2-11x+14$

EXERCICE 1.38

1) $P_1(x)=6x^3-19x^2+x+6$

$\pm 1, \pm 2, \pm 3, \pm 6$

$P(1)=6-19+1+6\neq 0 \quad P(-1)=-6-19-1+6\neq 0$

$P(2)=48-76+2+6\neq 0$

$P(-2)=-48-19-2+6\neq 0$

$P(3)=162-171+3+6=0$ ✓

6	-19	1	6	3
↓	18	-3	-6	
6	-1	-2	0	

$\Delta=1+48=49$

$P(x)=(x-3)(6x^2-x-2)$ $\quad x_{1,2}=\frac{1\pm 7}{12}=$ $\frac{8}{12}=\frac{2}{3}$; $-\frac{6}{12}=-\frac{1}{2}$

$P(x) = (x-3)\,6\left(x-\frac{2}{3}\right)\left(x+\frac{1}{2}\right) = (x-3)(3x-2)(2x+1)$

$x = 3 \quad m = 1$

$x = \frac{2}{3} \quad m = 1$

$x = \frac{1}{2} \quad m = 1$

2) $P_2(x) = x^4 - 5x^3 - 4x^2 + 16x - 8$

possibles roots $\quad \pm 1, \pm 2, \pm 4, \pm 8$

$P(1) = 0$

$$\begin{array}{ccccc|c} 1 & -5 & -4 & 16 & -8 & 1 \\ \downarrow & 1 & -4 & -8 & 8 & \\ 1 & -4 & -8 & 8 & 0 & \end{array}$$

$P_2(x) = (x-1)\underbrace{(x^3 - 4x^2 - 8x + 8)}_{Q(x)}$

r.p. $\pm 1 \pm 2 \pm 4 \pm 8$.

$Q(1) \neq 0. \qquad Q(-1) = -1 - 4 + 8 + 8 \neq 0$

$Q(2) = 8 - 16 - 16 + 8 \neq 0.$

$Q(-2) = -8 - 16 + 16 + 8 = 0$

$$\begin{array}{cccc|c} 1 & -4 & -8 & 8 & -2 \\ \downarrow & -2 & 12 & -8 & \\ 1 & -6 & 4 & 0 & \end{array}$$

$P(x) = (x-1)(x+2)(x^2 - 6x + 4)$

$\Delta = 36 - 16 = 20 \qquad x_{1,2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$

$P(x) = (x-1)(x+2)(x-(3+\sqrt{5}))(x-(3-\sqrt{5}))$

For all the roots $\quad m = 1$

3) $P_3(x) = x^4 + 2x^3 - x^2 + 4x + 12$

r.p. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$P(1) \neq 0 \qquad P(-1) = 1 - 2 - 1 - 4 + 12 \neq 0$

$P(2) = 16 + 16 - 4 + 8 + 12 \neq 0$.

$P(-2) = \cancel{16} - \cancel{16} - 4 - 8 + 12 = 0$.

$$\begin{array}{ccccc|c} 1 & 2 & -1 & 4 & 12 & -2 \\ \downarrow & -2 & 0 & 2 & -12 & \\ 1 & 0 & -1 & 6 & 0 & \end{array}$$

$P(x) = (x+2)\underbrace{(x^3 - x + 6)}_{Q(x)}$

p r $-2, \pm 3, \pm 6$.

$Q(-2) = -8 + 2 + 6 = 0$

$$\begin{array}{cccc|c} 1 & 0 & -1 & 6 & -2 \\ \downarrow & -2 & 4 & -6 & \\ 1 & -2 & 3 & 0 & \end{array}$$

$P(x) = (x+2)(x+2)(x^2 - 2x + 3) \qquad \Delta = 4 - 12 < 0$

$x = -2 \quad \underline{m = 2}$

EXERCICE 1.39

$2x^3 - 5x^2 - 4x + 3 = 0$

p.r $\pm 1, \pm 3, \pm \frac{3}{2}, \pm \frac{1}{2}$

$x = 1 \quad P(1) = 2 - 5 - 4 + 3 \neq 0$

$P(-1) = -2 - 5 + 4 + 3 \neq 0$

$P(3) = 54 - 45 - 12 + 3 = 0$

1 solution $x = 3$

$$\begin{array}{rrrr|l} 2 & -5 & -4 & 3 & 3 \\ \downarrow & 6 & 3 & -3 & \\ \hline 2 & 1 & -1 & 0 & \end{array}$$

$P(x) = (x-3)(2x^2+x-1) \qquad \Delta = 1+8 = 9$

$x_{1,2} = \dfrac{-1 \pm 3}{4} = \left\langle \begin{array}{l} -1 \\ 1/2 \end{array} \right.$

$P(x) = (x-3)\,2\,(x+1)(x-1/2) = (x-3)(2x-1)(x+1)$

EXERCICE 1.40

1) $P(x) = 2x^3+mx^2-3x-2$

It should be: $P(-2)=0 \Leftrightarrow -16+4m+6-2=0 \Leftrightarrow -12+4m=0$

$m=3$

2) $Q(x) = kx^3+k^2x^2+kx-4k^2+6$

$Q(2)=0 \Leftrightarrow 8k+\cancel{4k^2}+2k-\cancel{4k^2}+6=0 \Leftrightarrow$

$\Leftrightarrow 10k+6=0 \Leftrightarrow k=-\dfrac{6}{10} \Leftrightarrow k=-\dfrac{3}{5}$

3) $2x-4=2(x-2) \qquad P(x)=3x^3-4x^2-2px+8$

So. It has to be: $P(2)=6 \Leftrightarrow$

$\Leftrightarrow 24-16-4p+8=6 \Leftrightarrow 16-4p=6 \Leftrightarrow 10=4p \Leftrightarrow$

$p=5/2=2.5$

EXERCICE 1.41

1) $\begin{cases} -4x+3y=4 \\ x-\dfrac{3y}{4}=-1 \end{cases} \Leftrightarrow \begin{cases} -4x+3y=4 & (1) \\ x=\dfrac{3y}{4}-1 & (2) \end{cases}$

$(1) \Rightarrow -4\left(\dfrac{3y}{4}-1\right)+3y=4 \Leftrightarrow -3y+4+3y=4 \Leftrightarrow$

$0=0$. So we have infinite solutions.

In particular all the points of the line

$4x-3y+4=0$.

2) $\begin{cases} y - \frac{13(x-2)}{2} = 2x+1 & (1) \\ 3x - \frac{2(y-5)}{3} = 2y-4 & (2) \end{cases}$

(1) $2y - 13x + 26 = 4x + 2 \Leftrightarrow$

$\Leftrightarrow 17x - 2y - 24 = 0$

(2) $9x - 2y + 10 = 6y - 12 \Leftrightarrow$

$\Leftrightarrow 9x - 8y + 22 = 0$

$\begin{cases} 17x - 2y - 24 = 0 & *(-4) \\ 9x - 8y + 22 = 0 & *(1) \end{cases} \Leftrightarrow \begin{cases} -68x + 8y + 96 = 0 \\ 9x - 8y + 22 = 0 \end{cases}$

$-59x + 118 = 0 \Leftrightarrow$

$x = 2$

We replace in (1)

$17 \cdot 2 - 2y - 24 = 0 \Leftrightarrow 34 - 24 = 2y \Leftrightarrow y = 5$

The solution is $(2;5)$

EXERCICE 1.42

1) $\begin{cases} 3x + 5y = x_1 & (1) \\ x_2 - 10y = 4 & (2) \end{cases}$ We have $(2) = (-2) \cdot (1)$.

So $x_1 = -2$

$x_2 = -6$

2) $\begin{cases} -2x + 3y = 1 \\ x - ay = b & *2 \end{cases} \quad \begin{cases} -2x + 3y = 1 \\ 2x - 2ay = 2b \end{cases}$

$3y - 2ay = 1 + 2b \Leftrightarrow$

$\Leftrightarrow (3 - 2a)y = 1 + 2b$

$a = \frac{3}{2}$ and $b \neq -\frac{1}{2}$

EXERCICE 1.43

$$\begin{cases} x\cdot y = 567 \\ 2x+2y=96 \end{cases} \Leftrightarrow \begin{cases} xy = 567 \\ x+y=48 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} xy = 567 \\ y = 48-x \end{cases} \qquad x\cdot(48-x)=567 \Leftrightarrow$$

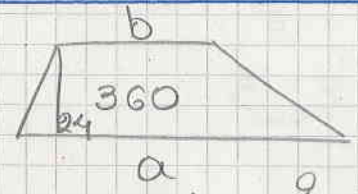
$$-x^2+48x-567=0 \Leftrightarrow$$

$$x^2-48x+567=0$$

$$\Delta = 36 \qquad x_{1,2} = \frac{48\pm 6}{2} = \begin{cases} 21 \\ 27 \end{cases}$$

So $x=21$ $\quad y=27$

EXERCICE 1.44



$$b = \frac{2}{3}a$$

$$360 = \frac{\left(a+\frac{2}{3}a\right)}{2}\cdot 24 \Leftrightarrow 360 = \frac{5}{3}a\cdot 12 \Leftrightarrow$$

$$360 = 5\cdot a\cdot 4 \Leftrightarrow a = 18$$

EXERCICE 1.45

1) $$\begin{cases} 2x-y+z=8 & ① \\ -x+3y+2z=1 & ② \\ 3x+y+3z=14 & ③ \end{cases} \xleftrightarrow{①\leftrightarrow 2} \begin{cases} -x+3y+2z=1 & *(-1) \\ 2x-y+z=8 \\ 3x+y+3z=14 \end{cases}$$

$$\begin{cases} x-3y-2z=-1 \\ 2x-y+z=8 \\ 3x+y+3z=14 \end{cases} \quad \begin{matrix} r_2 \to r_2-2r_1 \\ r_3\to r_3-3r_1 \end{matrix} \quad \begin{cases} x-3y-2z=-1 \\ 5y+5z=10 & (:5) \\ 10y+9z=17 \end{cases}$$

$$\begin{cases} x-3y-2z=-1 \\ y+z=2 \\ 10y+9z=17 \end{cases} \quad r_3\to r_3-10r_2 \quad \begin{cases} x-3y-2z=-1 \\ y+z=2 \\ -z=-3 \end{cases} \Leftrightarrow$$

$$\begin{cases} x-3y-2z=-1 \\ y+z=2 \\ z=3 \end{cases} \Leftrightarrow \begin{cases} x-3y-2z=-1 \\ y=-1 \\ z=3 \end{cases} \Leftrightarrow \begin{cases} x+3-6=-1 \\ y=-1 \\ z=3 \end{cases} \Leftrightarrow \begin{cases} x=2 \\ y=-1 \\ z=3 \end{cases}$$

2) $\begin{cases} -x+7y+2z=2 & (1) \\ 9x-5y-z=4 & (2) \\ 8x+3y=2.5 & (3) \end{cases}$ $(2)*2$

$$\begin{array}{r} -x+7y+2z=2 \\ +\ 18x-10y-2z=8 \\ \hline 17x-3y=10 \quad (4) \end{array}$$

(3), (4)

$$\begin{array}{r} 8x+3y=2.5 \\ +\ 17x-3y=10 \\ \hline 25x=12.5 \end{array}$$

$25x=12.5 \Leftrightarrow x=\frac{25}{2}\cdot\frac{1}{25} \Leftrightarrow x=\frac{1}{2}$

(4) gives $\frac{17}{2}-3y=10 \Leftrightarrow \frac{17}{2}-10=3y \Leftrightarrow$

$\Leftrightarrow 3y=-\frac{3}{2} \Leftrightarrow y=-\frac{1}{2}$

(1) gives $-\frac{1}{2}-\frac{7}{2}+2z=2 \Leftrightarrow -\frac{8}{2}+2z=2 \Leftrightarrow$

$\Leftrightarrow -4-2=-2z \Leftrightarrow 2z=6 \Leftrightarrow \underline{z=3}$

Solution $\left(\frac{1}{2}; -\frac{1}{2}; 3\right)$

3) $\begin{cases} -x+3y=1 & (1) \\ 5x-2y-z=0 & (2) \\ 3x+4y-z=2 & (3) \end{cases}$

$(2)-(3)$ $\quad 2x-6y=-2 \Leftrightarrow x-3y=-1 \quad (4)$

$$\begin{array}{r} -x+3y=1 \\ +\ x-3y=-1 \\ \hline 0=0 \end{array}$$

So the system is $\begin{cases} -x+3y=1 \\ 3x+4y-z=2 \end{cases}$

$x=3y-1$

$3(3y-1)+4y-z=2 \Leftrightarrow 9y-3+4y-z=2 \Leftrightarrow$

$\Leftrightarrow 13y-z=5 \Leftrightarrow z=13y-5$

Solution $(3t-1;\ t;\ 13t-5)$ $\quad t\in\mathbb{R}$

EXERCICE 1.46

1) $4x^4 - x^2 - 60 = 0$ (1)

$t = x^2 \geq 0$

(1) $\Leftrightarrow 4t^2 - t - 60 = 0$ $\quad \Delta = 1 + 4 \cdot 4 \cdot 60 = 961 = 31^2$

$t_{1,2} = \frac{1 \pm 31}{8} = \begin{cases} 4 \\ -\frac{30}{8} = -\frac{15}{4} \text{ imp} \end{cases}$

$x^2 = 4 \Leftrightarrow x = \pm 2$

2) $2x^4 - 5x^3 + 3x^2 = 0 \Leftrightarrow x^2(2x^2 - 5x + 3) = 0.$

$\Delta = 25 - 24 = 1$ $\quad x_{1,2} = \frac{5 \pm 1}{4} = \begin{cases} 6/4 = 3/2 \\ 1 \end{cases}$

$x_1 = 0 \quad x_2 = \frac{3}{2} \quad x_3 = 1$

3) $x^6 + 4x^3 + 3 = 0.$

$t = x^3$

$t^2 + 4t + 3 = 0.$ $\quad \Delta = 16 - 12 = 4$

$t_{1,2} = -\frac{4 \pm 2}{2} \begin{cases} -3 \\ -1. \end{cases}$

$x^3 = -3 \Leftrightarrow x_1 = -\sqrt[3]{3}$

$x^3 = -1 \Leftrightarrow x_2 = -1$

4) $x^5 + 2x^3 - 24x = 0 \Leftrightarrow x(x^4 + 2x^2 - 24) = 0$

$t = x^2 \geq 0$ $\quad t^2 + 2t - 24 = 0$ $\quad \Delta = 4 + 96 = 100.$

$t_{1,2} = -\frac{2 \pm 10}{2} = \begin{cases} -6 \text{ imp} \\ 4 \end{cases}$

$x^2 = 4 \Leftrightarrow x = \pm 2$

$x_1 = 0 \quad x_2 = 2 \quad x_3 = -2$

5) $x^3 - \frac{6^3}{x^3} = 19 \Leftrightarrow x^6 - 6^3 = 19x^3 \Leftrightarrow$

$\Leftrightarrow x^6 - 19x^3 - 6^3 = 0$

$x^3 = t$

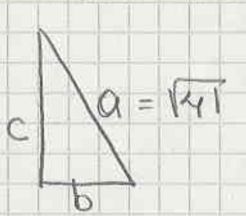
$t^2 - 19t - 6^3 = 0 \qquad \Delta = 1225 = 35^2$

$t_{1,2} = \frac{19 \pm 35}{2} = \begin{cases} -8 \\ 27 \end{cases}$

$x^3 = -8 \Leftrightarrow \underline{x_1 = -2}$

$x^3 = 27 \Leftrightarrow \underline{x_2 = 3}$

EXERICE 1.47



$A = 10$

$A = \frac{1}{2} b \cdot c \Leftrightarrow 20 = b \cdot c \Leftrightarrow b = \frac{20}{c}$

$a^2 = b^2 + c^2 \Leftrightarrow 41 = b^2 + c^2$

$41 = \frac{400}{c^2} + c^2 \Leftrightarrow 41c^2 = 400 + c^4 \Leftrightarrow$

$\Leftrightarrow c^4 - 41c^2 + 400 = 0 \qquad \Delta = 41^2 - 4 \cdot 400 = 81$

$c^2 = t \geq 0$

$t^2 - 41t + 400 = 0$

$t_{1,2} = \frac{41 \pm 9}{2} = \begin{cases} 25 \\ 16 \end{cases}$

$c^2 = 25 \Leftrightarrow \underline{x_1 = 5} \quad \underline{c_2 = -5}$

$c^2 = 16 \Leftrightarrow \underline{c_3 = 4} \quad \underline{c_4 = -4}$

EXERCICE 1.48

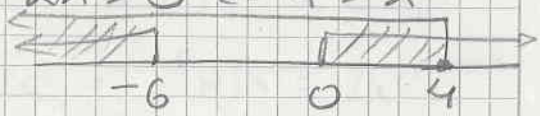
1) $2x + \sqrt{x(x+6)} = 8$

$x(x+6) \geq 0$

$x \leq -6$ ou $x \geq 0$

$\sqrt{x(x+6)} = 8 - 2x$

$8 - 2x \geq 0 \Leftrightarrow 4 \geq x$



$x(x+6) = (8-2x)^2 \Leftrightarrow$

$x^2 + 6x = 64 - 32x + 4x^2 \Leftrightarrow$

$\Leftrightarrow 3x^2 - 38x + 64 = 0$

$\Delta = 676 = 26^2 \quad x_{1,2} = \frac{38 \pm 26}{6} = \frac{64}{6} = \frac{32}{3}$ imp

$\frac{38-26}{6} = \frac{12}{6} = 2 = \cancel{x}$ ✓

2) $\sqrt{2+x} + 4 = \sqrt{10-3x}$

$2 + x \geq 0 \Leftrightarrow x \geq -2$

$\Rightarrow 2 + x + 16 + 8\sqrt{2+x} = 10 - 3x$

$10 - 3x \geq 0 \Leftrightarrow x \leq \frac{10}{3}$

$\Leftrightarrow 8\sqrt{2+x} = -4x - 8$

$-4x - 8 \geq 0 \Leftrightarrow -8 \geq 4x \Leftrightarrow$

$-2 \geq x$

$x = -2$

Pour $x = -2 \quad 8 \cdot 0 = 0$ ✓

Solution $x = -2$

EXERCICE 1.49

1) $\frac{5x}{x^2+9} = -1 \Leftrightarrow 5x = -x^2 - 9 \Leftrightarrow x^2 + 5x + 9 = 0$

$\Delta = 25 - 36 = -11 < 0$ No solutions!

2) $\frac{2x-4}{x} - \frac{1-x}{2x-1} = 9 \overset{\cdot x(2x-1)}{\Longleftrightarrow}$ $\quad x(2x-1) \neq 0 \Leftrightarrow x \neq \frac{1}{2} \cdot x \neq 0$

$(2x-4)(2x-1) - x(1-x) = 9x(2x-1) \Leftrightarrow$

$\Leftrightarrow 4x^2 - 8x - 2x + 4 - x + x^2 = 18x^2 - 9x \Leftrightarrow$

$\Leftrightarrow 5x^2 - 11x + 4 = 18x^2 - 9x \Leftrightarrow$

$\Leftrightarrow 13x^2 + 2x - 4 = 0 \qquad \Delta = 212$

$x_{1,2} = \frac{-2 \pm \sqrt{212}}{26} = \frac{-2 \pm 2\sqrt{53}}{26} = \begin{cases} \frac{-1+\sqrt{53}}{13} = x_1 & ✓ \\ \frac{-1-\sqrt{53}}{13} = x_2 & ✓ \end{cases}$

3) $\frac{5x}{x-3}+\frac{4}{x+3}=\frac{90}{x^2-9} \overset{\cdot(x-3)(x+3)}{\Longleftrightarrow}$ $(x-3)(x+3)\neq 0 \Leftrightarrow x\neq \pm 3$

$5x(x+3)+4(x-3)=90 \Leftrightarrow 5x^2+15x+4x-12=90 \Leftrightarrow$

$\Leftrightarrow 5x^2+19x-102=0$

$\Delta = 49^2 \qquad x_{1,2}=\frac{-19\pm 49}{10}$ $x_1=-\frac{68}{10}=-\frac{34}{5}$; $x_2=\frac{30}{10}=3$ ✗

EXERCICE 1.50

1) $|x-3|=4 \Leftrightarrow x-3=\pm 4$ $x=7$; $x=-1$

2) $|2x-1|=|x+3| \Leftrightarrow \begin{cases} 2x-1=x+3 \Leftrightarrow x=4 \\ \text{ou} \\ 2x-1=-x-3 \Leftrightarrow 3x=-2 \Leftrightarrow x=-\frac{2}{3} \end{cases}$

3) $|x^2-4|=21 \Leftrightarrow x^2-4=21 \Leftrightarrow x^2=25 \Leftrightarrow x=\pm 5$

$x^2-4=-21 \Leftrightarrow x^2=-17$ impossible

4) $x^2-|2x|=3$

- $2x\geq 0$: $x^2-2x=3 \Leftrightarrow x^2-2x-3=0 \quad \Delta=4+12=16$

$x_{1,2}=\frac{2\pm 4}{2}$ $\boxed{x_1=3}$ ✓ ; $x_2=-1$ ✗

- $2x<0$ $\quad x^2+2x-3=0 \quad \Delta=4+12=16$

$x_{1,2}=\frac{-2\pm 4}{2}=$ $\boxed{x_1=-3}$; $x_2=1$

EXERCICE 1.51

1) $\frac{8}{6-2x}=\frac{9}{4-3x} \Leftrightarrow$ $\qquad x\neq 3$; $4-3x\neq 0 \Leftrightarrow x\neq \frac{4}{3}$

$\Leftrightarrow \frac{8}{2(3-x)}=\frac{9}{4-3x} \Leftrightarrow \frac{4}{3-x}=\frac{9}{4-3x} \Leftrightarrow$

$\Leftrightarrow 4(4-3x)=9(3-x) \Leftrightarrow 16-12x=27-9x \Leftrightarrow$

$\Leftrightarrow 3x=11 \Leftrightarrow x=\frac{11}{3}$ ✓

2) $\frac{3x-12}{x-4} = 2 \Leftrightarrow$ $\quad x \neq 4$

$\Leftrightarrow \frac{3(x-4)}{x-4} = 2 \Leftrightarrow 3 = 2$ <u>Imp</u>

3) $(8x-2)(3x+4) = (4x+3)(6x-1) \Leftrightarrow$

$\Leftrightarrow \cancel{24x^2} - 6x + 32x - 8 = \cancel{24x^2} + 18x - 4x - 3 \Leftrightarrow$

$\Leftrightarrow 26x - 8 = 14x - 3 \Leftrightarrow 12x = 5 \Leftrightarrow \underline{x = 5/12}$

4) $ax + 1 = 2x - a \Leftrightarrow (a-2)x = -(1+a)$

- $a = 2 \quad 0 = -3$ <u>Imp</u>
- $a \neq 2 \quad x = -\frac{1+a}{a-2}$

5) $(x+1)^2 - 16 = 0 \Leftrightarrow (x+1)^2 = 16 \Leftrightarrow$ $x + 1 = 4 \Leftrightarrow x = 3$
$x + 1 = -4 \Leftrightarrow x = -5$

6) $(a-x)(b+x) = 0 \Leftrightarrow x = a \quad x = -b$

7) $\frac{x+3}{x-4} = 1 \Leftrightarrow x + 3 = x - 4 \Leftrightarrow$ $\quad x \neq 4$

$\Leftrightarrow 3 = -4$ <u>Imp</u>

8) $\frac{1-x}{4} - \frac{2x+3}{3} = \frac{x}{2} \overset{\cdot 12}{\Leftrightarrow} 3(1-x) - 4(2x+3) = 6x \Leftrightarrow$

$\Leftrightarrow 3 - 3x - 8x - 12 = 6x \Leftrightarrow -11x - 9 = 6x \Leftrightarrow$

$\Leftrightarrow 17x = -9 \Leftrightarrow x = -9/17$

9) $\frac{-2x^2}{2x-1} = -x - 1 \Leftrightarrow$ $\quad 2x - 1 \neq 0 \Leftrightarrow \underline{x \neq 1/2}$

$\Leftrightarrow -2x^2 = (-x-1)(2x-1) \Leftrightarrow -2x^2 = -2x - 2x + x + 1 \Leftrightarrow$

$\Leftrightarrow -2x^2 = -3x + 1 \Leftrightarrow 2x^2 - 3x + 1 = 0$

$\Delta = 9 - 8 = 1 \quad x_{1,2} = \frac{3 \pm 1}{4} = \begin{cases} x_1 = 1 \\ x_2 = \cancel{1/2} \end{cases}$

10) $5a + 2x = x - \frac{x}{a} \quad a \neq 0 \quad \Leftrightarrow 5a^2 + 2ax = ax - x \Leftrightarrow$

$\Leftrightarrow 5a^2 = ax - x - 2ax \Leftrightarrow 5a^2 = -ax - x \Leftrightarrow$

$5a^2 = -(a+1)x$

- $a = -1 \qquad 5 = 0$ Imp
- $a \neq -1 \qquad x = -\frac{5a^2}{(a+1)}$

EXERCICE 1.52

1) $\frac{8}{x+2} = 6 \Leftrightarrow 8 = 6(x+2) \Leftrightarrow \qquad x \neq -2$

$\Leftrightarrow 4 = 3x + 6 \Leftrightarrow 3x = -2 \Leftrightarrow x = -\frac{2}{3}$

2) $\sqrt{x^2+7} = 4 \Leftrightarrow x^2 + 7 = 16 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm 3$ ✓

3) $x + \sqrt{x+5} = 7 \qquad x + 5 \geq 0 \Leftrightarrow x \geq -5$

$\Leftrightarrow \sqrt{x+5} = 7 - x \qquad 7 - x \geq 0 \Leftrightarrow x \leq 7$

$x + 5 = (7-x)^2 \Leftrightarrow x + 5 = 49 - 14x + x^2$

$\Leftrightarrow x^2 - 15x + 44 = 0$

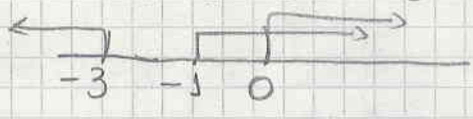
$\Delta = 49 \qquad x_{1,2} = \frac{15 \pm 7}{2} = \begin{cases} x_1 = 4 \ ✓ \\ x_2 = 11 \ \times \ \notin [-5; 7] \end{cases}$

4) $\sqrt{2x} + 3 - \sqrt{x+1} = 1 \Leftrightarrow \qquad x \geq 0$

$\Leftrightarrow \sqrt{2x} + 2 = \sqrt{x+1} \qquad x + 1 \geq 0 \Leftrightarrow x \geq -1$

$-x - 3 \geq 0 \Leftrightarrow -3 \geq x$

$\Leftrightarrow 2x + 4\sqrt{2x} + 4 = x + 1 \Leftrightarrow$



$\Leftrightarrow 4\sqrt{2x} = -x - 3$

Imp

5) $\frac{2x-1}{x+1} = \frac{2x+1}{x-1} \Leftrightarrow (2x-1)(x-1) = (2x+1)(x+1) \Leftrightarrow \qquad x \neq \pm 1$

$\Leftrightarrow \cancel{2x^2} - x - 2x + \cancel{1} = \cancel{2x^2} + x + 2x + \cancel{1} \Leftrightarrow$

$\Leftrightarrow 6x = 0 \Leftrightarrow x = 0$ ✓

6) $|-x+5| = |2x+3| \Leftrightarrow -x+5 = 2x+3 \Leftrightarrow 3x = 2 \Leftrightarrow \underline{x = 2/3}$

$-x+5 = -2x-3 \Leftrightarrow \underline{x = -8}$

7) $\dfrac{x\cancel{-5}}{3(\cancel{x-5})} = \dfrac{2}{x} \Leftrightarrow$ $\quad x \neq 0,\ x \neq 5$

$\Leftrightarrow x = 6$ ✓

8) 1, 4, 9 (2) $\quad x \neq 0$, $x \neq 1/2$

9) $x + \sqrt{6-x} = 0 \Leftrightarrow$ $\quad 6-x \geq 0 \Leftrightarrow x \leq 6$; $-x \geq 0 \Leftrightarrow x \leq 0$

$\Leftrightarrow \sqrt{6-x} = -x$

$\Leftrightarrow 6-x = x^2 \Leftrightarrow x^2 + x - 6 = 0$

$\Delta = 1 + 24 = 25 \qquad x_{1,2} = \dfrac{-1 \pm 5}{2}$ $\begin{cases} x_1 = -3 \ ✓ \\ x_2 = 2 \notin]-\infty, 0] \end{cases}$

10) $|x^2+3x-4| = |-x+5|$

$\Leftrightarrow$ • $x^2+3x-4 = -x+5 \Leftrightarrow x^2+4x-9 = 0$

$\Delta = 16 + 36 = 52 \qquad x_{1,2} = \dfrac{-4 \pm \sqrt{52}}{2} = \dfrac{-4 \pm 2\sqrt{13}}{2}$

$\underline{x_1 = -2+\sqrt{13}} \quad \underline{x_2 = -2-\sqrt{13}}$

• $x^2+3x-4 = x-5 \Leftrightarrow x^2+2x+1 = 0 \Leftrightarrow (x+1)^2 = 0 \Leftrightarrow$

$\underline{x_3 = -1}$

11) $-3 + 2|4x-6| = 11 \Leftrightarrow 2|4x-6| = 14 \Leftrightarrow$

$\Leftrightarrow |4x-6| = 7 \Leftrightarrow \begin{cases} 4x-6 = 7 \Leftrightarrow 4x = 13 \Leftrightarrow \underline{x = 13/4} \\ 4x-6 = -7 \Leftrightarrow 4x = -1 \Leftrightarrow \underline{x = -1/4} \end{cases}$

EXERCICE 1.53

1) $3x < -x+4 \Leftrightarrow 4x < 4 \Leftrightarrow x < 1$

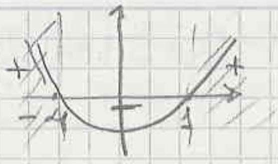
2) $x-1 \leq 3x-1 \Leftrightarrow 0 \leq 4x \Leftrightarrow x \geq 0$

3) $-2x+2 < x-5 \Leftrightarrow 2+5 < 3x \Leftrightarrow x > \frac{7}{3}$

4) $x-5 \geq 4x+9 \Leftrightarrow -5-9 \geq 3x \Leftrightarrow -\frac{14}{3} \geq x$

5) $-3x+4<x^2 \iff x^2+3x-4>0$

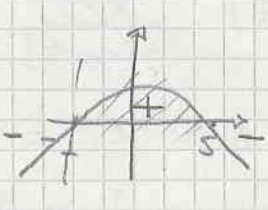
$\Delta = 9+16=25 \qquad x_{1,2} = \frac{-3\pm5}{2} < \begin{matrix}-4\\ 1\end{matrix}$



$x<-4$ ou $x>1$

6) $-x^2+4x\geq-5 \iff -x^2+4x+5\geq0$

$\Delta = 16+20=36 \qquad x_{1,2} = \frac{-4\pm6}{-2} < \begin{matrix}5\\ -1\end{matrix}$



$-1\leq x\leq 5$

EXERCICE 1.54

We should prove that $\Delta>0 \quad \forall p\in\mathbb{R}$

$\Delta = p^2+16>0 \quad \forall p\in\mathbb{R}$

EXERCICE 1.55

1) We have to solve : $\Delta=0 \iff (p-2)^2+16=0$ (1)

$\iff p^2-4x+4+16=0 \iff p^2-4x+20=0$

$\Delta_1=16-80<0$

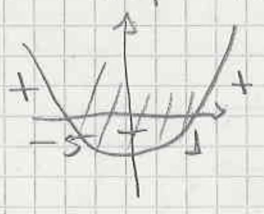
So the equation (1) has no solutions and so the is no value op p such that the given eq have 1 unique solution

2) We have to solve: $\Delta<0$

$\Delta<0 \iff (p+2)^2-4\cdot\frac{9}{4}<0 \iff p^2+4p+4-9<0 \iff$

$\iff p^2+4p-5<0$

$\Delta_1=16+20=36 \qquad p_{1,2} = \frac{-4\pm6}{2} = < \begin{matrix}-5\\ 1\end{matrix}$



$\boxed{-5<p<1}$

3) $x^2+b=b(x-2) \qquad x=5$

$25+b=b(5-2) \Leftrightarrow 25+b=b\cdot 3 \Leftrightarrow 2b=25 \Leftrightarrow \underline{b=12.5}$

EXERCICE 1.56

For every case we have to solve $\Delta<0$

1) $3x^2+dx+d=0$

$\Delta<0 \Leftrightarrow d^2-12d<0 \Leftrightarrow d(d-12)<0$

$0<d<2$ ou $\underline{d\in]0;2[}$

2) $2x^2+x+2=dx \Leftrightarrow 2x^2+(1-d)x+2=0$

$\Delta<0 \Leftrightarrow (1-d)^2-16<0 \Leftrightarrow (1-d-4)(1-d+4)<0$

$\Leftrightarrow (-d-3)(5-d)<0 \Leftrightarrow (d+3)(5-d)>0$

$d_1=-3 \quad d_2=5$

$\underline{-3<d<5}$ ou $\underline{d\in]-3;5[}$

3) $\frac{1}{4}(x+1)^2+2=dx \Leftrightarrow (x+1)^2+8=4dx \Leftrightarrow$

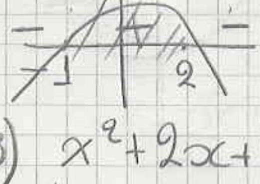
$\Leftrightarrow x^2+2x+1+8-4dx=0 \Leftrightarrow x^2+(2-4d)x+9=0$

$\Delta<0 \Leftrightarrow (2-4d)^2-36<0 \Leftrightarrow$

$\Leftrightarrow (2-4d-6)(2-4d+6)<0 \Leftrightarrow$

$\Leftrightarrow (-4-4d)(8-4d)<0 \Leftrightarrow -4(1+d)4(2-d)<0$

$\Leftrightarrow (1+d)(2-d)>0$



$-1<d<2$ ou $\underline{d\in]-1;2[}$

3) $x^2+2x+d-3=0$

$\Delta<0 \Leftrightarrow 4-4(d-3)<0 \Leftrightarrow 1-d+3<0 \Leftrightarrow \underline{4<d}$

EXERCICE 1.57

$f: \mathbb{Z} \to \mathbb{N} \quad x \longmapsto y = x^2+1$ yes

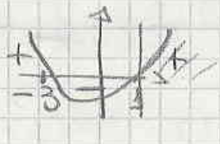
$g: \mathbb{Z} \to \mathbb{Q} \quad x \to y = \frac{1}{x}$ No; for $x=0$ there is no image.

$h: \mathbb{Q} \to \mathbb{N} \quad x \longmapsto y = 2$ yes

$i: \mathbb{N} \to \mathbb{R} \quad x \longmapsto y = x$ yes

$j: \mathbb{N}^* \longmapsto \mathbb{R} \quad x \to y = \sqrt{x^2+2x-3}$

$\Delta = 4+12 = 16 \quad x_{1,2} = \frac{-2 \pm 4}{2} = \begin{cases} -3 \\ 1 \end{cases}$



So generally $x \leq -3$ or $x \geq 1$
but as $x \in \mathbb{N}^*$: $x \geq 1$ So $D_j = \mathbb{N}^*$ yes

$K: \mathbb{R} \to \mathbb{R} \quad x \to y = \frac{3}{x^2-2}$ No because there is no image for $x = \pm\sqrt{2}$.

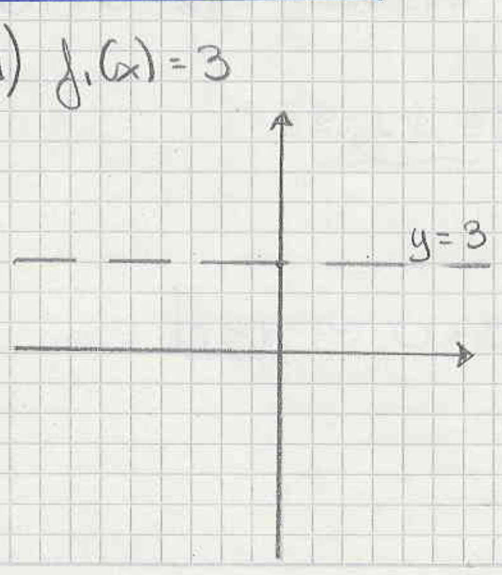
$l: \mathbb{Q} \to \mathbb{Q} \quad x \longmapsto y = \frac{3}{x^2-2}$ yes because $\pm\sqrt{2} \notin \mathbb{Q}$

$m: \mathbb{N} \to \mathbb{N} \quad x \to y = \frac{x^2+3x+2}{x+2}$ yes because $-2 \notin \mathbb{N}$

$n: \mathbb{Q} \to \mathbb{Q} \quad x \longmapsto y = \frac{x}{x^2-2}$ yes because $\pm\sqrt{2} \notin \mathbb{Q}$

EXERCICE 1.58

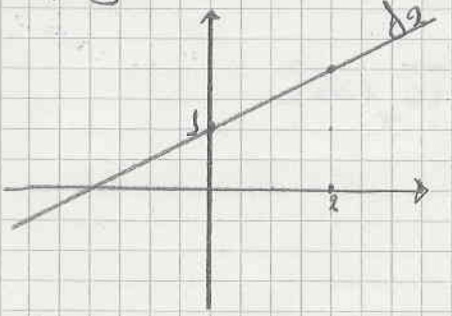
1) $f_1(x) = 3$



$D_1 = \mathbb{R}$

$R_1 = \mathrm{Im}(f_1) = \{3\}$

2) $f_2(x) = 0.5x + 1$



$D_2 = \mathbb{R}$

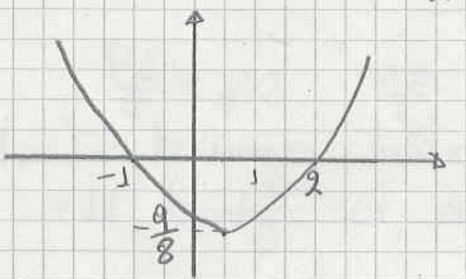
$R_2 = \mathrm{Im}(f_2) = \mathbb{R}$

3) $f_3(x) = \frac{1}{2}(x^2 - x - 2)$
$= \frac{1}{2}x^2 - \frac{1}{2}x - 1$

$\Delta = 1 + 8 = 9 \quad x_{1,2} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$

$x_S = -\frac{b}{2a} = \frac{\frac{1}{2}}{2 \cdot \frac{1}{2}} = \frac{1}{2}$

$y_S = f_3\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{4} - \frac{1}{2} - 2\right) =$
$= \frac{1}{2}\left(\frac{1 - 2 - 8}{4}\right) = \frac{-9}{8}$

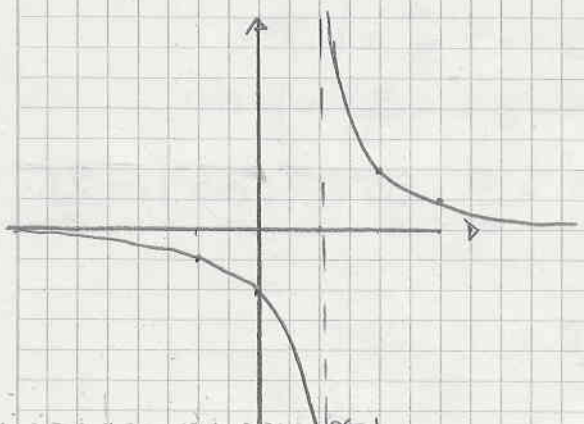


$D_{f_3} = \mathbb{R} \qquad R_3 = \mathrm{Im}(f_3) = [-\frac{9}{8}; +\infty[$

4) $f_4(x) = \frac{1}{x-1}$

$D_{f_4} = \mathbb{R} \setminus \{1\}$

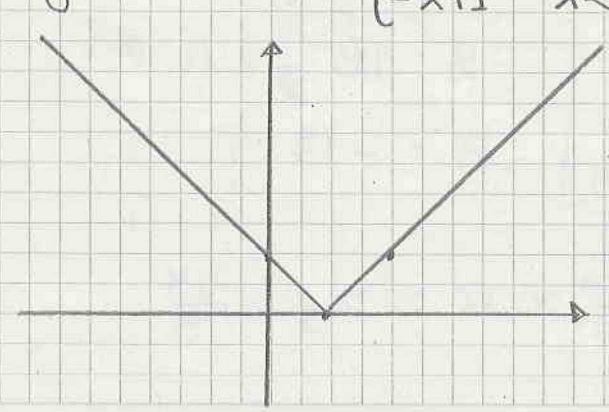
$\mathrm{Im}(f_4) = \mathbb{R} \setminus \{0\}$



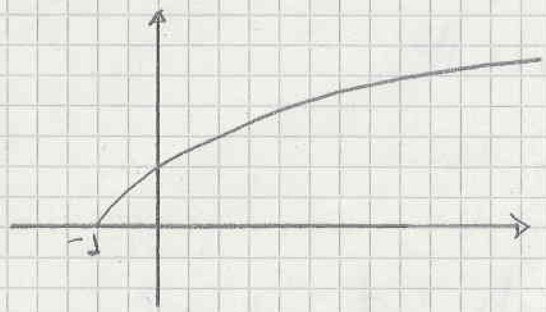
5) $f_5(x) = |x - 1| = \begin{cases} x - 1 & x \geq 1 \\ -x + 1 & x < 1 \end{cases}$

$D_{f_5} = \mathbb{R}$

$\mathrm{Im}(f_5) = [0, +\infty[$



6) $f_6(x) = \sqrt{x+1}$ $\quad x+1 \geq 0 \Leftrightarrow x \geq -1 \quad D_{f_6} = [-1, +\infty[$



$R_6 = \mathrm{Im}(f_6) = [0, +\infty[$

EXERCICE 1.59

1) $f(-11) = 1 \quad f(-7) = -3 \quad f(-4) = 4 \quad f(-1) = 3.5$

$f(1) = 4.5 \quad f(3) = 3 \quad f(9) = -1$

2) $f(x) = 0$

$x = -10$ or -8 or 8 or $x_1 = -\frac{26}{5} \quad x_2 = \frac{34}{3}$

In order to find the exact value around 5 (x_1) for withe $f(x_1)=0$ we have to find the equation of the line

$y = mx + h$

$(-7;-3) \quad -3 = -7m + h$

$(-4;2) \; (-) \; 2 = -4m + h$

$-5 = -3m \Leftrightarrow m = \frac{5}{3}$

$-3 = -7 \cdot \frac{5}{3} + h \Leftrightarrow$

$\Leftrightarrow -3 = -\frac{35}{3} + h \Leftrightarrow$

$\Leftrightarrow -\frac{9}{3} + \frac{35}{3} = h \Leftrightarrow h = \frac{26}{3}$

$y = \frac{5}{3}x + \frac{26}{3}$

$y = 0 \Leftrightarrow \frac{5}{3}x = -\frac{26}{3} \Leftrightarrow x_1 = -\frac{26}{5}$

The same for the other value x_2 around 11.

$y = mx + h$

$(10;-2) \quad -2 = 10m + h$

$(12;1) \; - \; 1 = 12m + h$

$-3 = -2m \Leftrightarrow m = \frac{3}{2}$

$-2 = \cancel{10}^{5} \frac{3}{\cancel{2}} + h \Leftrightarrow$

$h = -17$

$y = \frac{3}{2}x - 17 \qquad y = 0 \Leftrightarrow \frac{3}{2}x = 17 \Leftrightarrow x_2 = \frac{34}{3}$

3) $f([-12;12]) = [-3;5]$

$f(]2;5[) =]2;5[$

$f([-8;-2]) = [-3;2[\cup [3;4]$

$\{x \in \mathbb{R} \mid f(x) = 2\} = \{-12\} \cup [4;6]$

$\{x \in \mathbb{R} \mid f(x) > 2\} = [-4;4[$

4) $x = 2y \Leftrightarrow x = 2f(x) \Leftrightarrow f(x) = \frac{x}{2}$

$x = 4 : f(4) = 2 \qquad (4;2)$

EXERCICE 1.60

1) $f_1(x) = \frac{2x+1}{x-5}$ $\quad x-5 \neq 0 \Leftrightarrow x \neq 5 \quad D_{f_1} = \mathbb{R} \setminus \{5\}$

2) $f_2(x) = \sqrt{x-5}$ $\quad x-5 \geq 0 \Leftrightarrow x \geq 5 \quad D_{f_2} = [5;+\infty[$

3) $f_3(x) = \frac{(x-1)}{2(5x-3)}$ $\quad 5x-3 \neq 0 \Leftrightarrow x \neq \frac{3}{5} \quad D_{f_3} = \mathbb{R} \setminus \{3/5\}$

4) $f_4(x) = \frac{1}{x^2-2x+1} = \frac{1}{(x-1)^2}$ $\quad x-1 \neq 0 \quad D_{f_4} = \mathbb{R} \setminus \{1\}$

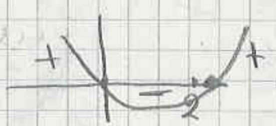
5) $f_5(x) = \frac{2x^2+x+1}{x^2-9}$ $\quad x^2-9 \neq 0 \Leftrightarrow x \neq \pm 3 \quad D_{f_5} = \mathbb{R} \setminus \{\pm 3\}$

6) $f_6(x) = \sqrt{2-3x}$ $\quad 2-3x \geq 0 \Leftrightarrow \frac{2}{3} \geq x \quad D_{f_6} =]-\infty;\frac{2}{3}]$

7) $f_7(x) = \frac{x^2}{(x-3)(x+4)}$ $\quad x \neq 3 \quad x \neq -4 \quad D_{f_7}(x) = \mathbb{R} \setminus \{3;-4\}$

8) $f_8(x) = \frac{1-x^2}{(x-4)^3}$ $\quad (x-4) \neq 0 \Leftrightarrow x \neq 4 \quad D_{f_8} = \mathbb{R} \setminus \{4\}$

9) $f_9(x) = \frac{1}{\sqrt{3x^2-6x}}$ $\quad 3x^2-6x > 0 \Leftrightarrow 3x(x-2) > 0$



$D_{f_9} =]-\infty;0[\cup]2;+\infty[$

10) $f_{10}(x) = \sqrt{x^3-x^2-5x-3}$ $\quad$ r.p. $\pm 1, \pm 3$ $\quad 1 \times \quad -1$ ✓

$x^3-x^2-5x-3 \geq 0$

1	−1	−5	−3	−1
↓	−1	2	3	
1	−2	−3	0	

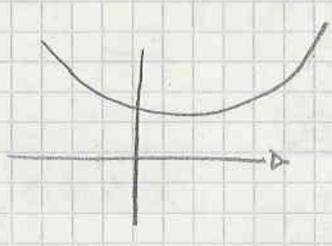
So $x^3-x^2-5x-3=(x+1)(x^2-2x-3)=(x+1)^2(x-3)$

$\Delta = 4+12=16$

$x_{1,2}=\dfrac{2\pm4}{2}=\begin{cases}3\\-1\end{cases}$

	$-\infty$		-1		3		$+\infty$
$(x+1)^2$		$+$	0	$+$		$+$	
$x-3$		$-$		$-$	0	$+$	
P		$-$	0	$-$	0	$+$	

$D_{f_{10}}=[3;+\infty[$



11) $f_{11}(x)=\sqrt{x^2-5x+7}$

$\Delta=25-28=-3<0$

So $x^2-5x+7>0 \ \forall x\in\mathbb{R}$

$D_{f_{11}}=\mathbb{R}$

12) $f_{12}(x)=\sqrt{x^2-1}$ $\quad x^2-1\geq0 \Leftrightarrow (x-1)(x+1)\geq0$

$D_{f_{12}}=]-\infty,-1]\cup[1,+\infty[$

EXERCICE 1.61

1) $f(x)=\dfrac{x^3-x+1}{x^2}$ $\quad D_f=\mathbb{R}\setminus\{0\}$

$f(-x)=\dfrac{-x^3+x+1}{x^2}\neq f(+x)$

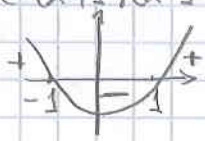
2) $g(x)=\dfrac{x^2+2}{x^3+x}$ $\quad x^3+x\neq0 \Leftrightarrow x(\underbrace{x^2+1}_{\neq0})\neq0 \Leftrightarrow x\neq0$

$D_g=\mathbb{R}\setminus\{0\}$

$g(-x)=\dfrac{(-x)^2+2}{(-x)^3+(-x)}=\dfrac{x^2+2}{-x^3-x}=-\dfrac{x^2+2}{x^3+x}=-g(x)$

odd

3) $h(x) = \dfrac{\sqrt{x^4-1}}{x^2+2}$ $\quad x^4-1 \geq 0 \Leftrightarrow (x^2+1)(x^2-1) \geq 0 \Leftrightarrow (x^2+1)(x-1)(x+1) \geq 0$



$$D_h =]-\infty, -1] \cup [1, +\infty[$$

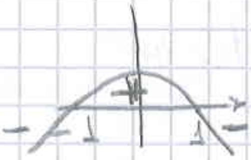
$$h(-x) = \frac{\sqrt{(-x)^4-1}}{(-x)^2+2} = \frac{\sqrt{x^4-1}}{x^2+2}$$

So h is even

4) $i(x) = \dfrac{x}{|x|}$ $\qquad D_i = \mathbb{R}^*$

$i(-x) = \dfrac{-x}{|-x|} = -\dfrac{x}{|x|} = -i(x)$ So i is odd

5) $j(x) = \sqrt{1-x^2}$ $\qquad 1-x^2 \geq 0 \Leftrightarrow (1-x)(1+x) \geq 0$



$$D_j = [-1, 1]$$

$j(-x) = \sqrt{1-(-x)^2} = \sqrt{1-x^2} = j(x)$ So j is even

6) $k(x) = \dfrac{1}{2x^2+x+1}$ $\qquad \Delta = 1-8 = -7 < 0 \Rightarrow 2x^2+x+1 \neq 0 \ \forall x \in \mathbb{R}$

$\Rightarrow D_k = \mathbb{R}$

$k(-x) = \dfrac{1}{2(-x)^2+(-x)+1} = \dfrac{1}{2x^2-x+1}$ Nothing

EXERCICE 1.62

1) $\dfrac{x^2-4}{(x+4)(x-1)} \geq 0 \Leftrightarrow \dfrac{(x-2)(x+2)}{(x+4)(x-1)} \geq 0$

	$-\infty$		-4		-2		1		2		$+\infty$
$(x-2)(x+2)$		+	|	+	0	−	|	−	0	+	
$(x+4)(x-1)$		+	0	−	|	−	0	+	|	+	
Q		+	||	−	0	+	||	−	0	+	

$x \in]-\infty; -4[\cup [-2; 1[\cup [2, +\infty[$

2) $\dfrac{4x^8-2x^7}{3x-5}<0 \Leftrightarrow$ $\qquad x \neq \dfrac{5}{3}$

$\Leftrightarrow \dfrac{2x^7(2x-1)}{3x-5}<0$

	$-\infty$	0		$1/2$		$5/3$	$+\infty$
x^7	$-$	0	$+$		$+$		$+$
$2x-1$	$-$		$-$	0	$+$		$+$
$3x-5$	$-$		$-$		$-$	0	$+$
Q	$-$	0	$+$	0	$-$	$\|$	$+$

$x \in]-\infty;0[\cup]1/2;5/3[$

3) $\dfrac{x+1}{x-1} > \dfrac{x-1}{x+1} \Leftrightarrow$ $\qquad x \neq \pm 1$

$\Leftrightarrow \dfrac{x+1}{x-1} - \dfrac{x-1}{x+1} > 0 \Leftrightarrow \dfrac{(x+1)^2-(x-1)^2}{(x-1)(x+1)} > 0 \Leftrightarrow$

$\Leftrightarrow \dfrac{(x+1+x-1)(x+1-x+1)}{(x-1)(x+1)} > 0 \Leftrightarrow \dfrac{2x\cdot 2}{(x-1)(x+1)} > 0 \Leftrightarrow$

$\dfrac{4x}{(x-1)(x+1)} > 0$

	$-\infty$	-1		0		1	$+\infty$
x	$-$		$-$	0	$+$		$+$
$(x-1)(x+1)$	$+$	0	$-$		$-$	0	$+$
Q	$-$	$\|$	$+$	0	$-$	$\|$	$+$

$x \in]-\infty,0[\cup]1,+\infty[$

4) $\dfrac{13}{2-x} \leq \dfrac{21x+3}{3x+1} \Leftrightarrow \dfrac{13}{2-x} - \dfrac{21x+3}{3x+1} \leq 0 \Leftrightarrow \dfrac{13(3x+1)-(2-x)(21x+3)}{(2-x)(3x+1)} \leq 0$

$\Leftrightarrow \dfrac{\cancel{39x}+13-\cancel{42x}+21x^2-6+\cancel{3x}}{(2-x)(3x+1)} \leq 0 \Leftrightarrow \dfrac{21x^2+7}{(2-x)(3x+1)} \leq 0$

	$-\infty$	$-1/3$		2	$+\infty$
$21x^2+7$	$+$		$+$		$+$
$(2-x)(3x+1)$	$-$	0	$+$	0	$-$
Q	$-$	$\|$	$+$	$\|$	$-$

$x \in]-\infty;-\frac{1}{3}[\cup]2;+\infty[$

EXERCICE 1.63

1) $f(x) = \frac{1}{2}x$ $\quad (0,0) \quad (2;1)$

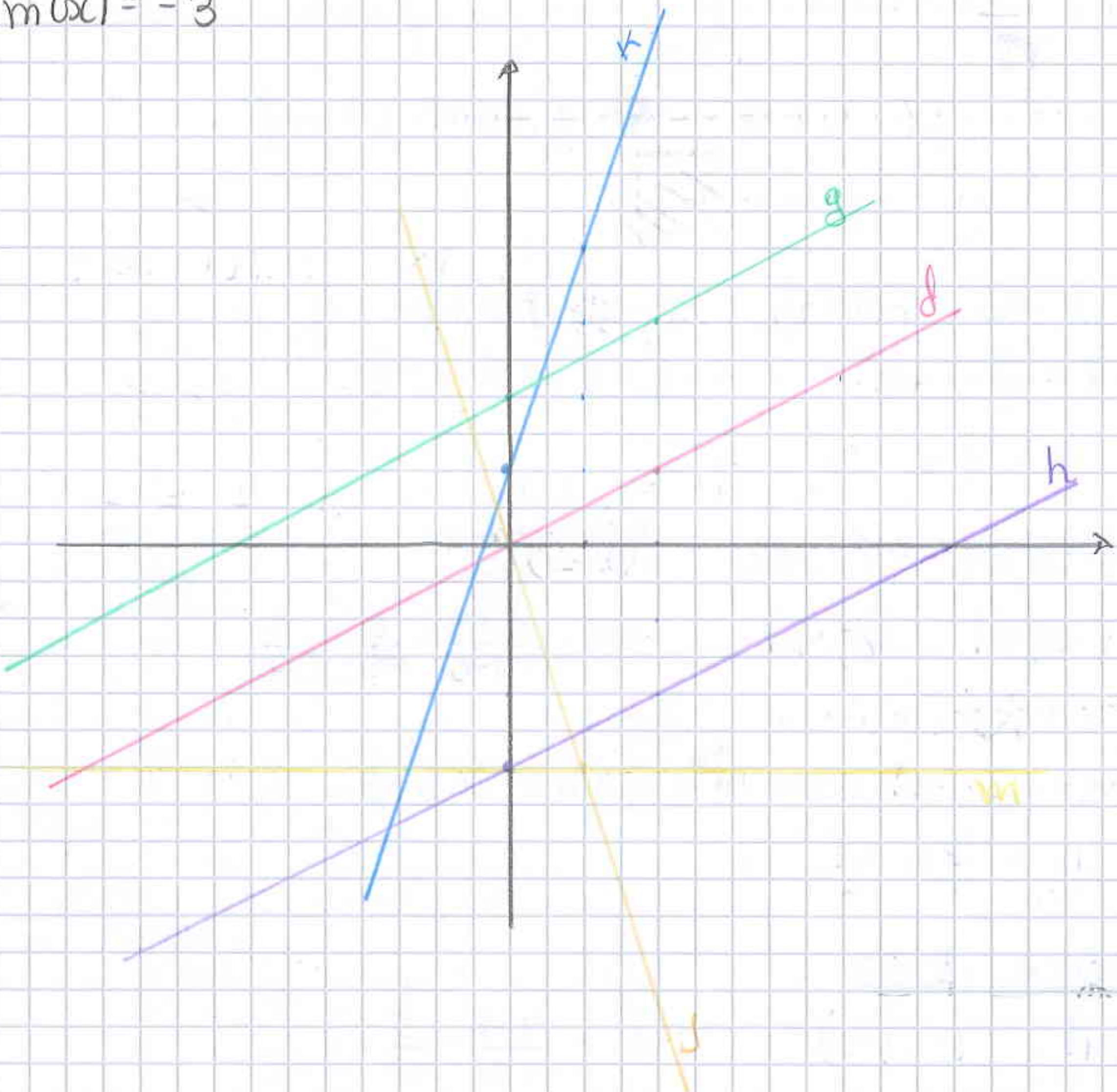
2) $g(x) = \frac{1}{2}x + 2$ $\quad (0,2) \quad (2;3)$

3) $h(x) = \frac{1}{2}x - 3$ $\quad (0,-3) \quad (2;-2)$

4) $j(x) = -3x$ $\quad (0,0) \quad (1;-3)$

5) $k(x) = 3x + 1$ $\quad (0,1) \quad (1;4)$

6) $m(x) = -3$



EXERCICE 1.64

1) $O(0,0)$ $P(2;6)$

$m = \frac{6-0}{2-0} = 3 \Rightarrow y = 3x + h$

But the line passes through $(0,0) \Rightarrow h = 0$

$y = 3x$

2) $A(-1;-4)$ $B(7;-8)$

$m = \frac{-8+4}{7+1} = \frac{-4}{8} = -\frac{1}{2} \Rightarrow y = -\frac{1}{2}x + h$

A: $-4 = -\frac{1}{2}(-1) + h \Leftrightarrow -4 = \frac{1}{2} + h \Leftrightarrow h = -\frac{9}{2}$

So ; $y = -\frac{1}{2}x - \frac{9}{2}$

3) $C(2;-2)$ $m = \frac{3}{5} \Rightarrow y = \frac{3}{5}x + h$

$-2 = \frac{3}{5}\cdot 2 + h \Leftrightarrow h = -2 - \frac{6}{5} \Leftrightarrow h = -\frac{16}{5}$

$y = \frac{3}{5}x - \frac{16}{5}$

4) $P(2;6)$ $C(2;-2)$ $x = 2$

5) $(-2;4)$ $(6;-2)$ $m = \frac{4+2}{-2-6} = \frac{6}{-8} = -\frac{3}{4}$

$y = -\frac{3}{4}x + h$

$4 = -\frac{3}{4}\cdot(-2) + h \Leftrightarrow 4 = \frac{3}{2} + h \Leftrightarrow h = \frac{5}{2}$

$y = -\frac{3}{4}x + \frac{5}{2}$

EXERCICE 1.65

$(4;4)$ $(-6;-1)$ $m = \frac{4+1}{4+6} = \frac{5}{10} = \frac{1}{2}$

$$\left.\begin{array}{l} y = \frac{1}{2}x + h \\ 4 = \frac{1}{\cancel{2}}\cdot \overset{2}{\cancel{4}} + h \Leftrightarrow 2 = h \end{array}\right\} y = \frac{1}{2}x + 2$$

EXERCICE 1.66

$f(x) = -2x+1 \quad A(2;1)$

1) $m_f = -2 \quad y = -2x+h$

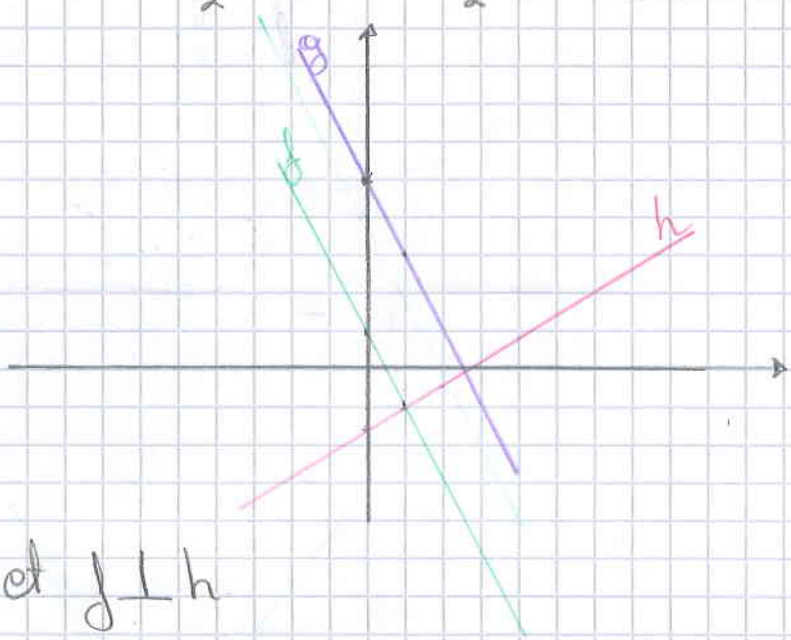
$A: \; 1 = -2 \cdot 2 + h \Leftrightarrow h = 5 \Big\} \; \underline{y = -2x+5} \; g$

2) $m = \frac{1}{2} \quad f(1) = -2+1 = -1$

$(1;-1) \quad y = \frac{1}{2}x + h$

$-1 = \frac{1}{2} + h \Leftrightarrow h = -\frac{3}{2} \Big\} \; \underline{y = \frac{1}{2}x - \frac{3}{2}} \quad h$

3)



$f \parallel g$ et $f \perp h$

EXERCICE 1.67

$\left.\begin{array}{l} f(x) = -3x+4 \\ g(x) = 5x+\frac{3}{2} \end{array}\right\}$ $f(x) = g(x) \Leftrightarrow -3x+4 = 5x+\frac{3}{2} \Leftrightarrow -6x+8 = 10x+3$

$\Leftrightarrow 16x = 5 \Leftrightarrow x = \frac{5}{16}$

$f(x) < g(x) \Leftrightarrow -3x+4 < 5x+\frac{3}{2} \Leftrightarrow 8x > \frac{5}{2} \Leftrightarrow x > \frac{5}{16}$

$\mathcal{A} = \{x \mid f(x) < g(x)\} = \left]\frac{5}{16}; +\infty\right[$

EXERCICE 1.68

$A(2;0) \quad B(-1;6) \quad m = \frac{6-0}{-1-2} = -2 \quad y = -2x+h$

$0 = -2 \cdot 2 + h \Leftrightarrow h = 4$

$\underline{f(x) = -2x+4}$

$m_f \cdot m_g = -1 \Leftrightarrow -2\,m_g = -1 \Leftrightarrow m_g = \frac{1}{2}$

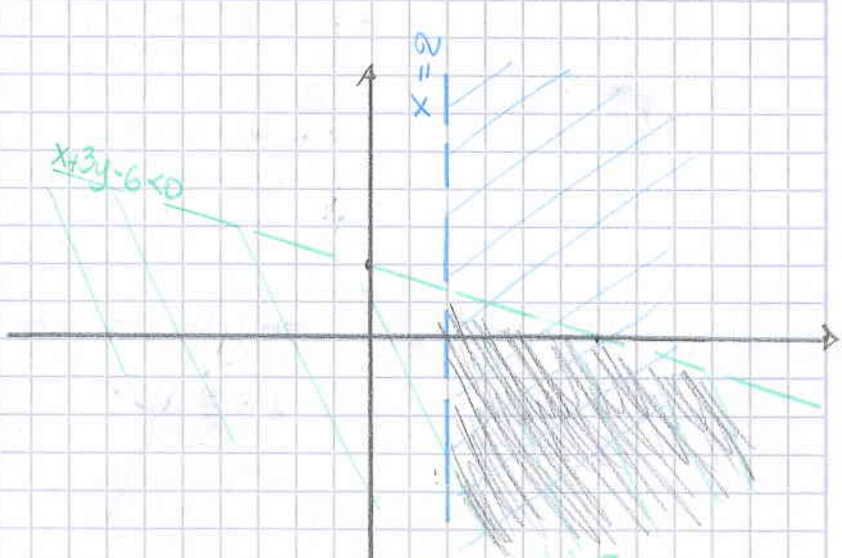
$g: \quad y = \frac{1}{2}x + h$

$C(1;-2) \quad -2 = -\frac{1}{2} + h \Leftrightarrow h = -2 + \frac{1}{2} \Leftrightarrow h = -\frac{3}{2}$

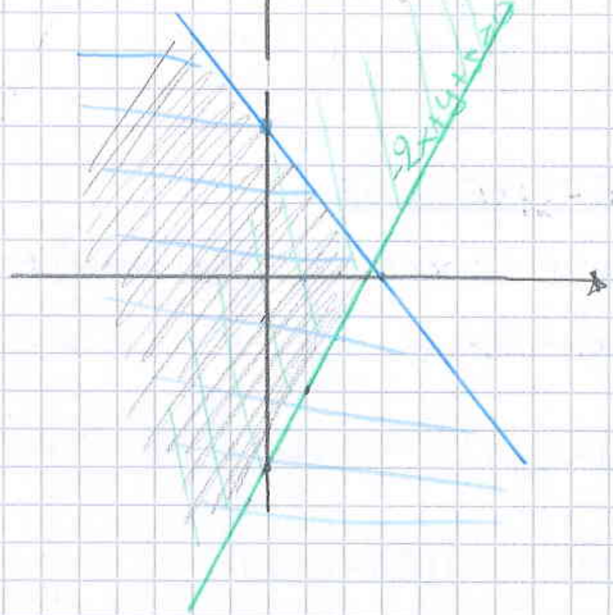
$g(x) = \frac{1}{2}x - \frac{3}{2}$

EXERCICE 1.69

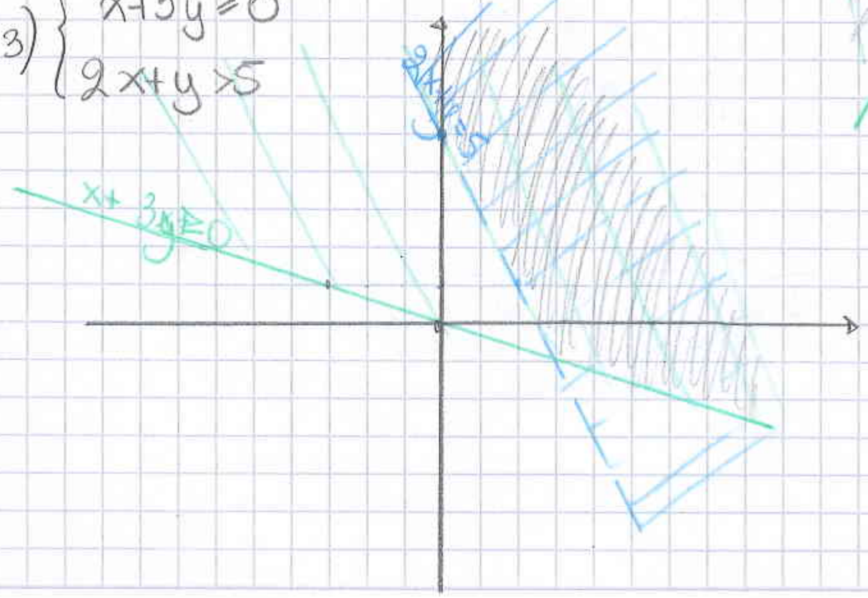
1) $\begin{cases} x + 3y - 6 < 0 \\ x > 2 \end{cases}$



2) $\begin{cases} -2x + y + 5 \geq 0 \\ 4x + 3y - 12 \leq 0 \end{cases}$

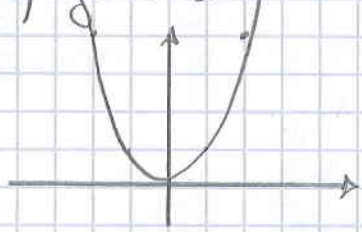


3) $\begin{cases} x + 3y \geq 0 \\ 2x + y > 5 \end{cases}$

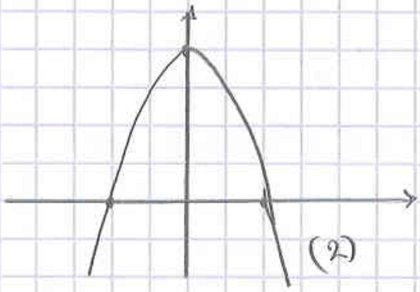


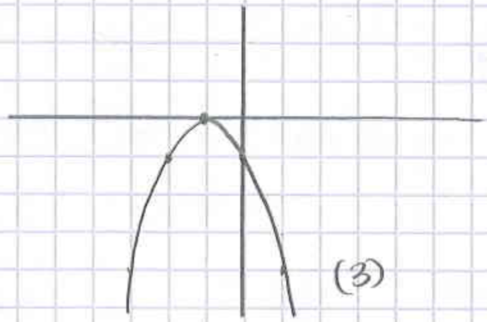
EXERCICE 1.70

1) $f(x) = x^2$ $\quad V(0,0) \quad I_x = I_y = (0,0)$



2) $f(x) = -x^2 + 4 = -(x-0)^2 + 4 \quad V(0;4) \quad I_y = (0;4) \quad I_x(0;2), I(0;-2)$



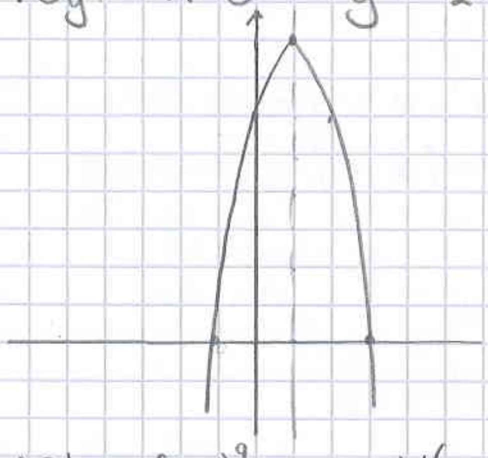


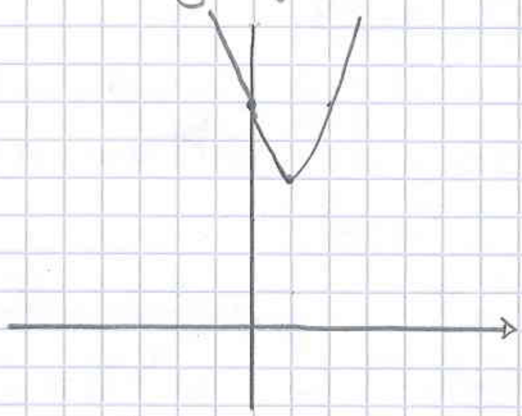
3) $f(x) = -(x+1)^2 \quad V(-1;0) \quad I_x(-1;0) \quad I_y(0;-1)$

4) $f(x) = -2(x+1)(x-3) \qquad x_v = \frac{-1+3}{2} = 1 \quad f(1) = 8 \quad V(1;8)$

$\cap Ox \quad y = 0 \Leftrightarrow x_1 = -1 \quad x_2 = 3 \quad I_x(-1;0) \quad (3;0)$

$\cap Oy: \quad x = 0 \Rightarrow y = -2 \cdot (-3) = 6 \quad I_y(0;6)$



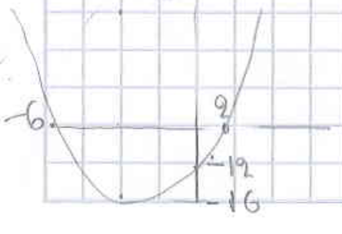


5) $f(x) = 2(x-1)^2 + 4 \quad V(1;4) \quad I_x \nexists \quad I_y(0;6)$

6) $f(x) = x^2 + 4x - 12 \quad x_v = \frac{-4}{2} = -2 \quad f(-2) = 4 - 8 - 12 = -16 \quad V(-2;16)$

$\cap Ox \quad f(x) = 0 \quad \Delta = 16 + 48 = 64 \quad x_{1,2} = \frac{-4 \pm 8}{2} \begin{cases} -6 & I_x(-6;0) \\ 2 & I_x(2;0) \end{cases}$

$\cap Oy \quad x = 0 \quad f(0) = -12$



Exercice 1.71

1) $f(x) = -\frac{1}{2}x^2 + x + 3$

$x_v = -\frac{b}{2a} = \frac{-1}{2(-\frac{1}{2})} = 1 \quad f(1) = -\frac{1}{2} + 1 + 3 = 4 - \frac{1}{2} = \frac{7}{2} \quad V(1; \frac{7}{2})$

$\cap O_x \quad f(x) = 0 \Leftrightarrow -\frac{1}{2}x^2 + x + 3 = 0 \Leftrightarrow x^2 - 2x - 6 = 0$

$\Delta = 4 + 24 = 28 \qquad x_{1,2} = \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

$I_x(1+\sqrt{7}; 0) \quad I_x(1-\sqrt{7}; 0)$

$\cap O_y: \quad x = 0 \Rightarrow f(0) = 3 \quad I_y(0; 3)$

$f \cap d: \quad -\frac{1}{2}x^2 + x + 3 = x + 1 \Leftrightarrow -x^2 + 2\cancel{x} + 6 = 2\cancel{x} + 2 \Leftrightarrow$

$\Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$

$x = 2 \qquad y = 3 \qquad I_1(2; 3)$

$x = -2 \qquad y = -1 \qquad I_2(-2; -1)$

2) $y = -\frac{1}{2}x^2 + x + 3 = -\frac{1}{2}(x - 1 - \sqrt{7})(x - 1 + \sqrt{7}) =$

$= -\frac{1}{2}(x-1)^2 + \frac{7}{2}$

Exercice 1.72

1) $\left.\begin{array}{l} f(x) = \frac{1}{2}x^2 + 1 \\ g(x) = 2x - 1 \end{array}\right\} \frac{1}{2}x^2 + 1 = 2x - 1 \Leftrightarrow x^2 + 2 = 4x - 2 \Leftrightarrow$

$x^2 - 4x + 4 = 0 \Leftrightarrow (x-2)^2 = 0 \Leftrightarrow x = 2$

$y = 3$

$I(2; 3)$

2) $\left\{\begin{array}{l} y = -x^2 + 2 \\ y = -3x + b \end{array}\right\} \begin{array}{l} -x^2 + 2 = -3x + b \Leftrightarrow \\ x^2 - 3x - 2 + b = 0 \end{array}$

It must be $\Delta = 0 \Leftrightarrow 9 - 4(b-2) = 0 \Leftrightarrow$

$\Leftrightarrow 9 - 4b + 8 = 0 \Leftrightarrow 17 = 4b \Leftrightarrow b = \frac{17}{4}$

Exercice 1.73

$A(1; 5) \quad B(2; 5) \quad C(-1; 11) \qquad y = ax^2 + bx + c$

$\left\{\begin{array}{ll} 5 = a + b + c \; (1) & (1) - (2) \Rightarrow 0 = -3a - b \Leftrightarrow 3a + b = 0 \\ 5 = 4a + 2b + c \; (2) & (2) - (3) \Rightarrow -6 = 3a + 3b \Leftrightarrow -2 = a + b \\ 11 = a - b + c \; (3) & \end{array}\right.$

$$\begin{cases} a+b=-2 \\ 3a+b=0 \end{cases} (-) \Leftrightarrow -2a=-2 \Leftrightarrow a=1 \quad b=-3$$

$$① \Leftrightarrow 5 = 1-3+c \Leftrightarrow \underline{c=7}$$

$$f(x) = x^2 - 3x + 7$$

EXERCICE 1.74

$V(3;3) \qquad (0;0)$

$$y = a(x-3)^2+3$$

$$0 = a(0-3)^2+3 \Leftrightarrow 0 = 9a+3 \Leftrightarrow a = -1/3$$

So: $y = -\frac{1}{3}(x-3)^2+3$

EXERCICE 1.75

$y = ax$ = linear line

$$y = \frac{x^2-2x+9}{4}$$

$$ax = \frac{x^2-2x+9}{4} \Leftrightarrow 4ax = x^2-2x+9 \Leftrightarrow$$

$$\Leftrightarrow x^2 - (2+4a)x + 9 = 0$$

$$\Delta = 0 \Leftrightarrow 2^2(1+2a)^2 - 4\cdot 9 = 0 \Leftrightarrow$$

$$\Leftrightarrow 4(1+4a+4a^2) - 36 = 0 \Leftrightarrow 1+4a+4a^2-9=0$$

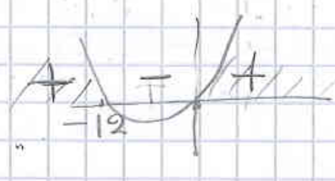
$$\Leftrightarrow 4a^2+4a-8=0 \Leftrightarrow a^2+a-2=0 \;\; < \begin{matrix} a=2 \\ a=-1 \end{matrix}$$

$y = 2x$ or $y = -x$

EXERCICE 1.76

$$y = 3x^2+ax-a$$

$$\Delta < 0 \Leftrightarrow a^2+12a<0 \Leftrightarrow a(a+12)<0$$



$$a \in \left]-12;0\right[$$

1.77.

$f(x) = 2x^2 - nx + 3n$

$X_V = 1. \Rightarrow \frac{n}{4} = 1 \Leftrightarrow n = 4$

So; $f(x) = 2x^2 - 4x + 12$

$f(1) = 2 - 4 + 12 = 10 \quad V(1;10)$

EXERCICE 1.78

$x^2 + mx + m - 0{,}75 = 0$

$\Delta = 0 \Leftrightarrow m^2 - 4(m - 0{,}75) = 0 \Leftrightarrow m^2 - 4m + 3 = 0 \Leftrightarrow \begin{matrix} m = 3 \\ m = 1 \end{matrix}$

For $m = 3 \quad x^2 + 3x + 2{,}25 = 0$

$\Delta = 0 \quad x_p = \frac{-3}{2} = -1{,}5 = x_1$

For $m = 1 \quad x^2 + x + 1{,}25 = 0 \quad x_2 = -\frac{1}{2} = -0{,}5 = x_2$

EXERCICE 1.79

$x_1 = 1 \quad x_2 = -3 \quad x_V = \frac{1-3}{2} = -1 \Big\} V(-1;8)$

$y = 8 \Rightarrow y_V = 8$

$y = a(x+1)^2 + 8$

$(1;0) \quad 0 = a(2)^2 + 8 \Leftrightarrow 4a = -8 \Leftrightarrow a = -2$

$y = -2(x+1)^2 + 8$

EXERCICE 1,80

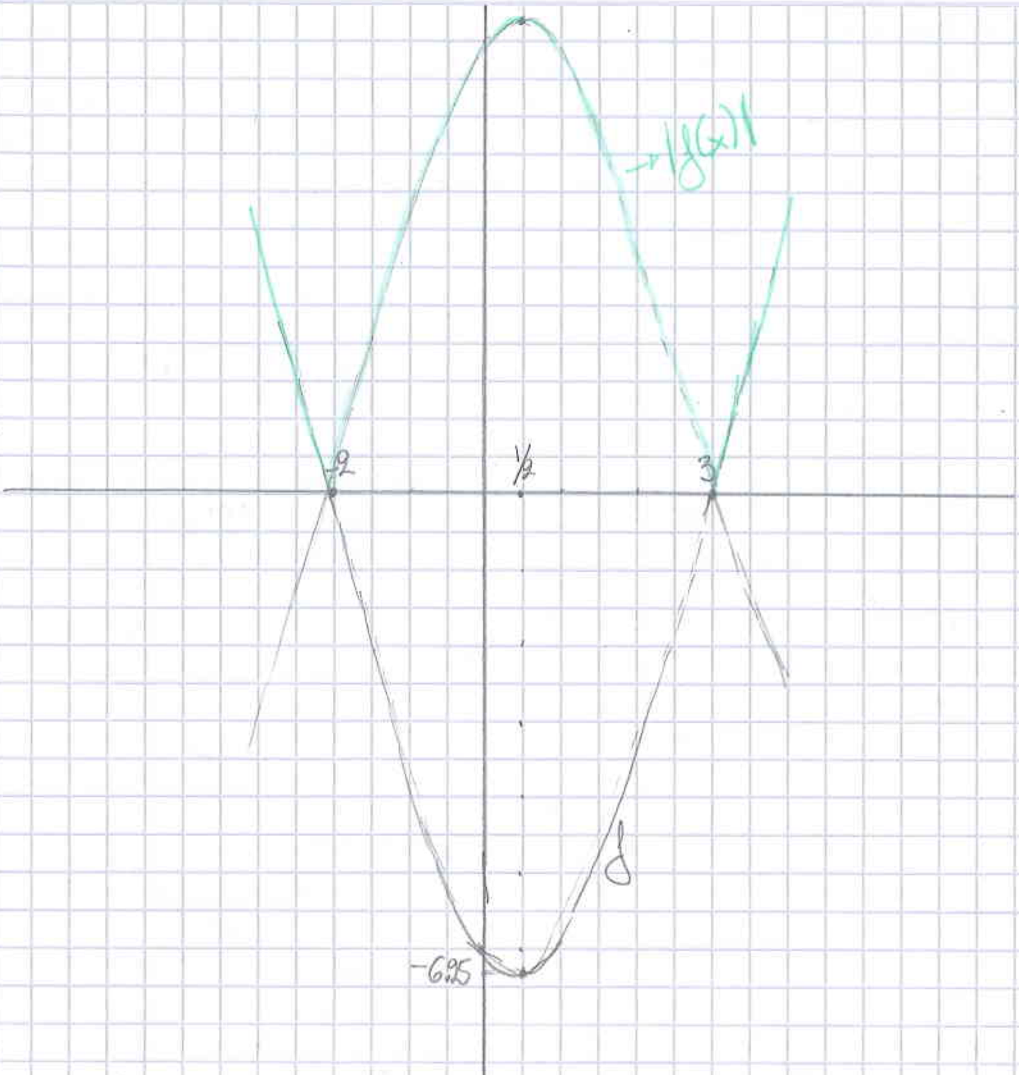
$f(x) = x^2 - x - 6 \quad g(x) = |f(x)|$

$x_V = \frac{1}{2} = 0{,}5 \quad f\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} - 6 = \frac{1 - 2 - 24}{4} = \frac{-25}{4} \approx -6{,}25$

$V_f\left(\frac{1}{2}; -6{,}25\right)$

$y = 0 \Leftrightarrow x^2 - x - 6 = 0 \begin{matrix} \nearrow 3 = x_1 \\ \searrow -2 = x_2 \end{matrix}$

$f(0) = -6$



$$g(x) = \begin{cases} x^2 - x - 6 & x \leq -2 \text{ or } x \geq 3 \\ -x^2 + x + 6 & -2 < x < 3 \end{cases}$$

EXERCICE 1.81

$f(x) = x^3 - x^2 - 2x = x(x^2 - x - 2) = 0$

$\Delta = 1 + 8 = 9 \quad x_{1,2} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$

$x_1 = 0 \quad x_2 = -1 \quad x_3 = 2$

EXERCICE 1.82

$f(x) = \frac{x^4 - 8x^2 - 9}{4}$

$D_f = \mathbb{R}$

$\cap O_x$: $f(x) = 0 \Leftrightarrow x^4 - 8x^2 - 9 = 0$

$x^2 = t \quad t^2 - 8t - 9 = 0 \quad \Delta = 64 + 36 = 100$

$t_{1,2} = \frac{8 \pm 10}{2} = \begin{cases} 9 \\ -1 \end{cases}$

$t_1 = 9 \Rightarrow x^2 = 9 \Leftrightarrow x = \pm 3$

$t_2 = -1$ imp

$I_{x1}(3;0) \quad I_{x2}(-3;0)$

$\cap O_y$: $x = 0 \quad f(0) = -\frac{9}{4} \quad I_y\left(0; -\frac{9}{4}\right)$ $\leftarrow -2,25$

· parity: $f(-x) = \frac{(-x)^4 - 8(-x)^2 - 9}{4} = \frac{x^4 - 8x^2 - 9}{4} = f(x)$

so f is even

· Range

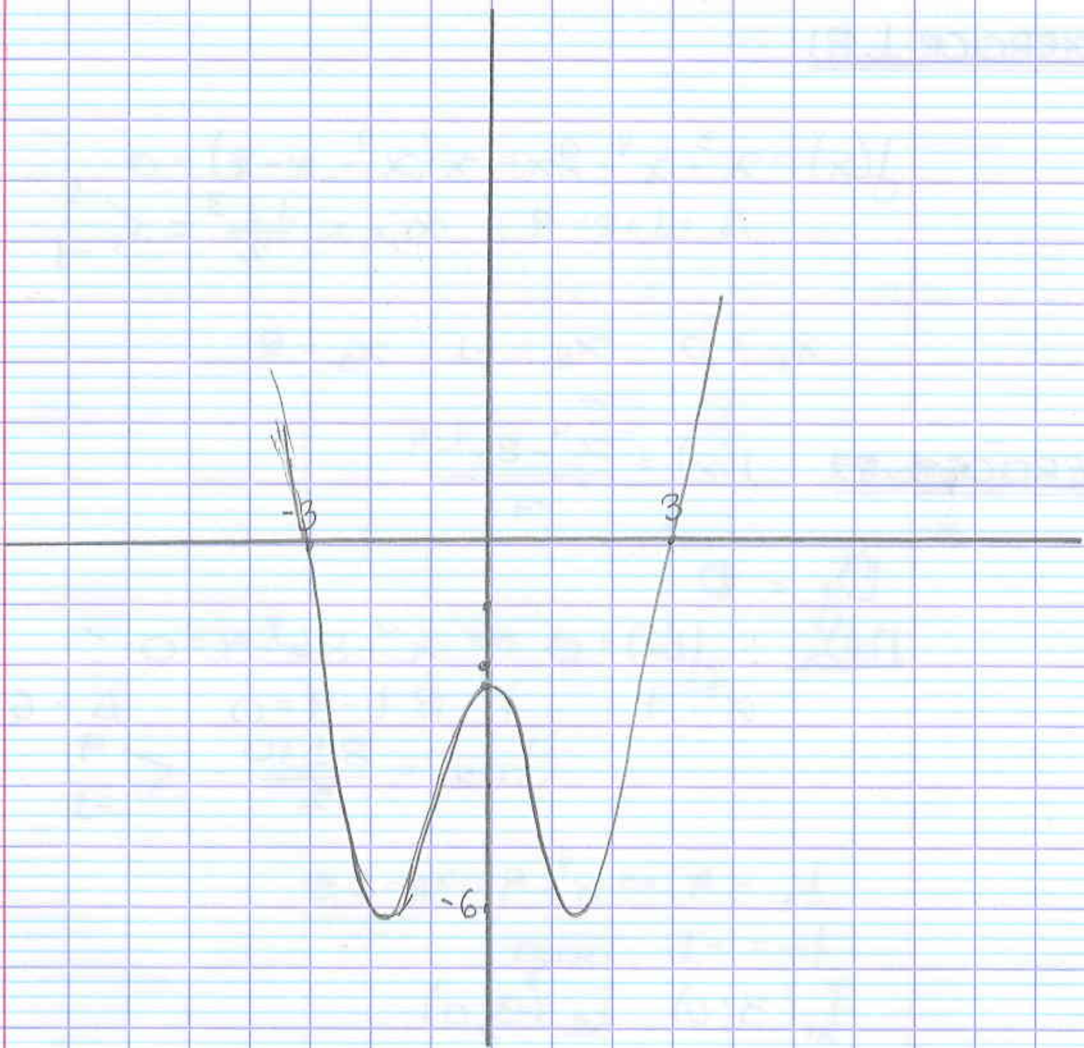
$y = \frac{x^4 - 8x^2 - 9}{4} \Leftrightarrow x^4 - 8x^2 - 9 = 4y \Leftrightarrow$

$x^4 - 8x^2 - (9 + 4y) = 0$

$f(D_f) = R_f = \{y \in \mathbb{R} \mid \exists x \in D_f : x^4 - 8x^2 - (9 + 4y) = 0\}$

$\Delta \geq 0 \Leftrightarrow 64 + 4(9 + 4y) \geq 0 \Leftrightarrow 64 + 36 + 16y \geq 0 \Leftrightarrow$

$y \geq -6.25 \quad R_f = [6,25; +\infty[$


-3
3
-6

Exercice 1.83

1) $x^3-8x-3=0$

p.v : $\pm 1 \quad \pm 3$

$x=1 \quad 1-8-3 \neq 0$ $\qquad x=3 \quad 27-24-3=0$ ✓

$x=-1 \quad -1+8-3 \neq 0.$

1	0	-8	-3	3
↓	3	9	3	
1	3	1	0	

$x^3-8x-3=0 \Leftrightarrow (x-3)(x^2+3x+1)=0$

$\Delta = 9-4=5 \quad x_{1,2} = \frac{-3\pm\sqrt{5}}{2} = \begin{cases} \frac{-3+\sqrt{5}}{2} \\ \frac{-3-\sqrt{5}}{2} \end{cases}$

$x_1=3 \quad x_2 = \frac{-3+\sqrt{5}}{2} \quad x_3 = \frac{-3-\sqrt{5}}{2}$

2) $x^3-8x-3 \geq 0$

x	$-\infty$		$\frac{-3-\sqrt{5}}{2}$		$\frac{-\sqrt{3}+\sqrt{5}}{2}$		3		$+\infty$
$x-3$		$-$		$-$		$-$	0	$+$	
x^2+3x+1		$+$	0	$-$	0	$+$		$+$	
E		$-$	0	$+$	0	$-$	0	$+$	

$x \in \left[\frac{-3-\sqrt{5}}{2} ; \frac{-\sqrt{3}+\sqrt{5}}{2}\right] \cup [3;+\infty[$

Exercice 1.84

$f(x) = y = x(x-1)(x^2+x+1)$

$\Delta = 1-4<0$ so $x^2+x+1>0 \ \forall x.$

x	$-\infty$		0		1		$+\infty$
x		$-$	0	$+$		$+$	
$x-1$		$-$		$-$	0	$+$	
x^2+x+1		$+$		$+$		$+$	
f		$+$	0	$-$	0	$+$	

Exercice 1.85 $f(x) = \frac{1}{x-3}$ $x \neq 3$ $d: x - 4y = 0 \Leftrightarrow x = 4y$

$$\begin{cases} y = \frac{1}{x-3} \\ x = 4y \end{cases} \Leftrightarrow y = \frac{1}{4y-3} \Leftrightarrow$$

$$\Leftrightarrow 4y^2 - 3y = 1 \Leftrightarrow 4y^2 - 3y - 1 = 0$$

$\Delta = 9 + 16 = 25$ $y_{1,2} = \frac{3 \pm 5}{8} = \begin{cases} 8 \\ -\frac{2}{8} = -\frac{1}{4} \end{cases}$

$y = 8$ $x = 32$ $I_1(32; 8)$
$y = -\frac{1}{4}$ $x = -\frac{4}{4} = -1$ $I_2(-1; -\frac{1}{4})$

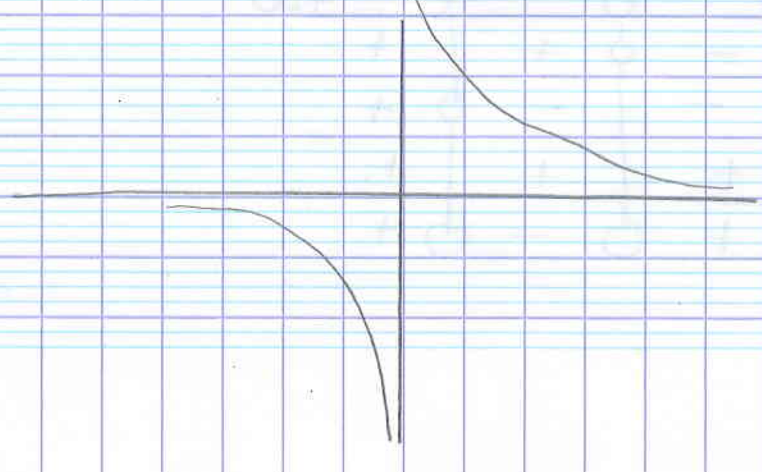
Exercice 1.86

1) $f(x) = \frac{1}{x}$

- $D_f = \mathbb{R}^*$
- O_x $f(x) = 0$ imp
  O_y: $x = 0$ imp.
- parity: $f(-x) = -f(x)$ odd
-

x	$-\infty$		0		$+\infty$
f		$-$	‖	$+$	

- $\lim_{x \to 0^-} f(x) = -\infty$ $\lim_{x \to +0^+} f(x) = +\infty$ VA: $x = 0$
- $\lim_{x \to \pm\infty} f(x) = 0$ $y = 0$ HA



2) $g(x) = \frac{1}{x^2}$

· $D_g = \mathbb{R}^*$

· O_x imp. O_y imp

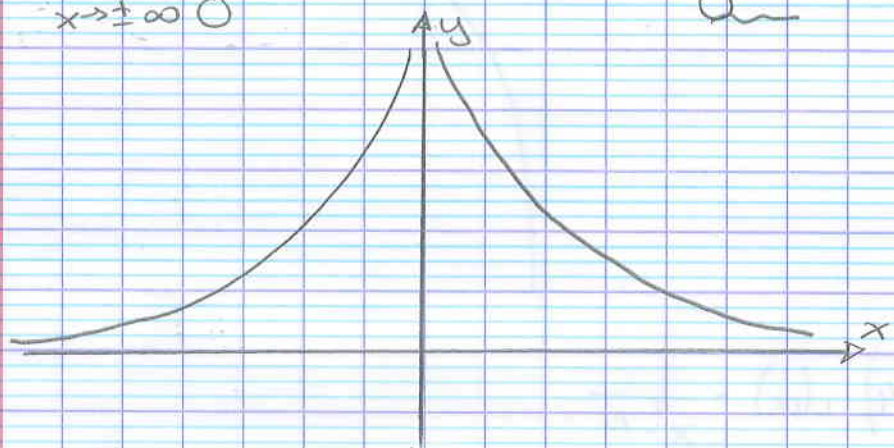
· parity $g(-x) = \frac{1}{(-x)^2} = g(x)$ even

·

x	$-\infty$	0	$+\infty$
g	$+$	//	$+$

· $\lim\limits_{x \to 0} g(x) = +\infty$ VA $x=0$

$\lim\limits_{x \to \pm\infty} g(x) = 0$ H.A $y=0$



3) $h(x) = \frac{2x-5}{3x+6}$ homographic

$D_h = \mathbb{R} \setminus \{-2\}$

· O_x: $h(x) = 0 \Leftrightarrow 2x-5=0 \Leftrightarrow x = 5/2$ $I_x\left(\frac{5}{2};0\right)$

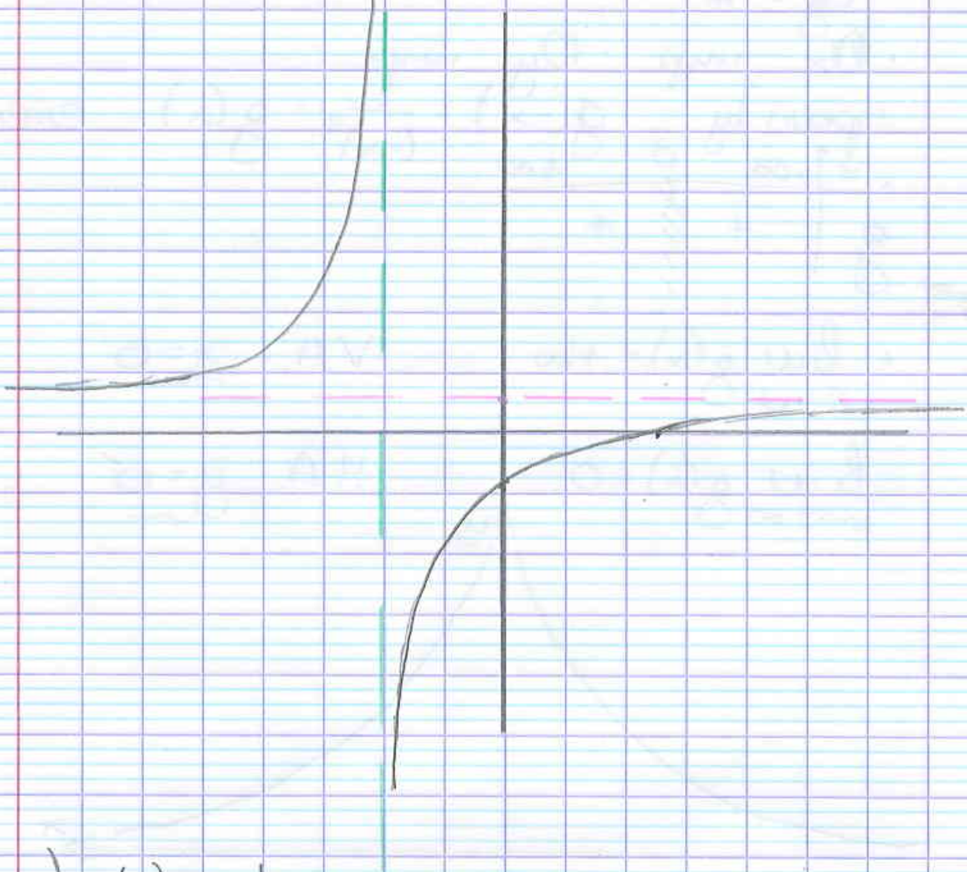
O_y: $h(0) = -\frac{5}{6}$ $I_y\left(0;-\frac{5}{6}\right)$

· parity: $h(-x) = \frac{-2x-5}{-3x+6}$ else

·

x		-2		$5/2$	$+\infty$
$2x-5$	$-$		$-$	0	$+$
$3x+6$	$-$	0	$+$		$+$
h	$+$	VA	$-$	0	$+$

• VA: $x=-2$
HA: $y=\frac{2}{3}$ $\pm\infty$



4) $i(x)=\frac{1}{x^2-4}$

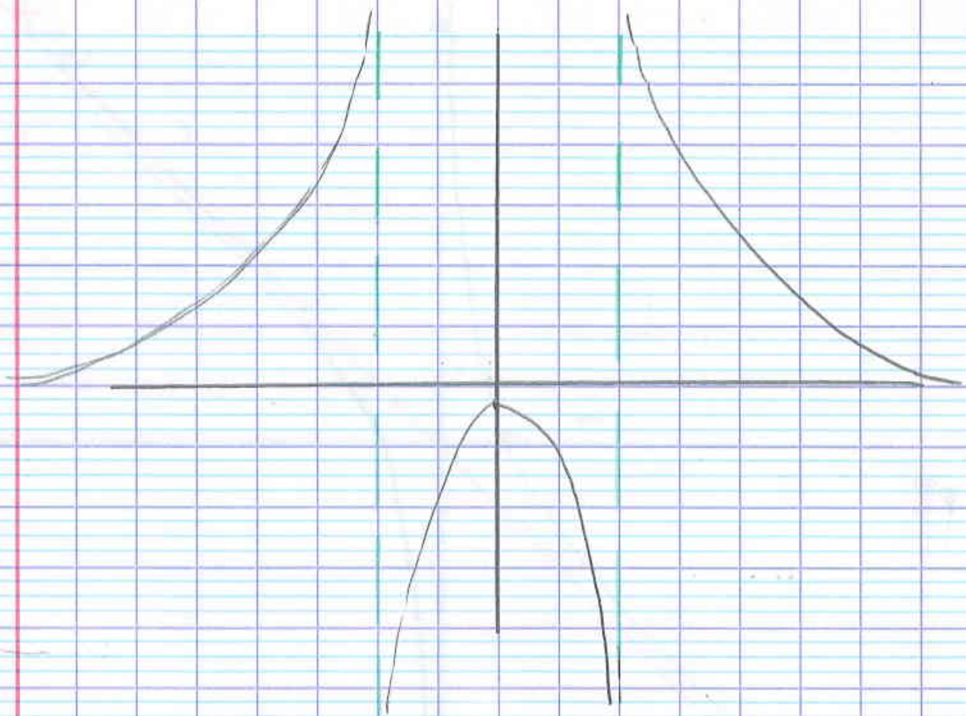
$D_i=\mathbb{R}\setminus\{\pm 2\}$

Ox: Imp. Oy $i(0)=-\frac{1}{4}$ $I_y(0;-\frac{1}{4})$

• parity $i(-x)=\frac{1}{(-x)^2-4}=\frac{1}{x^2-4}=i(x)$ even

x	$-\infty$		-2		2		$+\infty$
x^2-4		$+$	0	$-$	0	$+$	
i		$+$	$\|\|$	$-$	$\|\|$	$+$	

• VA $x=2$ $x=-2$
HA $y=0$



Exercice 1.87 1) $f(x) = x - \frac{1}{x} = \frac{x^2-1}{x}$

- $D_f = \mathbb{R}^*$
- O_x: $f(x) = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$ $\quad I_{1x}(1;0)$ $\quad I_{2x}(-1;0)$

  O_y: Imp

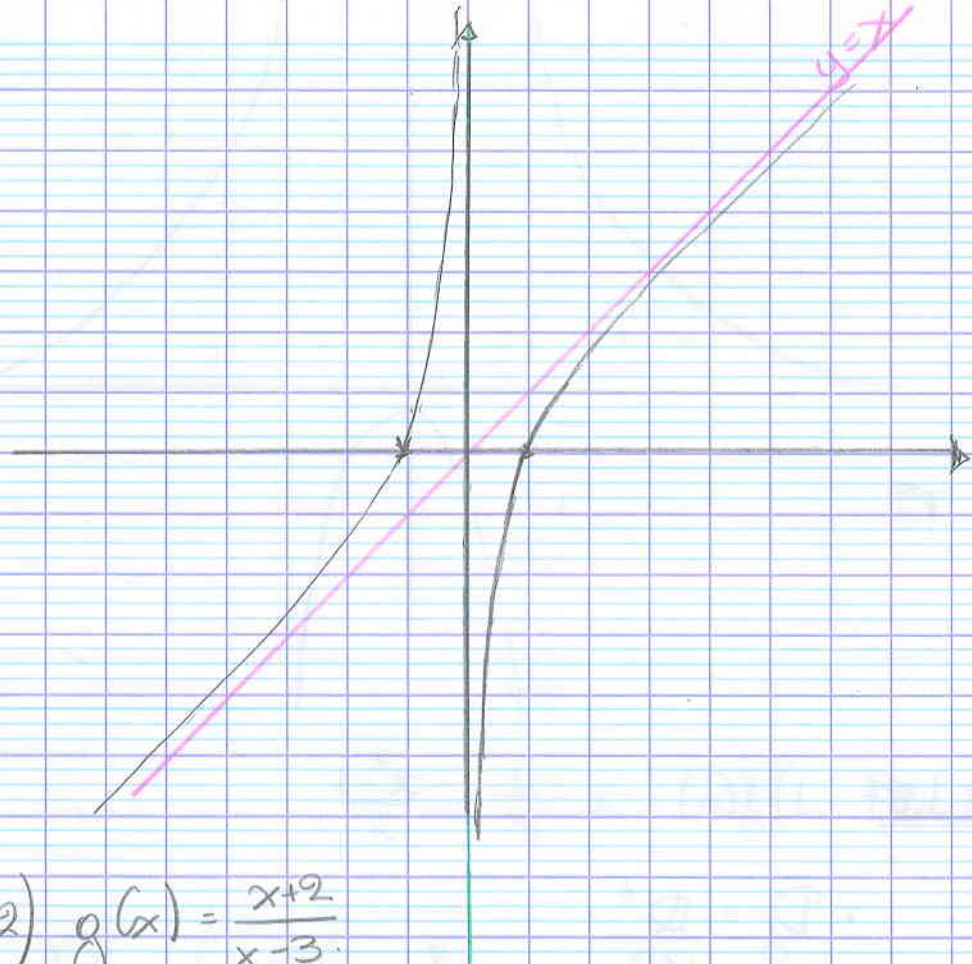
- parity: $f(-x) = \frac{(-x)^2-1}{-x} = -\frac{x^2-1}{x} = -f(x)$ odd

x	$-\infty$		-1		0		1		$+\infty$
x^2-1		$+$	0	$-$		$-$	0	$+$	
x		$-$		$-$	0	$+$		$+$	
f		$-$	0	$+$	$\|\|$	$-$	0	$+$	

- AV: $x = 0$
- AO:

$$\begin{array}{r|l} x^2 - 1 & x \\ \cline{2-2} -x^2 & x \\ \hline -1 & \end{array}$$

$y = x$



2) $g(x) = \dfrac{x+2}{x-3}$

$D_g = \mathbb{R} \setminus \{3\}$

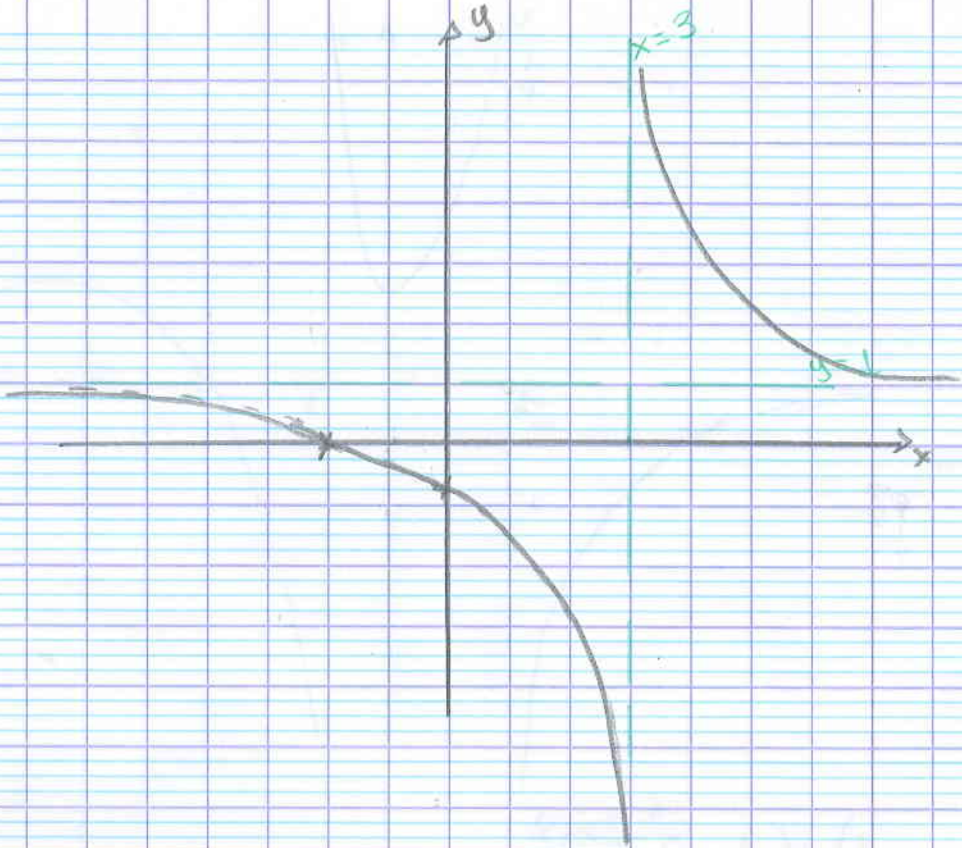
- Ox: $g(x) = 0 \Leftrightarrow x+2 = 0 \Leftrightarrow x = -2$ $\quad I_x(-2;0)$

  Oy: $g(0) = -\frac{2}{3}$ $\quad I_y(0; -\frac{2}{3})$

- parity dse

x	$-\infty$		-2		3		$+\infty$
$x+2$		$-$	0	$+$		$+$	
$x-3$		$-$		$-$	0	$+$	
g		$+$	0	$-$	0	$+$	

- VA: $x = 3$
- HA: $y = 1$



3) $h(x) = \frac{x-3}{x^2+x-2}$ $\Delta = 1+8 = 9$ $x_{1,2} = \frac{-1 \pm 3}{2} = \begin{cases} -2 \\ 1 \end{cases}$

$D_h = \mathbb{R} \setminus \{-2 ; 1\}$.

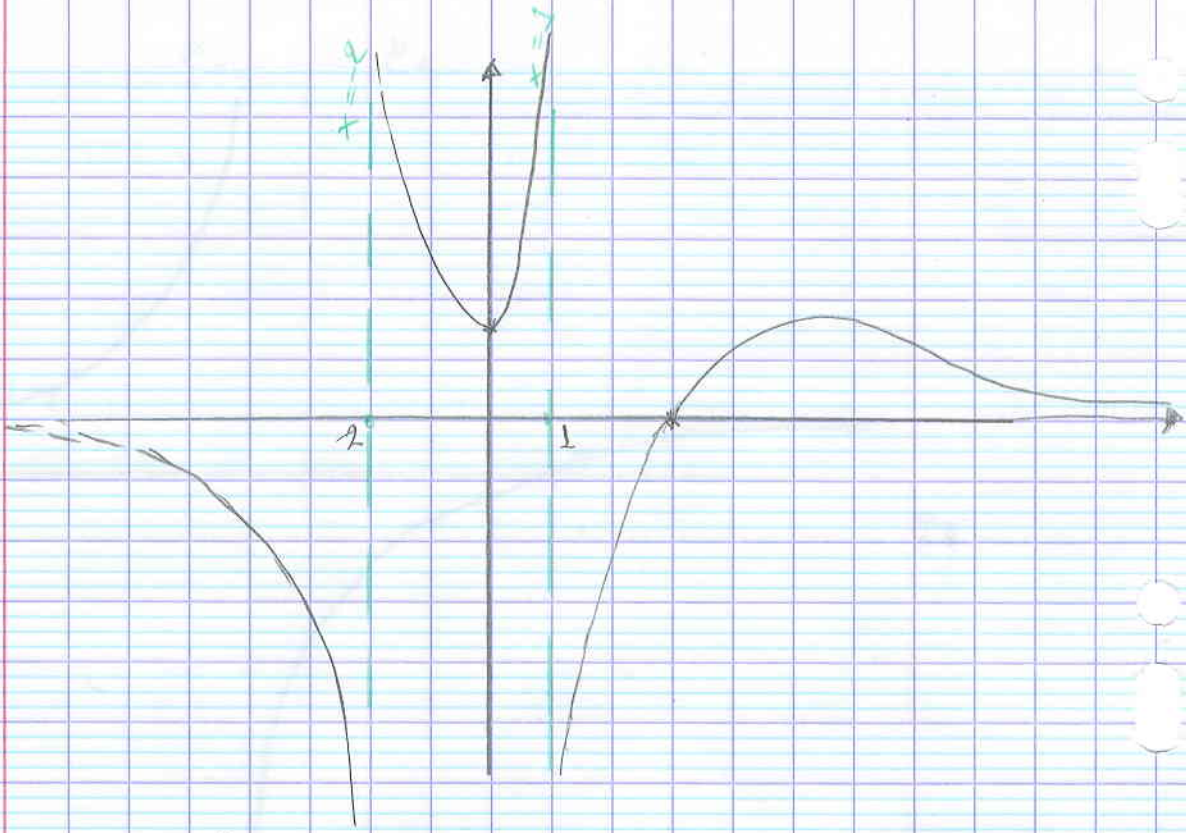
- O_x : $h(x) = 0 \Leftrightarrow x = 3$ $I_x(3;0)$
  O_y : $h(0) = +\frac{3}{2}$ $I_y(0;\frac{3}{2})$

- parity: else

-

x	$-\infty$		-2		1		3		$+\infty$
$x-3$		$-$		$-$		$-$	0	$+$	
x^2+x-2		$+$	0	$-$	0	$+$		$+$	
h		$-$	$\|\|$	$+$	$\|\|$	$-$	0	$+$	

- VA $x = 1$ $x = -2$
  HA $y = 0$



4) $i(x) = \dfrac{x^2-3x+2}{x+1}$

$D_i = \mathbb{R} \setminus \{-1\}$

O_x $\quad i(x)=0 \Leftrightarrow x^2-3x+2=0$ $\quad x_1 = 2 \quad I_{1x}(2;0)$; $\quad x_2 = 1 \quad I_{2x}(1;0)$

O_y $\quad i(0) = 2 \quad I_y(0;2)$

• parity else

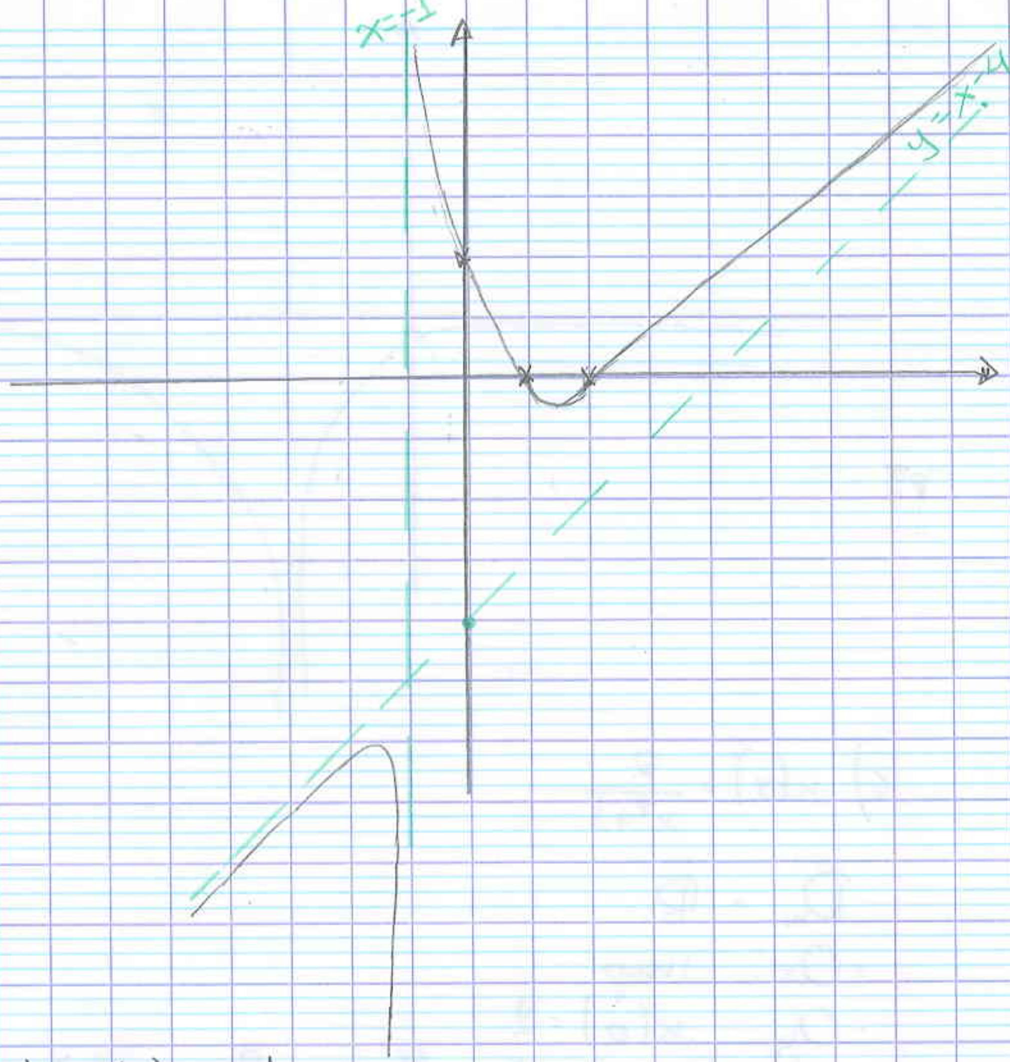
x	$-\infty$		-1		1		2		$+\infty$
x^2-3x+2		+		+	0	−	0	+	
$x+1$		−	0	+		+		+	
i		−	‖	+	0	−	0	+	

• VA $\quad x=-1$

• AO

$$\begin{array}{r|l} \cancel{x^2}-3x+2 & x+1 \\ -\cancel{x^2}-x & \overline{x-4} \\ \hline -\cancel{4x}+2 & \\ +\cancel{4x}+4 & \\ \hline 6 & \end{array}$$

$\underline{y = x-4}$ AO



5) $f(x) = \dfrac{-1}{(x-2)^2}$

$D_f = \mathbb{R} \setminus \{2\}$

• O_x: $f(x) = 0$ imp

O_y $f(0) = -\frac{1}{4}$ $\quad I_y\left(0; -\frac{1}{4}\right)$

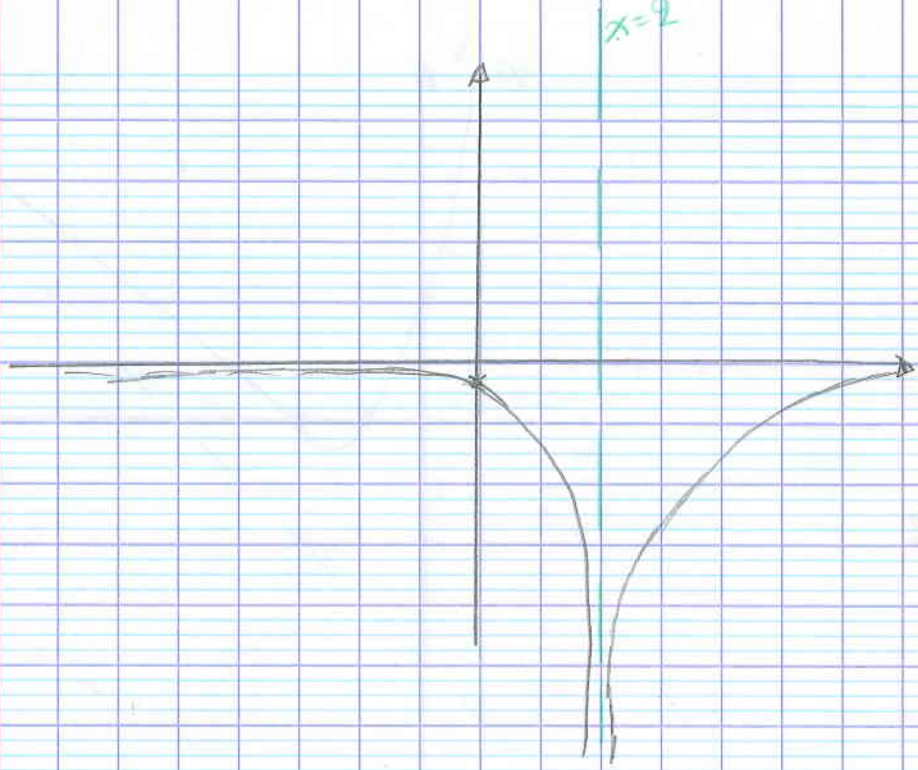
• parity: else

•

x	$-\infty$		2		$+\infty$
f		$-$	//	$-$	

• VA: $x = 2$

HA $y = 0$



6) $k(x) = \frac{2}{x^2+1}$

$D_k = \mathbb{R}$

· O_x : imp

· O_y $k(0) = 2$

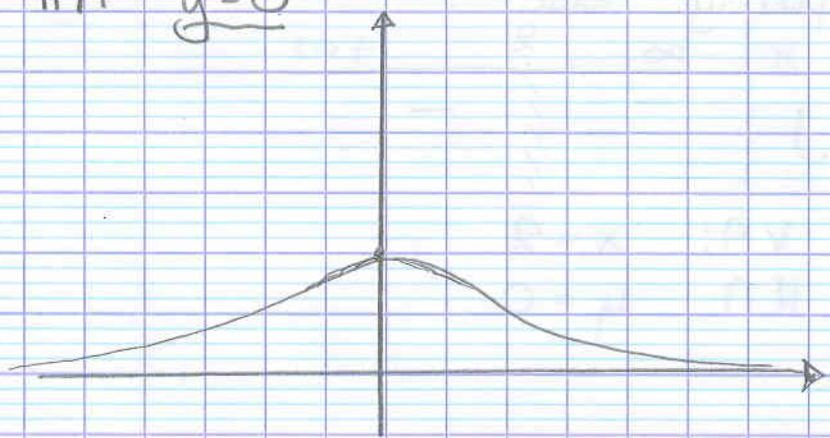
· parity $k(-x) = \frac{2}{(-x)^2+1} = \frac{2}{x^2+1} = k(x)$ even

·

x	$-\infty$		$+\infty$
k		$+$	

· AV $\nexists$

HA $y=0$



Exercice 1.88

$x^2+x-2=0$

$\Delta = 1+8 = 9$

$x_{1,2} = \frac{-1\pm 3}{2} \begin{cases} -2 \\ 1 \end{cases}$

$x \neq -2 \quad x \neq 1$

$y = f(x) \Leftrightarrow k = \frac{x-3}{x^2+x-2} \Leftrightarrow$

$\Leftrightarrow kx^2+kx-2k = x-3 \Leftrightarrow$

$kx^2+(k-1)x-2k+3=0$

$\Delta = 0 \Leftrightarrow (k-1)^2-4\cdot k(-2k+3)=0 \Leftrightarrow$

$\Leftrightarrow k^2-2k+1+8k^2-12k=0 \Leftrightarrow$

$\Leftrightarrow 9k^2-14k+1=0$

$\Delta' = 196-36=160 \quad k_{1,2} = \frac{14\pm\sqrt{160}}{18} =$

$= \frac{14\pm 4\sqrt{10}}{18} = \begin{cases} \frac{7+2\sqrt{10}}{9} \\ \frac{7-2\sqrt{10}}{9} \end{cases}$

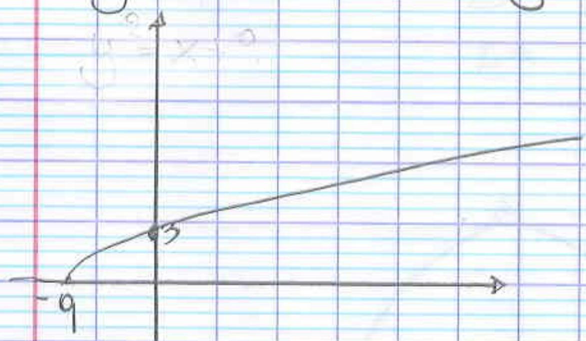
$k_1 = \frac{7+2\sqrt{10}}{9} \quad k_2 = \frac{7-2\sqrt{10}}{9}$

Exercice 1.89

1) $f(x) = \sqrt{x+9} \qquad x+9 \geq 0 \Leftrightarrow x \geq -9$

$D_f = [-9;+\infty[$

$y = \sqrt{x+9} \geq 0 \Rightarrow y \geq 0 \qquad R_f = [0,+\infty[$

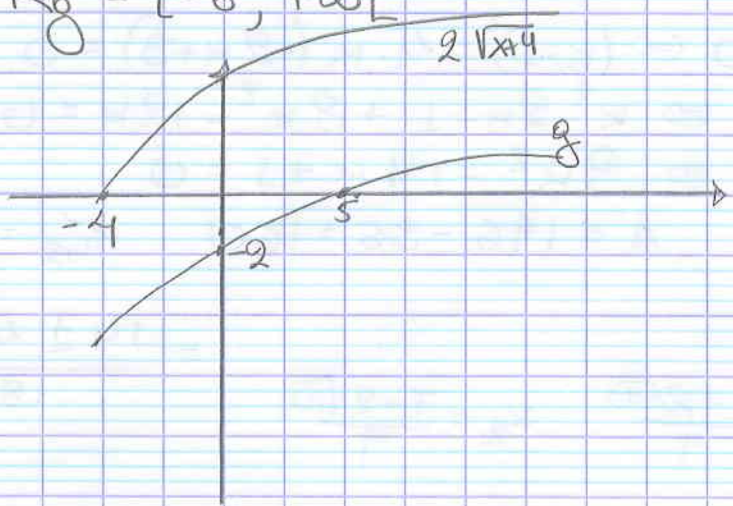


2) $g(x) = 2\sqrt{x+4} - 6$ $\qquad x+4 \geq 0 \Leftrightarrow x \geq -4$

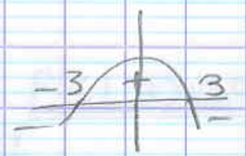
$D_g = [-4 ; +\infty[$

$y = 2\sqrt{x+4} - 6 \Leftrightarrow y+6 = 2\sqrt{x+4} \geq 0 \Leftrightarrow y \geq -6$

$R_g = [-6 ; +\infty[$

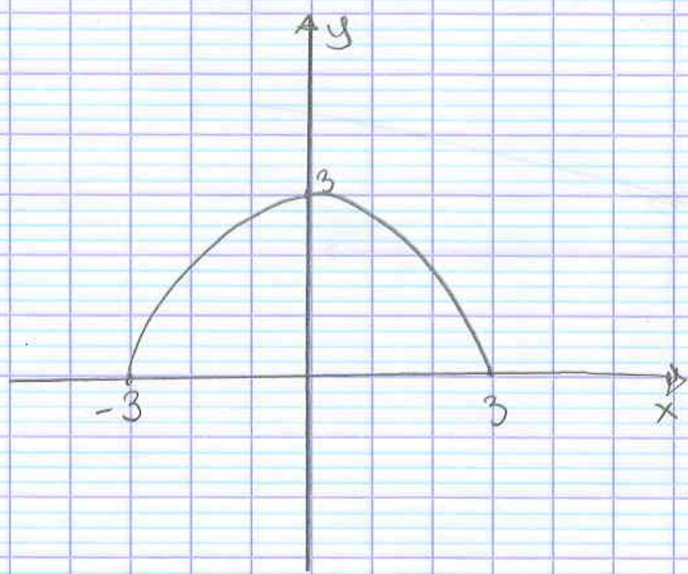


3) $h(x) = \sqrt{9-x^2}$ $\qquad 9-x^2 \geq 0$



$D_h = [-3 ; 3]$

$y = \sqrt{9-x^2} \geq 0 \Rightarrow y \geq 0 \qquad R_h = [0, +\infty[$



Exercice 1.90 $f(x) = \dfrac{x+b}{cx-3}$ $\quad P(4;-2)$ $\quad$ VA $x=6$

VA $x=6$: $c6-3=0 \Leftrightarrow \underline{c=\frac{1}{2}}$

$$f(x) = \frac{x+b}{\frac{x}{2}-3}$$

$$f(4) = -2 \Leftrightarrow \frac{4+b}{\frac{4}{2}-3} = -2 \Leftrightarrow \frac{4+b}{-1} = -2 \Leftrightarrow 4+b=2 \Leftrightarrow$$

$$\underline{b=-2}$$

Exercice 1.91 : $f(x) = \dfrac{2x+7}{x+3}$ $\quad g(x) = -x+\dfrac{3}{2}$

1) $D_f = \mathbb{R} \setminus \{-3\}$

$$\begin{array}{r|l} 2x+7 & x+3 \\ \cline{2-2} -2x-6 & 2 \\ \hline 1 & \end{array} \qquad f(x) = 2 + \frac{1}{x+3}$$

2) $f \cap Ox$: $f(x)=0 \Leftrightarrow x=-\frac{7}{2}$ $\quad I_{fx}\left(-\frac{7}{2};0\right)$

$f \cap Oy$: $f(0)=\frac{7}{3}$ $\quad I_{fy}\left(0;\frac{7}{3}\right)$

VA : $\underline{x=-3}$ $\quad$ HA : $\underline{y=2}$

$\underline{g}$ $D_g = \mathbb{R}$

$g \cap Ox$: $g(x)=0 \Leftrightarrow x=\frac{3}{2}$ $\quad I_{gx}\left(\frac{3}{2};0\right)$

$g \cap Oy$: $g(0)=\frac{3}{2}$ $\quad I_{gy}\left(0;\frac{3}{2}\right)$

VA $\nexists$ $\quad$ HA $\nexists$

$f(x) = g(x) \Leftrightarrow \frac{2x+7}{x+3} = -x + \frac{3}{2} \Leftrightarrow$

$\Leftrightarrow 2(2x+7) = -2x(x+3) + 3(x+3) \Leftrightarrow$

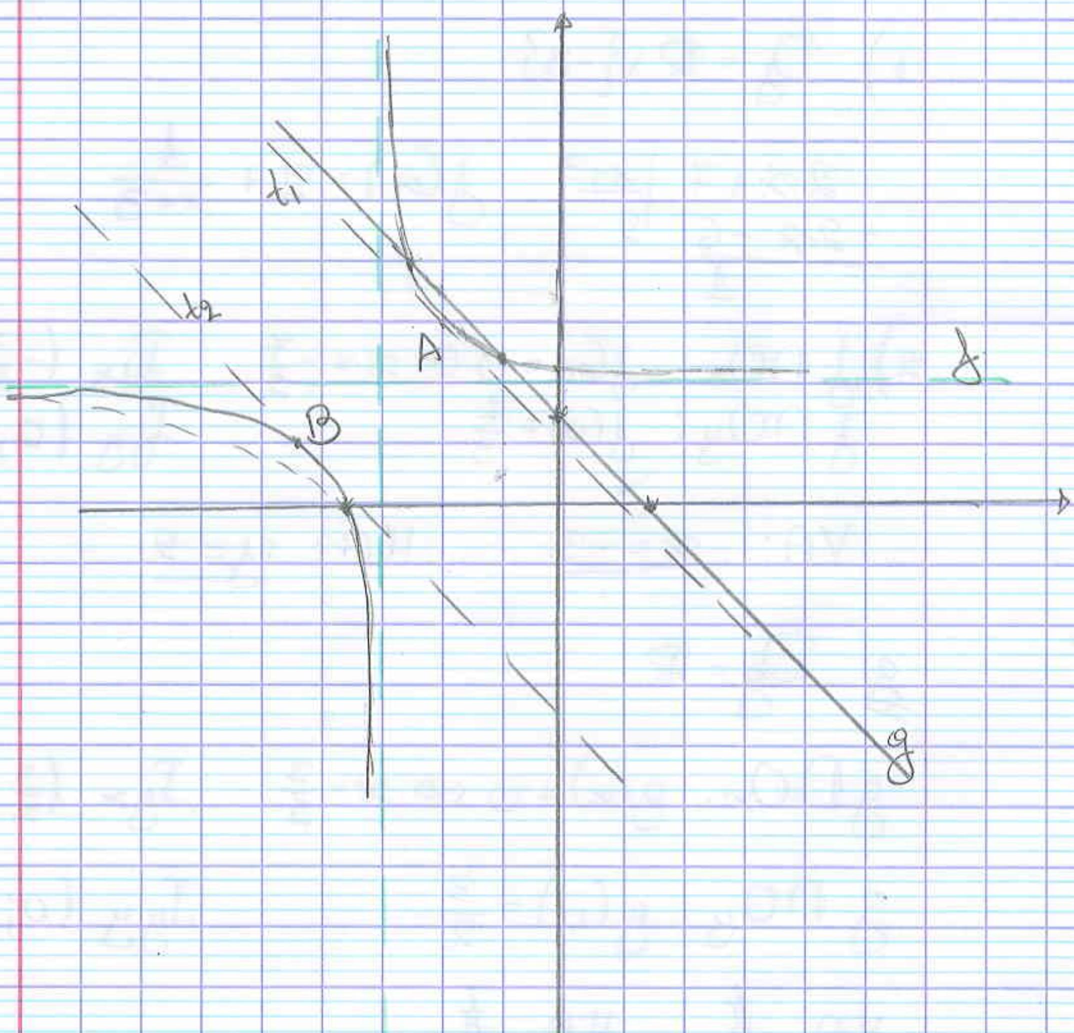
$\Leftrightarrow 4x + 14 = -2x^2 - 6x + 3x + 9 \Leftrightarrow$

$\Leftrightarrow 2x^2 + 7x + 5 = 0 \qquad \Delta = 49 - 40 = 9$

$x_{1,2} = \frac{-7 \pm 3}{4} = \begin{cases} -1 \\ -\frac{5}{2} \end{cases}$

$x = -1 \quad y = \frac{1}{2} \quad I_1\left(-1; \frac{1}{2}\right)$

$x = -\frac{5}{2} \quad y = \frac{5}{2} + \frac{3}{2} = 4 \quad I_2\left(-\frac{5}{2}; 4\right)$



$t: \begin{cases} y = -x + h \\ y = \dfrac{2x+7}{x+3} \end{cases}$ because $t // g$

$-x + h = \dfrac{2x+7}{x+3} \Leftrightarrow -x^2 - 3x + hx + 3h = 2x + 7 \Leftrightarrow$

$\Leftrightarrow -x^2 - 5x + hx + 3h - 7 = 0$

$\Leftrightarrow x^2 + 5x - hx - 3h + 7 = 0$

$x^2 + (5-h)x - 3h + 7 = 0 \quad (1)$

$\Delta = 0 \Leftrightarrow (5-h)^2 - 4(-3h+7) = 0 \Leftrightarrow$

$\Leftrightarrow 25 - 10h + h^2 + 12h - 28 = 0$

$h^2 + 2h - 3 = 0 \qquad \Delta = 4 + 12 = 16$

$h_{1,2} = \dfrac{-2 \pm 4}{2} = \begin{cases} -3 \\ 1 \end{cases} \qquad h_1 = -3 \quad h_2 = 1$

$t_1 : y = -x + 1$

$t_2 : y = -x - 3$

$t_1 \cap g$: $h = 1$. the eq (1) gives

$x^2 + 4x + 4 = 0 \Leftrightarrow (x+2)^2 = 0 \Leftrightarrow x = -2$

$x = -2 \quad y = 3 \quad A(-2 ; 3)$

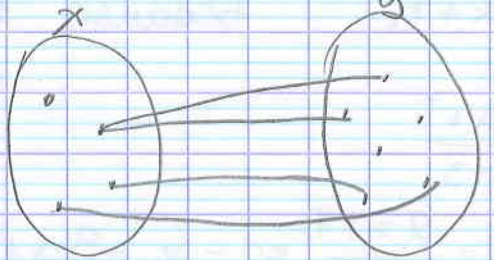
$h = -3 \quad (1) : x^2 + 8x + 16 = 0 \Leftrightarrow$

$(x+4)^2 = 0 \Leftrightarrow x = -4$

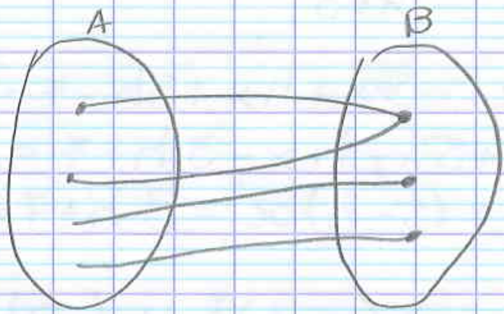
$x = -4 \quad y = 1 \quad B(-4 ; 1)$

EXERCICE 1.92

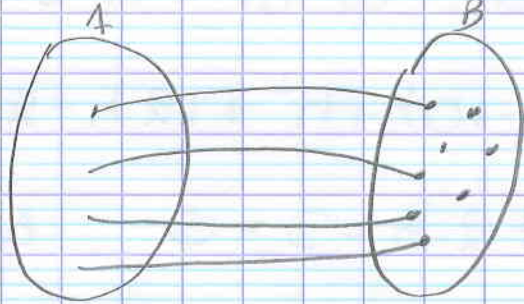
1)



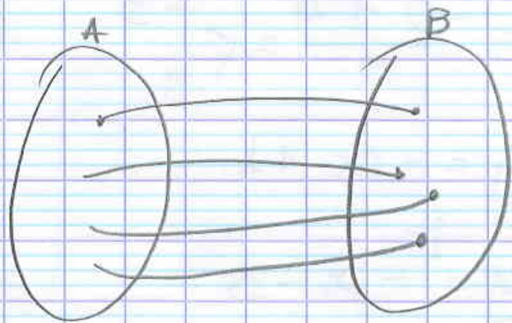
2)



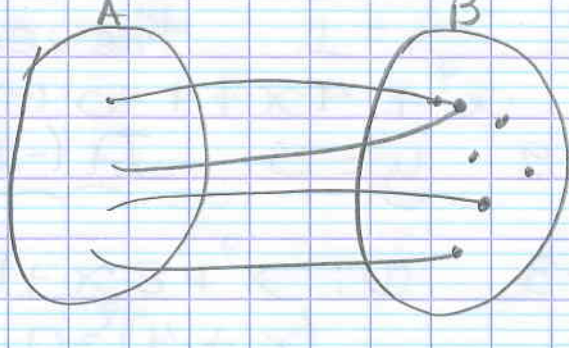
3)



4)

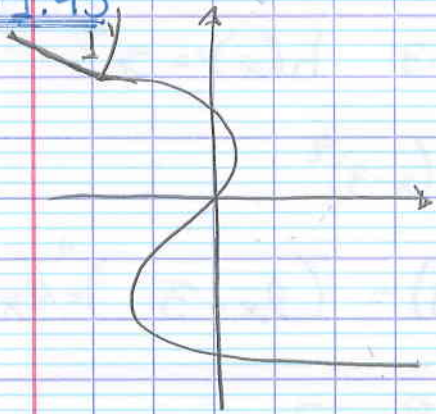


5)

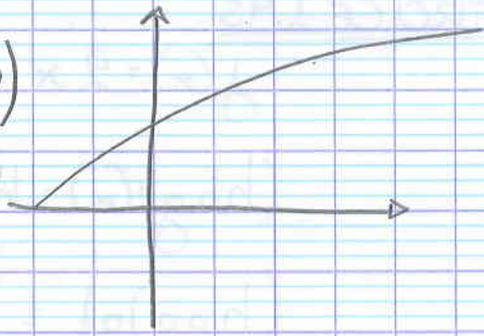


EXERCICE 1.93

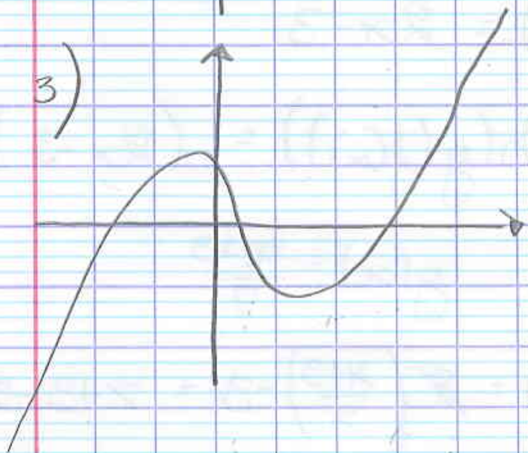
1)



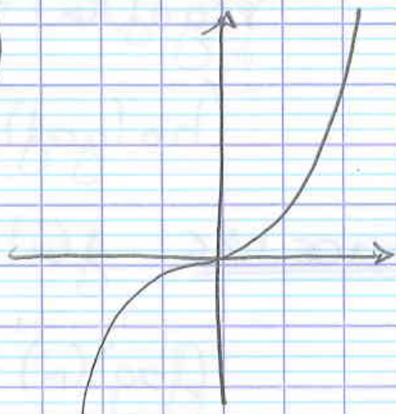
2)



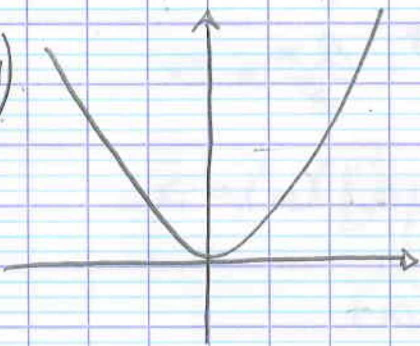
3)



4)



4)



EXERCICE 1.94

$f(x) = x^2 \quad g(x) = x+1$

$(f \circ g)(x) = f(g(x)) = (x+1)^2$

$(g \circ f)(x) = g(f(x)) = x^2+1$

EXERCICE 1.95

$f(x) = 2x \quad g(x) = x-3 \quad h(x) = x^2$

$(h \circ g)(x) = h(g(x)) = (x-3)^2$

$(h \circ g) \circ f = h(g(f(x))) = (2x-3)^2 = 4x^2 - 12x + 9$

$(g \circ f)(x) = g(f(x)) = 2x-3$

$(h \circ (g \circ f))(x) = h(g(f(x))) = (2x-3)^2 = 4x^2 - 12x + 9$

EXERCICE 1.96

$f(x) = 2x-3 \qquad g(x) = \frac{x+3}{2}$

$(f \circ g)(x) = f(g(x)) = \cancel{2}\left(\frac{x+3}{\cancel{2}}\right) - 3 = x+3-3 = x$

$g(f(x)) = \frac{2x \cancel{-3} \cancel{+3}}{2} = \frac{\cancel{2}x}{\cancel{2}} = x$

$(f \circ g)(x) = x \quad (g \circ f)(x) = x$

$\Longrightarrow f^{-1}(x) = g(x)$

EXERCICE 1.97

$f(x) = \frac{x+1}{2x-3} \qquad D_f = \mathbb{R} \setminus \{3/2\}$

$R_f = \mathbb{R} \setminus \{1/2\}$

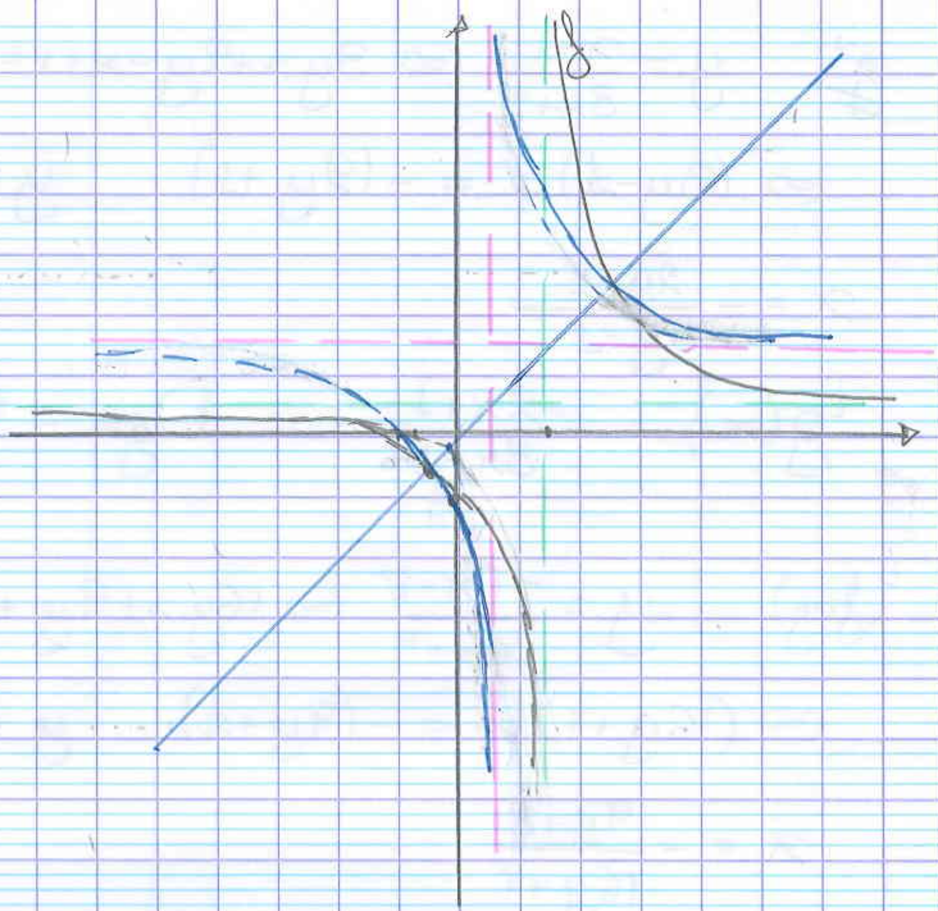
$y = \frac{x+1}{2x-3} \Leftrightarrow 2yx - 3y - x - 1 = 0 \Leftrightarrow$

$\Leftrightarrow (2y-1)x = 3y+1 \qquad y \neq 1/2$

$x = \frac{3y+1}{2y-1}$

$f^{-1}(x) = \frac{3x+1}{2x-1} \qquad D_{f^{-1}} = \mathbb{R} \setminus \{1/2\}$

$R_{f^{-1}} = \mathbb{R} \setminus \{3/2\}$



EXERCICE 1.98 $f(x) = \frac{2x-3}{x+5}$ $g(x) = \frac{x-1}{3x+2}$

$D_f = \mathbb{R} \setminus \{-5\}$ $D_g = \mathbb{R} \setminus \{-\frac{2}{3}\}$

$$(f \circ g)(x) = \frac{2 \cdot \left(\frac{x-1}{3x+2}\right) - 3}{\frac{x-1}{3x+2} + 5} = \frac{\frac{2x-2-9x-6}{\cancel{3x+2}}}{\frac{x-1+15x+10}{\cancel{3x+2}}} = \frac{-7x-8}{16x+9}$$

$\underline{y}$ $y = \frac{2x-3}{x+5} \Leftrightarrow yx + 5y = 2x - 3 \Leftrightarrow$

$\Leftrightarrow yx - 2x = -5y - 3 \Leftrightarrow$

$(y-2)x = -(5y+3)$ $y \neq 2$

$x = -\frac{5y+3}{y-2}$ $f^{-1}(x) = -\frac{5x+3}{x-2}$ $D_{f^{-1}} = \mathbb{R} \setminus \{2\}$

g^{-1}: $y = \dfrac{x-1}{3x+2} \Leftrightarrow 3yx + 2y - x + 1 = 0 \Leftrightarrow$

$\Leftrightarrow (3y-1)x = -(2y+1) \qquad y \neq 1/3$

$x = -\dfrac{2y+1}{3y-1}$

$g^{-1}(x) = -\dfrac{2x+1}{3x-1} \qquad D_{g^{-1}} = \mathbb{R} \setminus \{1/3\}$

$(f \circ g)^{-1}$ $y = \dfrac{-7x-8}{16x+9} \Leftrightarrow 16yx + 9y + 7x + 8 = 0$

$\Leftrightarrow (16y+7)x = -(9y+8) \qquad y \neq -\dfrac{7}{16}$

$x = -\dfrac{9y+8}{16y+7}$

$(f \circ g)^{-1}(x) = -\dfrac{9x+8}{16x+7}$

$g^{-1} \circ f^{-1}(x) = -\dfrac{2\left(-\dfrac{5x+3}{x-2}\right)+1}{3\left(-\dfrac{5x+3}{x-2}-1\right)} = -\dfrac{-\dfrac{10x+6}{x-2}+1}{-\dfrac{15x+9}{x-2}-1} =$

$= -\dfrac{\dfrac{-10x-6+x-2}{\cancel{x-2}}}{\dfrac{-15x-9-x+2}{\cancel{x-2}}} = -\dfrac{-9x-8}{-16x-7} =$

$= -\dfrac{9x+8}{16x+7}$

! $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

EXERCICE 1.99

$f(x) = \sqrt{1-x}$ $\qquad 1-x \geq 0 \Leftrightarrow 1 \geq x$

$D_f =]-\infty, 1]$

$y = \sqrt{1-x} \geq 0 \Leftrightarrow y \geq 0$

$R_f = [0, +\infty[$

$y = \sqrt{1-x} \Leftrightarrow y^2 = 1-x \Leftrightarrow x = 1-y^2$

$f^{-1}(x) = 1-x^2 \qquad D_{f^{-1}} = R_f = [0, +\infty[$

EXERCICE 1.100

$f(x) = \dfrac{ax+b}{cx+d}$

$y = \dfrac{ax+b}{cx+d} \Leftrightarrow cyx + yd = ax+b \Leftrightarrow$

$\Leftrightarrow (cy-a)x = b-yd$

$x = \dfrac{b-yd}{cy-a} \qquad f^{-1}(x) = \dfrac{-xd+b}{cx-a}$

$f(x) = f^{-1}(x) \Leftrightarrow \dfrac{ax+b}{cx+d} = \dfrac{-xd+b}{cx-a}$

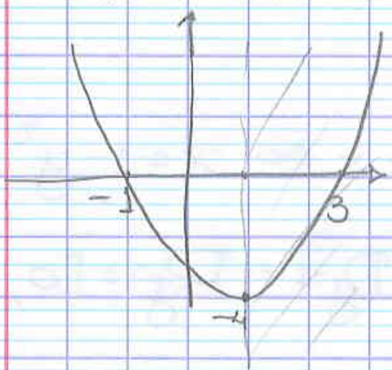
$\boxed{a = -d}$

Exercice 1.101 $f(x) = x^2 - 2x - 3$

Vertex $(1; -4)$

$x_v = -\frac{-2}{2} = 1 \qquad y_v = f(1) = -4$

$x^2 - 2x - 3 = 0 \qquad \Delta = 4 + 12 = 16 \qquad x_{1,2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \end{cases}$



Domain $= [1, +\infty[$

$R = [-4, +\infty[$

In this case: $y = x^2 - 2x - 3 \Leftrightarrow$

$\Leftrightarrow x^2 - 2x - 3 - y = 0$

$\Delta = 4 + 4(3+y) = 16 + 4y$

$x_{1,2} = \frac{2 \pm \sqrt{16+4y}}{2}$

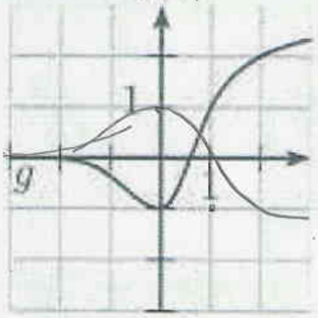
As $x \geq 1 \qquad x = \frac{2 + \sqrt{16+4y}}{2} = \frac{2 + 2\sqrt{4+y}}{2} =$

$= 1 + \sqrt{4+y}$

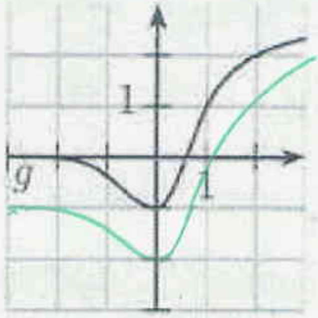
So $\quad f^{-1}(x) = 1 + \sqrt{4+x} \qquad D_{f^{-1}} = [-4; +\infty[$

1.102 : From the graph of the function g, sketch the graph of :

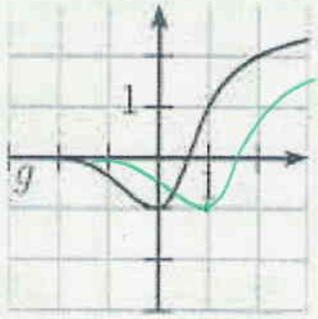
$-g(x)$



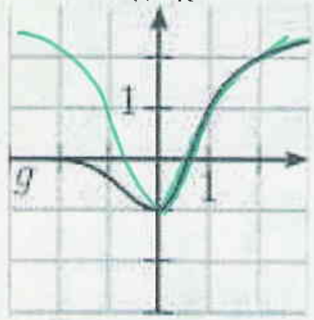
$g(x)-1$



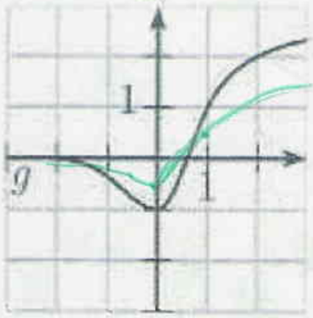
$g(x-1)$



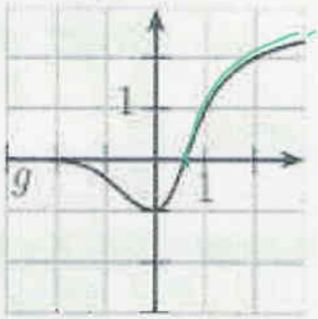
$g(|x|)$



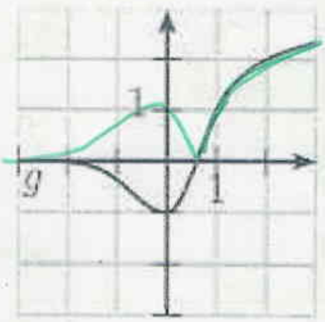
$\frac{g(x)}{2}$



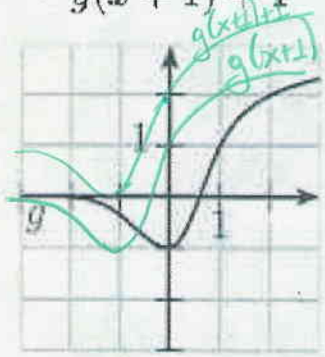
$\sqrt{g(x)}$



$|g(x)|$



$-g(x+1)+1$



EXERCICE 1.103

$f(x) = 10^x$

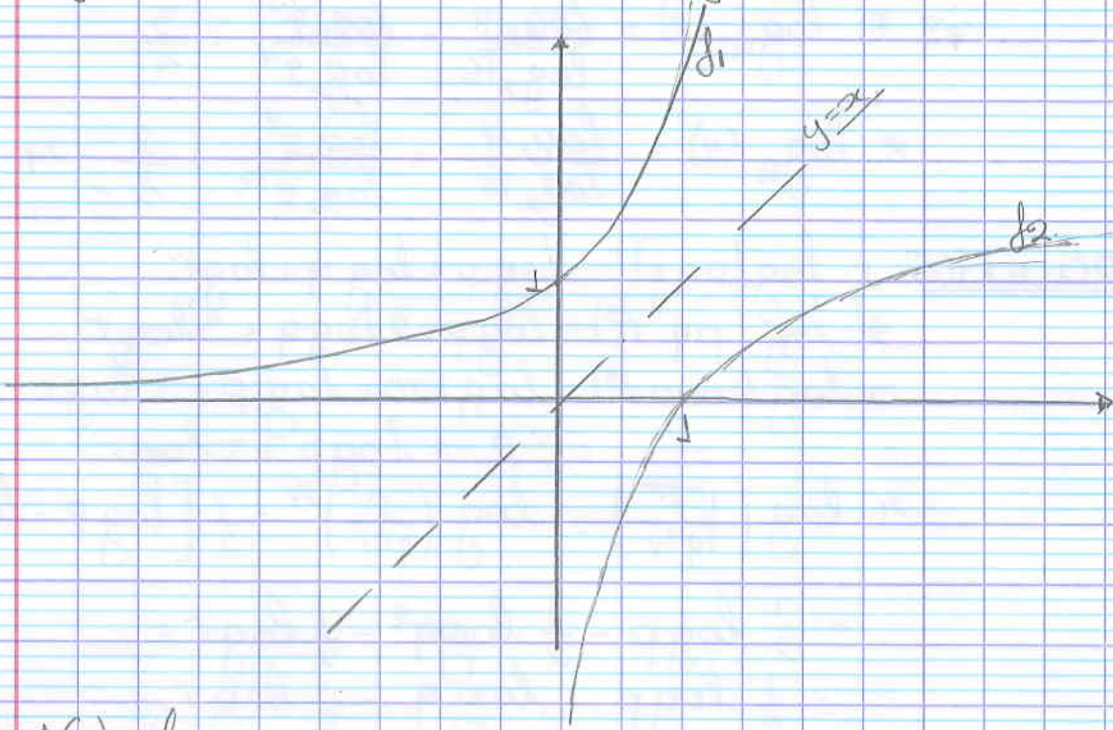
$D_f = \mathbb{R}$ $\quad R_f = (0, +\infty)$

signe: $f(x) > 0 \quad \forall x \in \mathbb{R}$

VA: $\emptyset$ $\quad$ HA: $y = 0$ pour $x \to -\infty$

$\cap Ox \quad \emptyset$ $\quad \cap Oy: \ y = 1. \quad (0; 1)$

f est croissante dans D_f



$f(x) = \log x$

$D_f =]0, +\infty[$ $\quad R_f = \mathbb{R}$

signe

x	0		1		$+\infty$
f	|	$-$	0	$+$	

VA: $x = 0$ $\quad \lim_{x \to 0} f(x) = -\infty$ $\quad$ HA $\emptyset$

$\cap Ox: \ x = 1 \quad (1; 0)$ $\quad \cap Oy: \ \emptyset$

f est croissante dans D_f

EXERCICE 1.104

1. $\log_2(16) = \log_2 2^4 = 4$
2. $\log_7\left(\frac{1}{49}\right) = \log_7 7^{-2} = -2$
3. $\log_4(1) = 0$
4. $\log_3(-9)$ impossible
5. $\log_5(5) = 1$
6. $\log_{27}\left(\frac{1}{3}\right) = \dfrac{\log_3 3^{-1}}{\log_3 3^3} = \dfrac{-1}{3}$
7. $\log_{16}(8) = \dfrac{\log_2 8}{\log_2 16} = \dfrac{\log_2 2^3}{\log_2 2^4} = \dfrac{3}{4}$
8. $\log_{\sqrt{2}}(4) = \dfrac{\log_2 4}{\log_2 \sqrt{2}} = \dfrac{\log_2 2^2}{\log_2 2^{1/2}} = \dfrac{2}{\frac{1}{2}} = 4$

EXERCICE 1.105

1. $\log(pqr) = \log p + \log q + \log r$
2. $\log(pq^2r^3) = \log p + 2\log q + 3\log r$
3. $\log(100pr^5) = \log 100 + \log p + 5\log r =$
   $= 2 + \log p + 5\log r$
4. $\log\left(\sqrt{\frac{p}{q^2 r}}\right) = \log\left(\frac{p}{q^2 r}\right)^{1/2} = \frac{1}{2}\left[\log p - \log(q^2 r)\right] =$
   $= \frac{1}{2}\log p - \frac{1}{2}\log q^2 - \frac{1}{2}\log r =$
   $= \frac{1}{2}\log p - \log q - \frac{1}{2}\log r$
5. $\log\left(\frac{qr^7p}{10}\right) = \log q + 7\log r + \log p - 1$
6. $\log\left(\sqrt{\frac{10p^{10}r}{q}}\right) = \frac{1}{2}\log\frac{10p^{10}r}{q} = \frac{1}{2}\left[\log 10p^{10}r - \log q\right]$
   $= \frac{1}{2}\left[\log 10 + 10\log p + \log r - \log q\right] =$
   $= \frac{1}{2} + 5\log p + \frac{1}{2}\log r - \frac{1}{2}\log q$

EXERCICE 1.106

1. $\log\left((a^5\sqrt[3]{b})/c\right) = \log(a^5\sqrt[3]{b}) - \log c =$
$= 5\log a + \frac{1}{3}\log b - \log c$

2. $\log(x+1) - \log(x-1) = \log\frac{x+1}{x-1}$

3. $2\log a - \frac{1}{2}\log b + 3\log c =$
$= \log a^2 - \log\sqrt{b} + \log c^3 = \log\frac{a^2\cdot c^3}{\sqrt{b}}$

4. $\log\left(\sqrt{a\cdot\sqrt[3]{b}}\right) = \frac{1}{2}\log(a\sqrt[3]{b}) = \frac{1}{2}\log a + \frac{1}{2}\cdot\frac{1}{3}\log b =$
$= \frac{1}{2}\log a + \frac{1}{6}\log b.$

EXERCICE 1.107 On pose $n = 87^{97} \Rightarrow \log_{87}(n) = 97$

$\Rightarrow \log_{87}(n) = \frac{\log(n)}{\log 87} = 97 \Rightarrow \log(n) = \log(87)\cdot 97$

$\Rightarrow n = 10^{97\cdot\log(87)}$

On a : $\log 87 = 1,939519253$ et $\log(87)\cdot 97 = 188,1333675$

Ainsi $n = 10^{188,1333675} \Rightarrow n = 87^{97}$ a 188 chiffres

De plus $n = 10^{188+0.1333675} = 10^{188}\cdot 10^{0.1333675} =$
$= 10^{188}\cdot 1.359946335$

Donc les 5 premier chiffres de $n = 87^{97}$ sont 13594

EXERCICE 1.108

1. $10^x = 9.56 \Leftrightarrow x = \log 9.56$
2. $10^{3x} = -5$ impossible

$x > 0$

3. $\log(x) = 2.5 \Leftrightarrow x = 10^{2.5}$

$4x-1>0 \Leftrightarrow x > 1/4$

4. $\log(4x-1) = -2 \Leftrightarrow 4x-1 = 10^{-2} \Leftrightarrow 4x = 1+10^{-2} \Leftrightarrow$
$\Leftrightarrow x = \frac{1+0.01}{4} \Leftrightarrow x = \frac{1.01}{4} > \frac{1}{4}$

$x > 0$

5. $2\log x = -4 \Leftrightarrow \log x = -2 \Leftrightarrow x = 10^{-2} = 0.01$

$x^2-21>0 \Leftrightarrow$
$x \in]-\infty, -\sqrt{21}[\cup]\sqrt{21}, +\infty[$

6. $\log(x^2-21) = 2 \Leftrightarrow x^2-21 = 10^2 \Leftrightarrow x^2 = 121 \Leftrightarrow x = \pm 11$ ✓

8. $\log(\log(x)) = 1 \Leftrightarrow \log(x) = 10^1 = 10 \Leftrightarrow$
$\Leftrightarrow x = 10^{10} > 1$ ✓

$\left.\begin{array}{l} x>0 \\ \log x > 0 \Rightarrow x > 10^0 = 1 \end{array}\right\} x > 1$

7. $10^{3x+2} = \sqrt{10} \Leftrightarrow 3x+2 = \log 10^{1/2} \Leftrightarrow 3x+2 = \frac{1}{2}$
$\Leftrightarrow 3x = -\frac{3}{2} \Leftrightarrow x = -\frac{1}{2}$

9. $2\cdot 2^{2x} - 9\cdot 2^x + 4 = 0$. On pose $t = 2^x > 0$
$2\cdot t^2 - 9\cdot t + 4 = 0 \quad \Delta = 81 - 32 = 49$
$t_{1,2} = \frac{9 \pm 7}{4} = \begin{cases} t_1 = 4 \\ t_2 = \frac{2}{4} = \frac{1}{2} \end{cases}$
$t_1 = 4 = 2^x \Rightarrow x = \log_2 4 \Rightarrow x = \log_2 2^2 \Leftrightarrow x_1 = 2$
$t_2 = \frac{1}{2} = 2^{-1} \Leftrightarrow 2^x = 2^{-1} \Leftrightarrow x_2 = -1$

10. $2^x = 10 \Leftrightarrow x = \log_2 10 = \log_2 (2\cdot 5) \Leftrightarrow$
$x = 1 + \log_2 5$

11. $1.03^x = 2 \Leftrightarrow x = \log_{1.03} 2 = \frac{\log 2}{\log 1.03} = 23.45$

$0.5x - 3 > 0$
$\Leftrightarrow 0.5x > 3 \Leftrightarrow$
$x > 6$

12. $\log(0.5x-3) = -1 \Leftrightarrow 0.5x - 3 = 10^{-1} \Leftrightarrow$
$\Leftrightarrow \frac{1}{2}x = 0.1 + 3 \Leftrightarrow x = 6.2 > 6$ ✓

13. $2.8^x = 5 \Leftrightarrow x = \log_{2.8} 5 = \frac{\log 5}{\log 2.8} = 1.56$

$x > 0$

14. $2\log(2x) = 9 \Leftrightarrow \log 2x = 4.5 \Leftrightarrow 2x = 10^{4.5} \Leftrightarrow$
$x = \frac{10^{4.5}}{2}$

15. $1000^{x+2} = 10^{5x+8} \Leftrightarrow 10^{3x+6} = 10^{5x+8} \Leftrightarrow$
$\Leftrightarrow 10^{3x}\cdot 10^6 = 10^{5x}\cdot 10^8 \Leftrightarrow \frac{10^6}{10^8} = \frac{10^{5x}}{10^{3x}} \Leftrightarrow$
$\Leftrightarrow 10^{-2} = 10^{2x} \Leftrightarrow 2x = -2 \Leftrightarrow x = -1$

$x > 0$

16. $\log^2(x) - \log(x) - 2 = 0$. On pose $\log x = t$
$t^2 - t - 2 = 0 \quad \Delta = 1 + 8 = 9 \quad t_{1,2} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$
$t_1 = 2 = \log x \Leftrightarrow x = 10^2$ ✓
$t_2 = -1 = \log x \Leftrightarrow x = 10^{-1} > 0$ ✓

17. $9\cdot 3^{2x} - 82\cdot 3^x + 9 = 0$ On pose $3^x = t > 0$
$9\cdot t^2 - 82\cdot t + 9 = 0 \quad \Delta = 80^2 \quad t_{1,2} = \frac{82 \pm 80}{18} = \begin{cases} 9 \\ \frac{1}{9} \end{cases}$
$t_1 = 9 = 3^x \Leftrightarrow x = 2$
$t_2 = 9^{-1} = 3^x \Leftrightarrow 3^x = 3^{-2} \Leftrightarrow x = -2$

18. $2^{x^2-8} = \frac{1}{16} \Leftrightarrow 2^{x^2-8} = 2^{-4} \Leftrightarrow x^2-8=-4 \Leftrightarrow x^2=4 \Leftrightarrow \underline{x=\pm 2}$

19. $\log(99+\log(8+\log(x-1)))=2$ *

$$\begin{cases} x-1>0 \Leftrightarrow x>1 \\ 8+\log(x-1)>0 \Leftrightarrow \log(x-1)>-8 \Leftrightarrow x-1>10^{-8} \Leftrightarrow \\ \qquad x>10^{-8}+1 \\ 99+\log(8+\log(x-1))>0 \Leftrightarrow \log(8+\log(x-1))>-99 \\ \Leftrightarrow 8+\log(x-1)>10^{-99} \Leftrightarrow \log(x-1)>-8+10^{-99} \Leftrightarrow \\ \Leftrightarrow x-1>10^{-8+10^{-99}} \Leftrightarrow x>10^{-8+10^{-99}}+1 \end{cases}$$

$x>10^{-8+10^{-99}}+1$

$* \Leftrightarrow 99+\log(8+\log(x-1))=10^2 \Leftrightarrow$

$\Leftrightarrow \log(8+\log(x-1))=1 \Leftrightarrow 8+\log(x-1)=10 \Leftrightarrow$

$\Leftrightarrow \log(x-1)=2 \Leftrightarrow x-1=10^2 \Leftrightarrow x=101$ ✓

20. $\log(x+1)+\log(x+5)=\log 96$ *

$\left.\begin{array}{l} x+1>0 \Leftrightarrow x>-1 \\ x+5>0 \Leftrightarrow x>-5 \end{array}\right\} \Rightarrow \underline{x>-1}$

$* \log(x+1)(x+5)=\log 96 \Leftrightarrow x^2+6x+5=96$

$x^2+6x-91=0 \qquad \Delta=20^2$

$x_{1,2}=\frac{-6\pm 20}{2}= \begin{cases} x_1=-13<-1 & ✗ \\ x_2=7>-1 & ✓ \end{cases}$

$\underline{x=7}$

21. $\log|x+1|+\log|x+5|=\log 96$ *

$x+1\neq 0 \Leftrightarrow x\neq -1$

$x+5\neq 0 \Leftrightarrow x\neq -5$

$* \log|(x+1)(x+5)|=\log 96 \Leftrightarrow$

$\Leftrightarrow |x^2+6x+5|=96 \Leftrightarrow$

a. $x^2+6x+5=96 \Leftrightarrow x_1=-13$ ✓ $x_2=7$ ✓

b. $x^2+6x+5=-96 \Leftrightarrow x^2+6x+101=0$

$\Delta=-368<$ Imp

$\underline{x_1=-13} \qquad \underline{x_2=7}$

22. $\log_x(0.0025) = 2$ *

$x > 0$, $x \neq 1$

* $0.0025 = x^2 \Leftrightarrow x = \pm\sqrt{0.0025} \Leftrightarrow$

$\Leftrightarrow x = \pm 0.05$ $\qquad$ $\underline{x = 0.05}$