1.31 Exercises

1. 1 : Compute without your calculator :

1) $2-5\cdot 4+18\div 6$
2) $2\cdot(5\cdot 9)+(-4)-18:2$
3) $(\sqrt{25}-3)^0-28:4+4(2+3\cdot(-4))^2$

1. 2 : Compute :

1) $A=\dfrac{5}{6}+\dfrac{6}{7}-\dfrac{7}{8}$
2) $B=\dfrac{13}{24}\cdot\dfrac{36}{7}$
3) $C=\dfrac{85}{49}:\dfrac{34}{21}$
4) $D=\dfrac{2}{3-\frac{5}{4}}$

1. 3 : Compute the following expressions and give your answer as an irreducible fraction.

1) $A=\dfrac{1}{4}+\left[\dfrac{1}{3}+\left[\dfrac{2}{3}-\left[\dfrac{1}{6}+\dfrac{1}{2}\right]\right]\right]$
2) $B=\dfrac{1}{4}-\left[\dfrac{1}{3}-\left[\dfrac{2}{3}-\left[\dfrac{1}{6}-\dfrac{1}{2}\right]\right]\right]$
3) $C=\dfrac{1}{2}-\left[\dfrac{2}{3}-\left[\dfrac{3}{4}+\left[\dfrac{4}{5}-\dfrac{5}{6}\right]\right]\right]$
4) $D=\dfrac{1}{2}-\left[\dfrac{2}{3}+\left[\dfrac{3}{4}-\left[\dfrac{4}{5}-\dfrac{5}{6}\right]\right]\right]$

1. 4 : Evaluate the following expressions with $x=2$ and $x=-3$.

1) $A(x)=([(x+5)\,x+3]\,x-8)\,x+3$
2) $B(x)=[(x+5)\,x+3]\,(x-8)\,x+3$
3) $C(x)=x\,(1-x\,[1-x\,(1-x)])$
4) $D(x)=(40-[30-(20+x)\,x]\,x)\,x$

1. 5 : Are the elements $\{0;\ 0.2;\ 5;\ 7;\ 1/7\}$ members of the sets
$S=\{x\,|\,x^2-12x+35=0\}$, $T=\{x\,|\,7x=1\}$ and $U=\{x\,|\,x^2-2x=0\}$?

1. 6 : We consider the sets $A=\{x\in\mathbb{N}^*\,|\,x<11\}$, $B=\{1;2;3;4;5\}$ and
$C=\{x\in\mathbb{R}\,|\,x=2n+1,\ n\in\mathbb{N}^*\text{ and }n<5\}$. Describe :

1) $D=A\backslash(B\cup C)$
2) $E=(A\cup C)\backslash(A\cup B)$
3) $F=(A\backslash C)\cap(A\backslash B)$

1. 7 : Describe the following sets using the interval notation. Then, represent them on the real number line and finally describe their complementary set.

1) $A=\{x\,|\,x\in\mathbb{R}\text{ and }-3<x<5\}$
2) $B=\{x\,|\,x\in\mathbb{R}\text{ and }x<1\}$
3) $C=\{x\,|\,x\in\mathbb{R}\,|\,x>3\text{ or }x\leqslant -1\}$
4) $D=\mathbb{R}$
5) $E=\{x\,|\,x\in\mathbb{R}\text{ and }|\,x\,|\leqslant 8\}$
6) $F=\{x\,|\,x\in\mathbb{R}\text{ and }1<x^2<5\}$
7) $G=\{x\,|\,x\in\mathbb{R}\text{ and }x\geqslant 10\}$
8) $H=\{x\,|\,x\in\mathbb{R}\text{ and }|\,x\,|>4\}$

1. 8 : Write the number $0.\overline{18}$ as a irreducible fraction.

1. 9 : Convert $a=0.043434343\ldots$ and $b=0.120\overline{234}$ to fractions.

1. 10 : Compute and simplify as much as possible.

1) $A = 1 - \left(\frac{7}{5} - \frac{3}{4}\right) : \left(\frac{2}{3} + \frac{1}{5}\right)$

2) $B = 3 - \left(\frac{13}{2} - \frac{5}{3}\right) : \left(\frac{7}{3} + \frac{5}{6}\right)$

3) $C = \left(2 + \frac{1}{a}\right) : \left(1 + \frac{2}{a}\right)$

4) $D = \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}}$

5) $E = \frac{\frac{1}{ab} + \frac{1}{ac}}{\frac{1}{ab} + \frac{1}{bc}}$

6) $F = \frac{2t}{4 - t^2} + \frac{1}{2 - t}$

7) $G = \frac{1}{b} - \frac{1}{bc}$

8) $H = \frac{x - y}{2x} - (1 - y)$

9) $I = \frac{1}{x - 1} - \frac{2}{x^2 - x}$

10) $J = \frac{1}{2 - x} \cdot \frac{x - 2}{4}$

11) $K = \frac{m^4 - 1}{(m + 1)^2}$

12) $L = \frac{\frac{3}{x} - \frac{8}{x}}{\frac{x}{2} + \frac{2}{x}}$

1. 11 : Simplify by extracting the greatest number from the root.

1) $A = \sqrt{12}$ 2) $B = \sqrt{72}$ 3) $C = \sqrt{216}$

1. 12 : Rationalize the denominator of the following fractions and simplify whenever it is possible.

1) $A = \frac{1}{\sqrt{3}}$

2) $B = \frac{\sqrt{27}}{\sqrt{15}}$

3) $C = \frac{12}{\sqrt{3}} - \sqrt{27}$

4) $D = \sqrt{63} - \sqrt{28}$

5) $E = \frac{3\sqrt{20} - 5\sqrt{15}}{\sqrt{5}}$

6) $F = \frac{\sqrt{12} - \sqrt{3}}{\sqrt{12}}$

7) $G = \frac{\sqrt{2}}{\sqrt{5} - \sqrt{2}}$

8) $H = \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}}$

9) $I = \frac{7\sqrt{5} - 5\sqrt{7}}{\sqrt{7} + \sqrt{5}}$

10) $J = \frac{7 + 2\sqrt{10}}{\sqrt{2} + \sqrt{5}} - \frac{7 - 2\sqrt{10}}{\sqrt{2} - \sqrt{5}}$

11) $K = \frac{\sqrt{5} + \sqrt{3}}{1 - \sqrt{5}} - \frac{\sqrt{5} - \sqrt{3}}{1 + \sqrt{5}}$

1. 13 : Simplify :

1) $A = \sqrt{5 - 2\sqrt{3}} \cdot \sqrt{5 + 2\sqrt{3}}$

2) $B = \sqrt{5 + \sqrt{21}} + \sqrt{5 - \sqrt{21}}$

3) $C = \sqrt{6 - 2\sqrt{5}} + \sqrt{6 + 2\sqrt{5}}$

1. 14 : Show that $\sqrt{6} - \sqrt{2} = 2\sqrt{2 - \sqrt{3}}$

1. 15 : Simplify :

1) $A = (2e^2 f^{-3})^4$

2) $B = \left(\sqrt[5]{\dfrac{p^2}{w^{-3}}}\right)^0$

3) $C = (100r^3 s^{-1}) : (4r^{-2}s^5 t)$

4) $D = \dfrac{\sqrt{x\sqrt{y}}}{\sqrt{y\sqrt{x}}}$

5) $E = \sqrt[4]{c^3} \cdot c$

6) $F = \left(125^{\frac{2}{3}} x^{-1} y^2\right)^{-\frac{1}{2}}$

7) $G = (a^{-3}b^7)^{\frac{1}{2}} : \left(a^{\frac{1}{4}} b^{\frac{3}{2}}\right)^{-2}$

8) $H = \sqrt[3]{\dfrac{\sqrt{2}}{64}}$

9) $I = \sqrt{\dfrac{a^{\frac{2}{3}}}{\sqrt[3]{a^{\frac{1}{2}}}}} \cdot \left(\left(a^{\frac{2}{3}}\right)^2\right)^{\frac{1}{3}}$

1. 16 : Simplify :

1) $A = a^4 \cdot a^{28}$

2) $B = x^{-25} \cdot x^{14}$

3) $C = u^{x-4} \cdot u^{6-x}$

4) $D = \dfrac{n^0}{n^{-5}}$

5) $E = \dfrac{h^4}{h^2}$

6) $F = b^{4n-7} \cdot b^{-n+7}$

7) $G = \dfrac{z^5}{z^{3-6n}}$

8) $H = \dfrac{e^x}{e^{-x}}$

9) $I = (-a)^5$

10) $J = (-x^6)^4$

11) $K = (a^5 b^{-8} x)^4$

12) $L = (-3x^4)^3$

13) $M = (x^{-3}y^5)^3$

14) $N = (ab^0 c^{-3})^{-n}$

15) $O = \dfrac{(-a)^6}{(-a)^3}$

16) $P = \dfrac{(-a)^{4n+3}}{(-a^n)^4}$

1. 17 : Simplify :

1) $A = \left(\dfrac{a^5}{b^6}\right)^{-4} \left(\dfrac{a^4 b^{-4}}{c^7}\right)^3$

2) $B = \left(\dfrac{x^3}{y^4}\right)^6 \left(\dfrac{x^2 z^{-2}}{y^{-1}}\right)^{-3}$

3) $C = \left(81^{-\frac{1}{2}} \cdot (3x)^3\right)^2 \cdot (y^{-5}xy)^2$

4) $D = \dfrac{(x^4y^3)^n}{(x^2y^4)^{2n-1}}$

5) $E = \dfrac{(a^{n-1}b^n)^2}{(a^2b^2)^{n-1}}$

6) $F = \dfrac{a^{-2}b^{-10}}{a^7b^{-8}} \cdot \dfrac{a^4b^{-3}}{(a^{-3}b)^2}$

7) $G = \dfrac{(16x^4y^2)^n}{(4x^2y)^{2n-1}}$

8) $H = \dfrac{\sqrt{(2^6 \cdot x^{-12} \cdot y^{24})^{\frac{1}{3}}}}{(x^{-2} \cdot y)^{-4}}$

1. 18 : Simplify :

1) $A = \dfrac{3a}{4b} - \dfrac{4b}{3a}$

2) $B = \dfrac{x - \frac{4}{x}}{1 - \frac{2}{x}}$

3) $C = a\sqrt{a\sqrt{a\sqrt{a}}}$

4) $D = \dfrac{a+2}{2a-6} - \dfrac{a-2}{2a+6}$

5) $E = \dfrac{(a^2)^{3n}}{\sqrt[3]{a^{6n-3}}}$

6) $F = \left(\dfrac{4a^{-2}}{9x^2}\right)^{-\frac{1}{2}}$

7) $G = \dfrac{(a^{-2})^3}{\sqrt{b} \cdot a^{\frac{2}{3}}} \cdot \dfrac{\sqrt{b^{-5}}}{\sqrt[3]{a}}$

1. 19 : Expand the following products.

1) $A = (3a + 2b)(2a + 3b)$
2) $B = (5x + 4)(9 - 11x)$
3) $C = (a + 2b + 3c)(3a + 2b + c)$
4) $D = (x - 5y + 2)(2x + y - 2)$

1. 20 : Using special products, expand :

1) $A = (3x + 2y)^2$
2) $B = [3(x + y)]^2$
3) $C = 3(x + y)^2$
4) $D = (a+b+c)(a+b-c)$
5) $E = (4x - \sqrt{6}y)^2$
6) $F = (x + \frac{1}{x})(x - \frac{1}{x})$
7) $G = (\sqrt{2} - \sqrt{5})^2$
8) $H = (x - 3y)^3$
9) $I = (3x - y)(3x + y)$
10) $J = (2a + 3b)^4$

1. 21 : Complete :

1) $A = 4x^2 + ... + ... = (.... + 5)^2$
2) $B = ... + 14x + ... = (x + ...)^2$
3) $C = x^2 - .... + 100 = (.... - ....)^2$
4) $D = (3x + ...)^2 = ... + ... + 25$
5) $E = 64 + ... + 9x^4 = (.... + ...)^2$
6) $F = ... - 12x + 4 = (.... - ....)^2$
7) $G = x^2 - 16 = (.... + ...)(.... - ....)$
8) $H = ... - 24x + ... = (2x - ....)^2$

1. 22 : Simplifiy the expression $(x + 1)^2 - (x - 1)^2$, then calculate $10001^2 - 9999^2$.

1. 23 : Solve the equations $x^2 - 5 = 0$, $x^2 + 3x = 0$ and $(x - 3)^2 - 3 = 0$.

1. 24 : By completing the square, solve the following equations.

1) $x^2 + 3x - 4 = 0$
2) $x^2 + 3x + 4 = 0$
3) $x^2 + 4x + 4 = 0$
4) $2x^2 - 5x + 2 = 0$
5) $ax^2 + bx + c = 0$

1. 25 : Factorize the expressions above and then deduce the value(s) of x for which they are equal to zero.

1) $A(x) = x^2 + 6x + 9$
2) $B(x) = 4x^2 - 4x + 1$
3) $C(x) = x^2 - 121$
4) $D(x) = x^2 - 10x + 25$
5) $E(x) = 16x^4 - 1$
6) $F(x) = 9x^2 + 12x + 4$
7) $G(x) = 2x^3 - 18x$
8) $H(x) = (2x + 1)(x - 2) + x(2x + 1)$
9) $I(x) = (x^2 + 1)(x - 5) + 2x(x - 5)$
10) $J(x) = (x^2 - 5)(x^2 - 1) - 4(x^2 - 1)$

1. 26 : Factorize :

1) $A(x) = 4x^2 + 4xy + y^2$
2) $B(x) = x^2 + 8x + 16$
3) $C(x) = x^2 - 4y^2$
4) $D(x) = 5xy^3 - 10xy^2 + 5xy$
5) $E(x) = (x^2 - 1)(3x^2 - 4)$

1. 27 : If $\Delta \geq 0$, the expression $ax^2 + bx + c$ can be written like $a(x - x_1)(x - x_2)$ where x_1 and x_2 are the roots of $ax^2 + bx + c = 0$. Prove the Vieta's relations $x_1 + x_2 = -b/a$ and $x_1 \cdot x_2 = c/a$.

1. 28 : Solve :

1) $25x^2 + 10x + 2 = 1$
2) $2x^2 + 5x - 3 = 0$
3) $x^2 + kx - k^2 = 0$
4) $x^2 - 2\sqrt{3}x + 1 = 0$
5) $ax^2 + (1 - a^2)x - a = 0$
6) $x^4 - 16x^3 = 0$

1. 29 : In each case, determine the value of the parameter d so that the equation has a unique solution.

1) $3x^2 + dx + d = 0$
2) $2x^2 + x + 2 = dx$
3) $x^2 + 2x + d = 3$
4) $\frac{1}{4}(x + 1)^2 + 2 = dx$

1. 30 : Simplify :

1) $\dfrac{2x^2 - x - 3}{2x^2 - 7x + 6}$
2) $\dfrac{x^2 + x - 2}{x^2 - x - 2}$
3) $\dfrac{(3x^2 + 10x - 8)(5x^2 - 9x - 2)}{(5x^2 + 16x + 3)(3x^2 - 8x + 4)}$

1. 31 : Look for all the integer roots of the polynomials $P(x) = x^3 - 2x^2 - 5x + 6$ and $Q(x) = x^3 + x^2 - 3x + 9$ among the divisors of their respective constant term.

1. 32 : Determine, using polynomial long division, the quotient and the remainder of the following divisions.

1) $A = \dfrac{x^4 + 2x^3 - 1}{x^2 - 1}$
2) $B = \dfrac{7x^4 - 3x^2 + 2x - 4}{x^2 - 4}$
3) $C = \dfrac{x^6 - 3x^5 + 2}{x^3 + 3x}$
4) $D = \dfrac{2x^3 - 11x^2 + 23x - 26}{2x - 5}$

1. 33 : Look for all the rational roots of the polynomials $P(x) = 4x^2 + 3x - 1$ and $Q(x) = 6x^2 - x - 2$.

1. 34 : Without using long polynomial division or Horner, decide whether $P(x) = x^3 + 2x^2 - 2x - 12$ is divisible by $x - 3$, $x + 1$ or $x - 2$.

1. 35 : Use polynomial long division to calculate the quotient and the remainder of the following divisions. Check your result with Horner' scheme.

1) $A = \dfrac{x^2 - 2x + 1}{x - 1}$
2) $B = \dfrac{2x^2 + 3x + 4}{x + 1}$
3) $C = \dfrac{3x^2 + 2x + 5}{x + 3}$
4) $D = \dfrac{x^3 + 2x^2 + 3x + 4}{x + 2}$

1. 36 : Check that $P(x) = 2(x+1)^3(3x-5)^2(2x+3)(x^2-3x+5)(2x^2+2x+1)$ is completely factored and give the roots with their multiplicity.

1. 37 : Answer the following :

1) The polynomial $P(x) = 4x^3 + 32x^2 - 9x - 72$ is divisible by $x + 8$. Factorize completely $P(x)$ and give the solutions of the equation $4x^3 + 32x^2 - 9x - 72 = 0$.

2) Factorize completely the polynomial $P(x) = -9x^3 - 9x^2 + 4x$.

3) Find a polynomial whose roots are 5, 3 and -13. Is there only one possible answer ?

4) Find a polynomial $P(x)$ of degree 3 such that $P(0) = 14$, $P(x)$ is divisible by $x - 1$, $x = -2$ is a root of $P(x)$ and the remained of the division of $P(x)$ by $x - 2$ is -12.

1. 38 : Factorize completely the following polynomials. In each case, give the roots with their multiplicity.

1) $P_1(x) = 6x^3 - 19x^2 + x + 6$

2) $P_2(x) = x^4 - 5x^3 - 4x^2 + 16x - 8$

3) $P_3(x) = x^4 + 2x^3 - x^2 + 4x + 12$

1. 39 : Find a solution of the equation $2x^3 - 5x^2 - 4x + 3 = 0$. Then, factorize completely the polynomial $P(x) = 2x^3 - 5x^2 - 4x + 3$.

1. 40 :

1) Determine the value of m such that the polynomial $P(x) = 2x^3 + mx^2 - 3x - 2$ is divisible by $x + 2$.

2) Given $Q(x) = kx^3 + k^2x^2 + kx - 4k^2 + 6$. Determine the value(s) of k such that $Q(x)$ is divisible by $x - 2$.

3) The remainder of the division of $P(x) = 3x^3 - 4x^2 - 2px + 8$ by $2x - 4$ is 6. Calculate the value of the parameter p.

1. 41 : Solve the following systems.

1) $\begin{cases} -4x + 3y = 4 \\ x - \dfrac{3y}{4} = -1 \end{cases}$

2) $\begin{cases} y - \dfrac{13\,(x-2)}{2} = 2x + 1 \\ 3x - \dfrac{2\,(y-5)}{3} = 2y - 4 \end{cases}$

1. 42 : Fill in the blanks so that :

1) The two equations are equivalent : $\begin{cases} 3x + 5y = \_\_\_ \\ \_\_\_ - 10y = 4 \end{cases}$

2) The two equations are incompatible : $\begin{cases} -2x + 3y = 1 \\ x - \_\_\_\, y = \_\_\_ \end{cases}$

1. 43 : The area of a rectangular field is 567m^2. The fence which delimits that field has a length of 96m. Determine its dimensions.

1. 44 : The area of a trapezoid is 360cm^2, its height measures 24cm. One of its parallel sides measures 2/3 of the other one. Determine the length of the parallel sides.

1. 45 : Solve the following systems :

1) $\begin{cases} 2x - y + z = 8 \\ -x + 3y + 2z = 1 \\ 3x + y + 3z = 14 \end{cases}$ 2) $\begin{cases} -x + 7y + 2z = 2 \\ 9x - 5y - z = 4 \\ 8x + 3y = 2.5 \end{cases}$ 3) $\begin{cases} -x + 3y = 1 \\ 5x - 2y - z = 0 \\ 3x + 4y - z = 2 \end{cases}$

1. 46 : Solve :

1) $4x^4 - x^2 - 60 = 0$
2) $2x^4 - 5x^3 + 3x^2 = 0$
3) $x^6 + 4x^3 + 3 = 0$
4) $x^5 + 2x^3 - 24x = 0$
5) $x^3 - \dfrac{6^3}{x^3} = 19$

1. 47 : Determine the lengths of the sides of a right triangle whose hypothenuse is $a = \sqrt{41}$ and whose area is 10.

1. 48 : Solve

1) $2x + \sqrt{x(x+6)} = 8$
2) $\sqrt{2+x} + 4 = \sqrt{10 - 3x}$
3) $\sqrt{x} + 3 + \sqrt{x+1} = 5$
4) $\sqrt{x-3} - \sqrt{x+2} = \sqrt{x+18}$

1. 49 : Solve :

1) $\dfrac{5x}{x^2+9} = -1$
2) $\dfrac{2x-4}{x} - \dfrac{1-x}{2x-1} = 9$
3) $\dfrac{5x}{x-3} + \dfrac{4}{x+3} = \dfrac{90}{x^2-9}$

1. 50 : Solve :

1) $|\,x - 3\,| = 4$
2) $|\,2x - 1\,| = |\,x + 3\,|$
3) $|\,x^2 - 4\,| = 21$
4) $x^2 - |\,2x\,| = 3$

1. 51 : Solve, in terms of x, the equations here below :

1) $\dfrac{8}{6-2x} = \dfrac{9}{4-3x}$
2) $\dfrac{3x-12}{x-4} = 2$
3) $(8x-2)(3x+4) = (4x+3)(6x-1)$
4) $ax + 1 = 2x - a$
5) $(x+1)^2 - 16 = 0$
6) $(a-x)(b+x) = 0$
7) $\dfrac{x+3}{x-4} = 1$
8) $\dfrac{1-x}{4} - \dfrac{2x+3}{3} = \dfrac{x}{2}$
9) $\dfrac{-2x^2}{2x-1} = -x - 1$
10) $5a + 2x = x - \dfrac{x}{a}$

1. 52 : Some more equations to solve.

1) $\dfrac{8}{x+2} = 6$
2) $\sqrt{x^2+7} = 4$
3) $x + \sqrt{x+5} = 7$
4) $\sqrt{2x} + 3 - \sqrt{x+1} = 1$
5) $\dfrac{2x-1}{x+1} = \dfrac{2x+1}{x-1}$
6) $|-x+5| = |2x+3|$
7) $\dfrac{x-5}{3x-15} = \dfrac{2}{x}$
8) $\dfrac{2x-4}{x} - \dfrac{1-x}{2x-1} = 9$
9) $x + \sqrt{6-x} = 0$
10) $|x^2+3x-4| = |-x+5|$
11) $-3 + 2 \cdot |4x-6| = 11$

1. 53 : Find the solutions of the following inequalities.

1) $3x < -x + 4$
2) $x - 1 \leq 3x - 1$
3) $-2x + 2 < x - 5$
4) $x - 5 \geq 4x + 9$
5) $-3x + 4 < x^2$
6) $-x^2 + 4x \geq -5$

1. 54 : Prove that the equation $x^2 + px - 4 = 0$ has always two solutions, regardless the value of p.

1. 55 :

1) For which value(s) of p, does the equation $x^2 + (p-2)x - 4 = 0$ have a unique solution ?
2) For which value(s) of p, the equation $x^2 + (p+2)x + \dfrac{9}{4} = 0$ has no solutions ?
3) The equation $x^2 + b = b(x-2)$ has $x = 5$ for root. Determine the value of b.

1. 56 : In each case, determine the values of d such that the given equation has no solutions.

1) $3x^2 + dx + d = 0$
2) $2x^2 + x + 2 = dx$
3) $x^2 + 2x + d = 3$
4) $\dfrac{1}{4}(x+1)^2 + 2 = dx$

1. 57 : Among the expressions below, which ones are functions ? Justify.

$f: \mathbb{Z} \longrightarrow \mathbb{N}$
$x \longmapsto y = x^2+1$

$g: \mathbb{Z} \longrightarrow \mathbb{Q}$
$x \longmapsto y = \dfrac{1}{x}$

$h: \mathbb{Q} \longrightarrow \mathbb{N}$
$x \longmapsto y = 2$

$i: \mathbb{N} \longrightarrow \mathbb{R}$
$x \longmapsto y = x$

$j: \mathbb{N}^* \longrightarrow \mathbb{R}$
$x \longmapsto y = \sqrt{x^2+2x-3}$

$k: \mathbb{R} \longrightarrow \mathbb{R}$
$x \longmapsto y = \dfrac{3}{x^2-2}$

$l: \mathbb{Q} \longrightarrow \mathbb{Q}$
$x \longmapsto y = \dfrac{3}{x^2-2}$

$m: \mathbb{N} \longrightarrow \mathbb{N}$
$x \longmapsto y = \dfrac{x^2+3x+2}{x+2}$

$n: \mathbb{Q} \longrightarrow \mathbb{Q}$
$x \longmapsto y = \dfrac{x}{x^2-2}$

1. 58 : Sketch the graph of every function here below for $-6 \leq x \leq 6$. Then, find their domain and range.

1) $f_1(x) = 3$

2) $f_2(x) = 0.5x + 1$

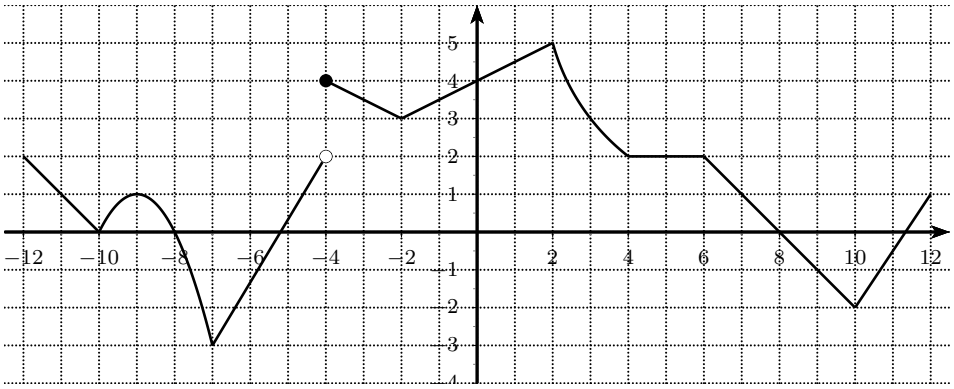
3) $f_3(x) = \dfrac{1}{2}(x^2 - x - 2)$

4) $f_4(x) = \dfrac{1}{x-1}$

5) $f_5(x) = |x-1|$

6) $f_6(x) = \sqrt{x+1}$

1. 59 : A function $f : [-12; 12] \longrightarrow \mathbb{R}$ has the following graph :



1) Find the following images : $f(-11)$, $f(-7)$, $f(-4)$, $f(-1)$, $f(1)$, $f(3)$, $f(9)$

2) Determine the zeros of f.

3) Identify the following sets (use interval notation)

$f([-12; 12])$ $f([-8; -2])$ $\{x \in \mathbb{R} \mid f(x) > 2\}$
$f(]2; 5[)$ $\{x \in \mathbb{R} \mid f(x) = 2\}$

4) Find the point(s) of the graph whose abscissa is the double of the ordinate.

1. 60 : Find the domain of the following :

1) $f_1(x) = \dfrac{2x+1}{x-5}$

2) $f_2(x) = \sqrt{x-5}$

3) $f_3(x) = \dfrac{(x-1)}{2(5x-3)}$

4) $f_4(x) = \dfrac{1}{x^2-2x+1}$

5) $f_5(x) = \dfrac{2x^2+x+1}{x^2-9}$

6) $f_6(x) = \sqrt{2-3x}$

7) $f_7(x) = \dfrac{x^2}{(x-3)(x+4)}$

8) $f_8(x) = \dfrac{1-x^2}{(x-4)^3}$

9) $f_9(x) = \dfrac{1}{\sqrt{3x^2-6x}}$

10) $f_{10}(x) = \sqrt{x^3-x^2-5x-3}$

11) $f_{11}(x) = \sqrt{x^2-5x+7}$

12) $f_{12}(x) = \sqrt{x^2-1}$

1. 61 : Among the following functions, determine the odd ones and the even ones.

1) $f(x) = \dfrac{x^3-x+1}{x^2}$

2) $g(x) = \dfrac{x^2+2}{x^3+x}$

3) $h(x) = \dfrac{\sqrt{x^4-1}}{x^2+2}$

4) $i(x) = \dfrac{x}{|x|}$

5) $j(x) = \sqrt{1-x^2}$

6) $k(x) = \dfrac{1}{2x^2+x+1}$

1. 62 : In order to solve the inequality $\frac{3}{x-1} > \frac{2x}{x+1}$ which is equivalent to $\frac{3}{x-1} - \frac{2x}{x+1} > 0$, we establish the table of signs of the function $f(x) = \frac{3}{x-1} - \frac{2x}{x+1} = \frac{-2x^2+5x+3}{(x-1)(x+1)}$. By using the same method, solve :

1) $\dfrac{x^2-4}{(x+4)(x-1)} \geq 0$

2) $\dfrac{4x^8-2x^7}{3x-5} < 0$

3) $\dfrac{x+1}{x-1} > \dfrac{x-1}{x+1}$

4) $\dfrac{13}{2-x} \leq \dfrac{21x+3}{3x+1}$

1. 63 : Represent, in the same set of axes, the following lines :

1) $f(x) = \dfrac{1}{2}x$

2) $g(x) = \dfrac{1}{2}x + 2$

3) $h(x) = \dfrac{1}{2}x - 3$

4) $j(x) = -3x$

5) $k(x) = 3x + 1$

6) $m(x) = -3$

1. 64 : Find the equations of the line :

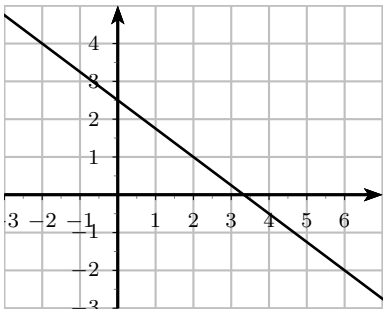
1) Which passes through $O(0;0)$ and $P(2;6)$.

2) Which passes through $A(-1;-4)$ and $B(7;-8)$.

3) Which passes through $C(2;-2)$ and whose gradient is $\frac{3}{5}$.

4) Which passes through P and C.

5) Opposite.



1. 65 : Find the equation of the line f such that $f(4) = 4$ and $f(-6) = -1$.

1. 66 : Given the line $f(x) = -2x + 1$ and the point $A(2; 1)$.

1) Find the equation of the line g whose graph passes through A and whose graph is is parallel to the one of f.

2) Find the equation of the line h whose gradient is $\frac{1}{2}$ and that intersects the graph of f at $x = 1$.

3) Represent these three functions in the same coordinate system. Observation ?

1. 67 : Given $f(x) = -3x+4$ and $g(x) = 5x+\frac{3}{2}$ two functions. Solve algebraically the equation $f(x) = g(x)$. Then, represent, using the interval notation, the set $A = \{x \mid f(x) < g(x)\}$.

1. 68 : Determine the expression of the line f passing through $A(2; 0)$ and $B(-1; 6)$. Then determine the equation of the line g whose graph passes through $C(-1; -2)$ and that is perpendicular to the one of f.

1. 69 : Solve the following :

1) $\begin{cases} x + 3y - 6 < 0 \\ x > 2 \end{cases}$ 2) $\begin{cases} -2x + y + 5 \geqslant 0 \\ 4x + 3y - 12 \leqslant 0 \end{cases}$ 3) $\begin{cases} x + 3y \geqslant 0 \\ 2x + y > 5 \end{cases}$

1. 70 : After having calculated the coordinates of the intersections with the axes and the vertex of the quadratics here below, sketch their graph.

1) $f_1 : x \longmapsto y = x^2$
2) $f_2 : x \longmapsto y = -x^2 + 4$
3) $f_3 : x \longmapsto y = -(x + 1)^2$
4) $f_4 : x \longmapsto y = -2(x + 1)(x - 3)$
5) $f_5 : x \longmapsto y = 2\,(x - 1)^2 + 4$
6) $f_6 : x \longmapsto y = x^2 + 4x - 12$

1. 71 :

1) Determine the vertex of the parabola $f : x \longmapsto y = -\frac{1}{2}x^2 + x + 3$. Then, determine its intersection points with the line $d : x - y + 1 = 0$.

2) Complete $y = -\frac{1}{2}x^2 + x + 3 = ...(x.....)(x.....) = ...(x.....)^2 + ...$

1. 72 :

1) Determine the intersection points between the parabola $f : x \longmapsto y = \frac{1}{2}x^2 + 1$ and the line $d_1 : y = 2x - 1$.

2) Calculate the value of b so that the parabola $y = -x^2 + 2$ is tangent to the line $d_2 : y = -3x + b$.

1. 73 : Determine the equation of the parabola which passes through the points $A(1; 5)$, $B(2; 5)$ and $C(-1; 11)$.

1. 74 : Find the equation of the parabola with vertex $V(3;3)$ that passes through the origin.

1. 75 : Determine the equation of the linear lines which are tangent to the parabola P : $x \longmapsto y = \dfrac{x^2 - 2x + 9}{4}$?

1. 76 : For which values of a does the graph of $f : x \longmapsto y = 3x^2 + ax - a$ have no intersection with the x-axis ?

1. 77 : Given $f(x) = 2x^2 - nx + 3n$ a parabola. Determine the value of n so that the axis of symmetry of that parable is the vertical line $x = 1$. Then, calculate the coordinates of its vertex.

1. 78 : For which values of m does the equation $x^2 + mx + m - 0,75 = 0$ have a repeated root ? What are these solutions ?

1. 79 : A parabola intersects the x-axis at $x = 1$ and $x = -3$. Moreover, it is tangent to the line with equation $y = 8$. Determine the coordinates of its vertex and its equation.

1. 80 : Sketch the graphs of $f : x \longmapsto y = x^2 - x - 6$ and $g : x \longmapsto y = |f(x)|$. Then, write $g(x)$ without the absolute value.

1. 81 : Find the zeros of the function $f : x \longmapsto y = x^3 - x^2 - 2x$

1. 82 : Sketch the graph of $f : x \longmapsto y = \dfrac{x^4 - 8x^2 - 9}{4}$ after having determined the domain, the intersections with the axes, the parity and the range of f.

1. 83 : Find the values of x so that :

1) $x^3 - 8x - 3 = 0$

2) $x^3 - 8x - 3 \geq 0$

1. 84 : Establish the table of signs of the function
$f : x \longmapsto y = x(x-1)(x^2 + x + 1)$

1. 85 : Calculate the coordinates of the intersection points between the graph of $f(x) = \dfrac{1}{x-3}$ and the line $d : x - 4y = 0$.

1. 86 : Analyse the functions (Domain, intersections with the axes, parity, table of signs, asymptotic behavior, table of values, graph) :

1) $f(x) = \dfrac{1}{x}$

2) $g(x) = \dfrac{1}{x^2}$

3) $h(x) = \dfrac{2x-5}{3x+6}$

4) $i(x) = \dfrac{1}{x^2 - 4}$

1. 87 : Analyse the functions (Domain, intersections with the axes, parity, table of signs, asymptotic behavior, table of values, graph) :

1) $f(x) = x - \dfrac{1}{x}$

2) $g(x) = \dfrac{x+2}{x-3}$

3) $h(x) = \dfrac{x-3}{x^2+x-2}$

4) $i(x) = \dfrac{x^2-3x+2}{x+1}$

5) $j(x) = \dfrac{-1}{(x-2)^2}$

6) $k(x) = \dfrac{2}{x^2+1}$

1. 88 : For which values of k do the line $y = k$ and the graph of $f(x) = \dfrac{x-3}{x^2+x-2}$ have only one intersection point ?

1. 89 : Find the domain of the following functions. Then, sketch their graph and give their range.

1) $f(x) = \sqrt{x+9}$

2) $g(x) = 2\sqrt{x+4} - 6$

3) $h(x) = \sqrt{9-x^2}$

1. 90 : Calculate b and c so that the graph of $f(x) = \dfrac{x+b}{cx-3}$ passes through the point $P(4;-2)$ and has the vertical asymptote $x = 6$.

1. 91 : We consider an homographic function $f(x) = \dfrac{2x+7}{x+3}$ and a linear function $g(x) = -x + \dfrac{3}{2}$.

1) Determine the domain of the function f and show, by using the long division, that $f(x) = 2 + \dfrac{1}{x+3}$.

2) Sketch the graph of f (hyperbola) and g in the same set of axes. Calculate the intersection points between these graphs and with the axes. Give the equation of the asymptotes.

3) Sketch the graph of the tangents, t_1 and t_2, to the graph of f which are parallel to the line g.

4) Determine the equations of these tangents and calculate the coordinates of their intersection points with the hyperbola.

1. 92 : Represent in diagrams containing at least 4 elements per set :

1) A correspondence that is not an application.

2) An application that is surjective but not injective.

3) An application that is injective but not surjective.

4) An application that is a bijection.

5) An application that is not surjective nor injective.

1. 93 : Sketch the graph of a correspondence from $\mathbb{R}$ to $\mathbb{R}$ which :

1) Is not a function.

2) Is an injective function but not surjective.

3) Is an surjective function but not injective.

4) Is a bijection.

5) Is neither injective nor surjective.

1. 94 : Given two functions $f : x \longmapsto y = x^2$ and $g : x \longmapsto y = x + 1$. Calculate $f \circ g$ and $g \circ f$.

1. 95 : Given three functions $f : x \longmapsto y = 2x$, $g : x \longmapsto y = x - 3$ and $h : x \longmapsto y = x^2$. Calculate $h \circ g$, $(h \circ g) \circ f$, $g \circ f$ and $h \circ (g \circ f)$.

1. 96 : Given the functions $f : x \longmapsto y = 2x - 3$ and $g : x \longmapsto y = \dfrac{x+3}{2}$. Calculate $f \circ g$ and $g \circ f$. Observation ?

1. 97 : Calculate the inverse function of $f : x \longmapsto y = \dfrac{x+1}{2x-3}$ and then, sketch the graph of f and ${}^{i}f$ in the same coordinate system.

1. 98 : We consider the two homographic functions $f(x) = \dfrac{2x-3}{x+5}$ and $g(x) = \dfrac{x-1}{3x+2}$. Determine $f \circ g$, ${}^{i}f$, ${}^{i}g$, ${}^{i}(f \circ g)$ and ${}^{i}g \circ {}^{i}f$. Observations ?

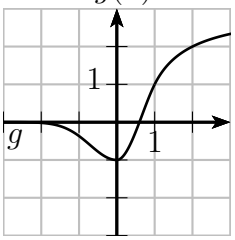
1. 99 : Give the domain and the range of the function $f : x \longmapsto y = \sqrt{1-x}$ so that it is a bijection. Determine its inverse function.

1. 100 : Given $f : x \longmapsto y = \dfrac{ax+b}{cx+d}$. Find conditions on a, b, c and d so that $f(x) = {}^{i}f(x)$.

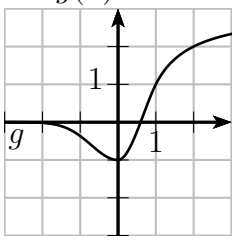
1. 101 : Determine the domain and the range of $f : x \longmapsto y = x^2 - 2x - 3$ so that it is a bijection. Then determine ${}^{i}f(x)$.

1. 102 : From the graph of the function g, sketch the graph of :

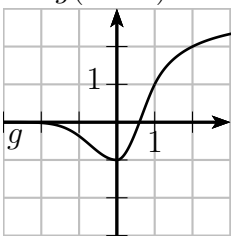
$-g(x)$



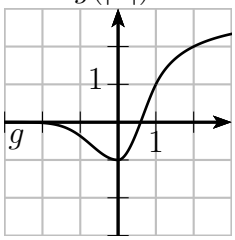
$g(x) - 1$



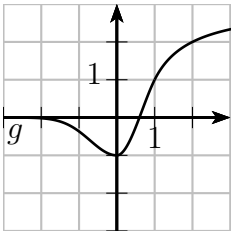
$g(x - 1)$



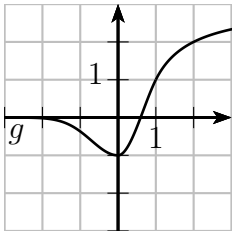
$g(|x|)$



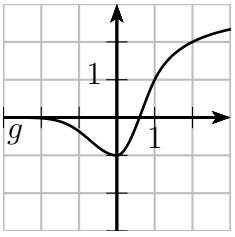
$\frac{g(x)}{2}$



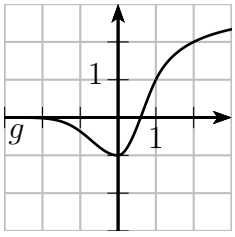
$\sqrt{g(x)}$



$|g(x)|$



$-g(x + 1) + 1$



1. 103 : Study the functions $f_1(x) = 10^x$ and $f_2(x) = \log(x)$.

1. 104 : Calculate without calculator :

1) $\log_2(16)$
2) $\log_7\left(\frac{1}{49}\right)$
3) $\log_4(1)$
4) $\log_3(-9)$
5) $\log_5(5)$
6) $\log_{27}\left(\frac{1}{3}\right)$
7) $\log_{16}(8)$
8) $\log_{\sqrt{2}}(4)$

1. 105 : Rewrite using $\log(p)$, $\log(q)$ and $\log(r)$.

1) $\log(pqr)$
2) $\log(pq^2r^3)$
3) $\log(100pr^5)$
4) $\log\left(\sqrt{\frac{p}{q^2r}}\right)$
5) $\log\left(\frac{qr^7p}{10}\right)$
6) $\log\left(\sqrt{\frac{10p^{10}r}{q}}\right)$

1. 106 : Rewrite the following :

1) $\log\left(\left(a^5\sqrt[3]{b}\right)/c\right)$
2) $\log(x+1) - \log(x-1)$
3) $2\log(a) - \frac{1}{2}\log(b) + 3\log(c)$
4) $\log\left(\sqrt{a \cdot \sqrt[3]{b}}\right)$

1. 107 : How many digits are there in the number 87^{97} ? Give the first five digits.

1. 108 : Solve the following :

1) $10^x = 9,56$
2) $10^{3x} = -5$
3) $\log(x) = 2,5$
4) $\log(4x-1) = -2$
5) $2\log(x) = -4$
6) $\log(x^2-21) = 2$
7) $10^{3x+2} = \sqrt{10}$
8) $\log(\log(x)) = 1$
9) $2 \cdot 2^{2x} - 9 \cdot 2^x + 4 = 0$
10) $2^x = 10$
11) $1,03^x = 2$
12) $\log(0,5x-3) = -1$
13) $2,8^x = 5$
14) $3\log(2x) = 9$
15) $1000^{x+2} = 10^{5x+8}$
16) $\log^2(x) - \log(x) - 2 = 0$
17) $9 \cdot 3^{2x} - 82 \cdot 3^x + 9 = 0$
18) $2^{x^2-8} = \dfrac{1}{16}$
19) $\log(99 + \log(8 + \log(x-1))) = 2$
20) $\log(x+1) + \log(x+5) = \log 96$
21) $\log|x+1| + \log|x+5| = \log 96$
22) $\log_x(0.0025) = 2$

1.32 Solutions

1. 1 : 1) -15 2)77 3)394

1. 2 : $A = \frac{137}{168}$ $B = \frac{39}{14}$ $C = \frac{15}{14}$ $D = \frac{8}{7}$

1. 3 : $A = \frac{7}{12}$ $B = \frac{11}{12}$ $C = \frac{11}{20}$ $D = -\frac{19}{20}$

1. 4 :

1) $A(2) = 55$ $A(-3) = 0$
2) $B(2) = -201$ $B(-3) = -96$
3) $C(2) = -10$ $C(-3) = -120$
4) $D(2) = 136$ $D(-3) = -849$

1. 5 : $0 \in U$, $0, 2 \notin S,\ T,\ U$ $5 \in S$ $7 \in S$ $\frac{1}{7} \in T$

1. 6 : $D = F = \{6; 8; 10\}$ $E = \varnothing$

1. 7 :

1) $A =]-3\,;\,5\,[$
$\overline{A} =]-\infty\,;\,-3\,] \cup [\,5\,;\,\infty\,[$

2) $B =]-\infty\,;\,1\,[$
$\overline{B} = [\,1\,;\,\infty\,[$

3) $C =]-\infty\,;\,-1\,] \cup]\,3\,;\,\infty[$
$\overline{C} =]-1\,;\,3\,]$

4) $D =]-\infty\,;\,\infty\,[$
$\overline{D} = \varnothing$

5) $E = [\,-8\,;\,8]$
$\overline{E} =]-\infty\,;\,-8\,[\cup]\,8\,;\,\infty\,[$

6) $F = \left]\,-\sqrt{5}\,;\,-1\,\right[\cup \left]\,1\,;\,\sqrt{5}\,\right[$
$\overline{F} = \left]-\infty\,;\,-\sqrt{5}\right] \cup [\,-1\,;\,1\,] \cup \left[\,\sqrt{5}\,;\,\infty\,\right[$

7) $G = [\,10\,;\,\infty\,[$
$\overline{G} =]-\infty\,;\,10\,[$

8) $H =]-\infty\,,\,-4\,[\ \cup\ ]\,4\,;\,\infty\,[$
$\overline{H} = [\,-4\,;\,4\,]$

1. 8 : —

1. 9 : $a = \frac{43}{990}$ $b = \frac{6673}{55'500}$

1. 10 :

1) $A = \frac{1}{4}$
2) $B = \frac{28}{19}$
3) $C = \frac{2a+1}{a+2}$
4) $D = \frac{b+a}{b-a}$
5) $E = \frac{c+b}{c+a}$
6) $F = \frac{3t+2}{4-t^2}$
7) $G = \frac{c-1}{bc}$
8) $H = \frac{-x-y+2xy}{2x}$
9) $I = \frac{x-2}{x(x-1)}$
10) $J = -\frac{1}{4}$
11) $K = \frac{(m-1)(m^2+1)}{m+1}$
12) $L = \frac{-10}{x^2+4}$

1. 11 : $A = 2\sqrt{3}$ $B = 6\sqrt{2}$ $C = 6\sqrt{6}$

1. 12 :

1) $A = \frac{\sqrt{3}}{3}$
2) $B = \frac{3\sqrt{5}}{5}$
3) $C = \sqrt{3}$
4) $D = \sqrt{7}$
5) $E = 6 - 5\sqrt{3}$
6) $F = \frac{1}{2}$
7) $G = \frac{\sqrt{10}+2}{3}$
8) $H = 3\sqrt{2} - 2\sqrt{3}$
9) $I = 6\sqrt{35} - 35$
10) $J = 2\sqrt{5}$
11) $K = -\frac{\sqrt{3}+5}{2}$

1. 13 : $A = \sqrt{13}$ $B = \sqrt{14}$ $C = \sqrt{20}$

1. 14 : —

1. 15 :

1) $A = 16e^8 f^{-12}$
2) $B = 1$
3) $C = 25r^5 s^{-6} t^{-1}$
4) $D = \sqrt[4]{\frac{x}{y}}$
5) $E = c^{\frac{7}{4}}$
6) $F = 5^{-1} x^{\frac{1}{2}} y^{-1}$
7) $G = a^{-1} b^{\frac{13}{2}}$
8) $H = 2^{\frac{-11}{6}}$
9) $I = a^{\frac{25}{36}}$

1. 16 :

1) $A = a^{32}$
2) $B = x^{-11}$
3) $C = u^2$
4) $D = n^5$
5) $E = h^2$
6) $F = b^{3n}$
7) $G = z^{2+6n}$
8) $H = e^{2x}$
9) $I = -a^5$
10) $J = x^{24}$
11) $K = a^{20} b^{-32} x^4$
12) $L = -27x^{12}$
13) $M = x^{-9} y^{15}$
14) $N = \frac{c^{3n}}{a^n}$
15) $O = -a^3$
16) $P = -a^3$

1. 17 :

1) $A = \frac{b^{12}}{a^8 c^{21}}$
2) $B = \frac{x^{12} z^6}{y^{27}}$
3) $C = \frac{9x^8}{y^8}$
4) $D = \frac{x^2}{y^{5n-4}}$
5) $E = b^2$
6) $F = ab^{-7}$
7) $G = 4x^2 y$
8) $H = 2x^{-10} y^8$

1. 18 :

1) $A = \frac{9a^2 - 16b^2}{12ab}$
2) $B = x + 2$
3) $C = a^{\frac{15}{8}}$
4) $D = \frac{5a}{a^2 - 9}$
5) $E = a^{4n+1}$
6) $F = \frac{3ax}{2}$
7) $G = \frac{1}{a^7 b^3}$

1. 19 :

1) $A = 6a^2 + 13ab + 6b^2$
2) $B = -55x^2 + x + 36$
3) $C = 3a^2 + 4b^2 + 3c^2 + 8ab + 10ac + 8bc$
4) $D = 2x^2 - 5y^2 + 2x + 12y - 9xy - 4$

1. 20 :

1) $A = 9x^2 + 12xy + 4y^2$
2) $B = 9x^2 + 18xy + 9y^2$
3) $C = 3x^2 + 6xy + 3y^2$
4) $D = a^2 + 2ab + b^2 - c^2$
5) $E = 16x^2 - 8\sqrt{6}xy + 6y^2$
6) $F = x^2 - \frac{1}{x^2}$
7) $G = 7 - 2\sqrt{10}$
8) $H = x^3 - 9x^2y + 27xy^2 - 27y^3$
9) $I = 9x^2 - y^2$
10) $J = 16a^4 + 96a^3b + 216a^2b^2 + 216ab^3 + 81b^4$

1. 21 :

1) $A = 4x^2 + 20x + 25 = (2x + 5)^2$
2) $B = x^2 + 14x + 49 = (x + 7)^2$
3) $C = x^2 - 20x + 100 = (x - 10)^2$
4) $D = (3x + 5)^2 = 9x^2 + 30x + 25$
5) $E = 64 + 48x^2 + 9x^4 = (8 + 3x^2)^2$
6) $F = 9x^2 - 12x + 4 = (3x - 2)^2$
7) $G = x^2 - 16 = (x + 4)(x - 4)$
8) $H = 4x^2 - 24x + 36 = (2x - 6)^2$

1. 22 : 40'000

1. 23 : $x_{1,2} = \pm\sqrt{5}$ $x_1 = 0$ $x_2 = -3$ $x_{1,2} = 3 \pm \sqrt{3}$

1. 24 :

1) $x_1 = -4$ $x_2 = 1$
2) No solutions in $\mathbb{R}$
3) $x = -2$
4) $x_1 = 2$ $x_2 = 0.5$
5) $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. 25 :

1) $A(x) = (x + 3)^2$
   $x = -3$
2) $B(x) = (x - 11)(x + 11)$
   $x_{1,2} = \pm 11$
3) $C(x) = (4x^2 + 1)(2x - 1)(2x + 1)$
   $x_{1,2} = \pm\frac{1}{2}$
4) $D(x) = 2x(x - 3)(x + 3)$
   $x_{1,2} = \pm 3$ $x_3 = 0$
5) $E(x) = (x - 5)(x + 1)^2$
   $x_1 = -1$ $x_2 = 5$
6) $F(x) = (2x - 1)^2$
   $x = \frac{1}{2}$
7) $G(x) = (x - 5)^2$
   $x = 5$
8) $H(x) = (3x + 2)^2$
   $x = -\frac{2}{3}$
9) $I(x) = 2(2x + 1)(x - 1)$
   $x_1 = -\frac{1}{2}$ $x_2 = 1$
10) $J(x) = (x + 1)(x - 1)(x + 3)(x - 3)$
    $x_{1,2} = \pm 3$ $x_{3,4} = \pm 1$

1. 26 :

1) $A(x) = (2x + y)^2$
2) $B(x) = (x + 4)^2$
3) $C(x) = (x - 2y)(x + 2y)$
4) $D(x) = 5xy(y - 1)^2$
5) $E(x) = 3(x + 1)(x - 1) \cdot \left(x + \frac{2\sqrt{3}}{3}\right)\left(x - \frac{2\sqrt{3}}{3}\right)$

1. 27 : —

1. 28 :

1) $x = -\frac{1}{5}$
2) $x_1 = \frac{1}{2} \quad x_2 = -3$
3) $x_1 = \frac{k\left(\sqrt{5}-1\right)}{2} \quad x_2 = \frac{-k\left(\sqrt{5}+1\right)}{2}$
4) $x_1 = \sqrt{3}+\sqrt{2} \quad x_2 = \sqrt{3}-\sqrt{2}$
5) $x_1 = a \quad x_2 = -\frac{1}{a}$
6) $x_1 = 16 \quad x_2 = 0$

1. 29 :

1) $d_1 = 0 \quad d_{12} = 12$
2) $d_1 = 5 \quad d_2 = -3$
3) $d = 4$
4) $d_1 = 2 \quad d_2 = -1$

1. 30 : 1) $\frac{x+1}{x-2}$ 2) $\frac{(x-1)(x+2)}{(x+1)(x-2)}$ 3) $\frac{x+4}{x+3}$

1. 31 : P: $x_1 = 1 \quad x_2 = -2 \quad x_3 = 3$ Q:$x = -3$

1. 32 :

1) $Q(x) = x^2 + 2x + 1$
$R(x) = 2x$
2) $Q(x) = 7x^2 + 25$
$R(x) = 2x + 96$
3) $Q(x) = x^3 - 3x^2 - 3x + 9$
$R(x) = 9x^2 - 27x + 2$
4) $Q(x) = x^2 - 3x + 4$
$R(x) = -6$

1. 33 : $x_1 = -1 \quad x_2 = \frac{1}{4}$ $x_1 = \frac{2}{3} \quad x_2 = -\frac{1}{2}$

1. 34 : $P(x)$ divisible by $x - 2$

1. 35 :

1) $Q(x) = x - 1 \quad R(x) = 0$
2) $Q(x) = 2x + 1 \quad R(x) = 3$
3) $Q(x) = 3x - 7 \quad R(x) = 26$
4) $Q(x) = x^2 + 3 \quad R(x) = -2$

1. 36 : $x_1 = -1$ multiplicity 3 $x_2 = \frac{5}{3}$ multiplicity 2 $x_3 = \frac{-3}{2}$ multiplicity 1

1. 37 :

1) $P(x) = 4(x+8)\left(x-\frac{3}{2}\right)\left(x+\frac{3}{2}\right)$ $x_1 = -8 \quad x_{2,3} = \pm\frac{3}{2}$
2) $P(x) = -9x\left(x-\frac{1}{3}\right)\left(x+\frac{4}{3}\right)$ $x_1 = 0 \quad x_2 = \frac{1}{3} \quad x_3 = -\frac{4}{3}$
3) $P(x) = (x-5)(x-3)(x+13)$ No, infinite number of possibilities
4) $P(x) = 2x^3 - 5x^2 - 11x + 14$

1. 38 :

1) $P_1(x) = 6\,(x-3)\left(x+\frac{1}{2}\right)\left(x-\frac{2}{3}\right)$ $x_1 = 3$ multiplicity 1 $x_2 = -\frac{1}{2}$ multiplicity 1 $x_3 = \frac{2}{3}$ multiplicity 1

2) $P_2(x) = (x-1)\,(x+2)\left(x-(3+\sqrt{5})\right)\left(x-(3-\sqrt{5})\right)$ $x_1 = 1$ multiplicity 1 $x_2 = -2$ multiplicity 1 $x_3 = 3+\sqrt{5}$ multiplicity 1 $x_4 = 3-\sqrt{5}$ multiplicity 1

3) $P_3(x) = (x+2)^2\,(x^2-2x+3)$ $x_1 = -2$ multiplicity 2

1. 39 : $P(x) = 2\,(x+1)\,(x-0.5)\,(x-3)$

1. 40 : 1) $m = 3$ 2) $k = -0.6$ 3) $p = 2.5$

1. 41 : 1) Infinite number of solutions 2) $x = 2 \quad y = 5$

1. 42 : 1) $\begin{cases} 3x+5y = \underline{-2} \\ \underline{-6x} - 10y = 4 \end{cases}$ 2) $x - 1.5y = k$ with $k \neq -0.5$

1. 43 : $x_1 = 21\text{m} \quad y_1 = 27\text{m} \quad x_2 = 27\text{m} \quad y_2 = 21\text{m}$

1. 44 : 18cm and 12cm

1. 45 :

1) $x = 2 \quad y = -1 \quad z = 3$ 2) $x = \frac{1}{2} \quad y = -\frac{1}{2} \quad z = 3$ 3) Infinite number of solutions

1. 46 :

1) $x_{1,2} = \pm 2 \quad x_{3,4} =$ impossible
2) $x_1 = 0 \quad x_2 = \frac{3}{2} \quad x_3 = 1$
3) $x_1 = -1 \quad x_2 = \sqrt[3]{-3}$
4) $x_1 = 0 \quad x_{2,3} = \pm 2 \quad x_{4,5} =$ impossible
5) $x_1 = 3 \quad x_2 = -2$

1. 47 : $a_1 = 5 \quad b_1 = 4 \quad a_2 = 4 \quad b_2 = 5$

1. 48 : 1) $x = 2$ 2) $x = -2$ 3) $x = \frac{9}{16}$ 4) $x = 7$

1. 49 : 1) No real solutions 2) $x_{1,2} = \frac{-1 \pm 1\sqrt{53}}{13}$ 3) $x = -\frac{34}{5}$

1. 50 :

1) $x_1 = 7 \quad x_2 = -1$
2) $x_1 = 4 \quad x_2 = -\frac{2}{3}$
3) $x_{1,2} = \pm 5$
4) $x_1 = \pm 3$

1. 51 :

1) $x = -\frac{11}{3}$
2) No solutions
3) $x = \frac{5}{12}$
4) $a \neq 2 \quad x = \frac{-a-1}{a-2}$
5) $x_1 = 3 \quad x_2 = -5$
6) $x_1 = a \quad x_2 = -b$
7) No solutions
8) $x = -\frac{9}{17}$
9) $x = 1$
10) $a \notin \{-1; 0\} \quad x = -\frac{5a^2}{a+1}$

1. 52 :

1) $x = -\frac{2}{3}$
2) $x = \pm 3$
3) $x = 4$
4) No solutions
5) $x = 0$
6) $x_1 = \frac{2}{3} \quad x_2 = -8$
7) $x = 6$
8) $x_{1,2} = \frac{-1 \pm \sqrt{53}}{13}$
9) $x = -3$
10) $x_{1,2} = -2 \pm \sqrt{13}$
$x_3 = -1$
11) $x_1 = \frac{13}{4} \quad x_2 = -\frac{1}{4}$

1. 53 :

1) $x < 1$
2) $x \geq 0$
3) $x > \frac{7}{3}$
4) $x \leq \frac{-14}{3}$
5) $x \in]-\infty; -4[\cup]1; \infty[$
6) $x \in [-1; 5]$

1. 54 : $\Delta = p^2 + 16 > 0$

1. 55 : 1) Impossible 2) $p \in]-5; 1[$ 3) $b = \frac{25}{2}$

1. 56 :

1) $d \in]0; 12[$
2) $d \in]-3; 5[$
3) $d > 4$
4) $d \in]-1; 2[$

1. 57 :

f : yes
g : no
h : yes
i : yes
j : yes
k : no
l : yes
m : yes
n : yes

1. 58 :

1) $D = \mathbb{R} \quad f_1(D) = \{3\}$
2) $D = \mathbb{R} \quad f_2(D) = \mathbb{R}$
3) $D = \mathbb{R} \quad f_3(D) = [-1,125; +\infty[$
4) $D = \mathbb{R} \setminus \{1\} \quad f_4(D) = \mathbb{R}^*$
5) $D = \mathbb{R} \quad f_5(D) = \mathbb{R}_+$
6) $D = [-1; +\infty[\quad f_6(D) = [0; +\infty[$

1. 59 :

1) $1 \quad -3 \quad 4 \quad \frac{7}{2} \quad \frac{9}{2} \quad 3 \quad -1$

2) $x_1 = -10 \quad x_2 = -8 \quad x_3 = -\frac{26}{5} \quad x_4 = 8 \quad x_5 = \frac{34}{3}$

3) $[-3;5] \quad [2;5[\quad [-3;2[\cup [3;4] \quad \{-12\} \cup [4;6] \quad [-4;4[$

4) $(4;2)$

1. 60 :

1) $D_1 = \mathbb{R} \setminus \{5\}$
2) $D_2 = [5;+\infty[$
3) $D_3 = \mathbb{R} \setminus \left\{\frac{3}{5}\right\}$
4) $D_4 = \mathbb{R} \setminus \{1\}$
5) $D_5 = \mathbb{R} \setminus \{\pm 3\}$
6) $D_6 = \left]-\infty; \frac{2}{3}\right]$
7) $D_7 = \mathbb{R} \setminus \{-4;+3\}$
8) $D_8 = \mathbb{R} \setminus \{4\}$
9) $D_9 =]-\infty;0[\cup]2;\infty[$
10) $D_{10} = [3;+\infty] \cup \{-1\}$
11) $D_{11} = \mathbb{R}$
12) $D_{12} = \mathbb{R} \setminus]-1;1[$

1. 61 :

1) f is else
2) g is odd
3) h is even
4) i is odd
5) j is even
6) k is else

1. 62 :

1) $x \in]-\infty;-4[\cup [-2;1[\cup [2;\infty[$
2) $x \in]-\infty;0[\cup \left]\frac{1}{2};\frac{5}{3}\right[$
3) $x \in]-1;0[\cup]1;\infty[$
4) $x \in \left]-\infty;-\frac{1}{3}\right[\cup]2;\infty[$

1. 63 : —

1. 64 :

1) $y = 3x$
2) $y = -\frac{1}{2}x - \frac{9}{2}$
3) $y = \frac{3}{5}x - \frac{16}{5}$
4) $x = 2$
5) $y = -\frac{3}{4}x + \frac{5}{2}$

1. 65 : $y = \frac{1}{2}x + 2$

1. 66 : 1) $g : y = -2x + 5$ 2) $h : y = \frac{1}{2}x - \frac{3}{2}$ 3) $f \perp h$

1. 67 : $x = \frac{5}{16} \quad A = \left]\frac{5}{16};\infty\right[$

1. 68 : $f : y = -2x + 4 \quad g : y = \frac{1}{2}x - \frac{3}{2}$

1. 69 : —

1. 70 :

1) $f_1 : x = 0 \qquad V(0;0)$

2) $f_2 : x_{1,2} = \pm 2 \qquad V(0;4)$

3) $f_3 : x = -1 \qquad V(-1;0)$

4) $f_4 : x_1 = -1 \qquad x_2 = 3, V(1;8)$

5) $f_5 : V(1;4)$

6) $f_6 : x_1 = 2 \qquad x_2 = -6 \qquad V(-2;-16)$

1. 71 : $V\left(1;\frac{7}{2}\right) \qquad I_1(2;3) \quad I_2(-2;-1)$
$y = -\frac{1}{2}x^2 + x + 3 = -\frac{1}{2}(x - 1 + \sqrt{7})(x - 1 - \sqrt{7}) = -\frac{1}{2}(x-1)^2 + \frac{7}{2}$

1. 72 : 1) $I(2;3) \qquad$ 2) $b = \frac{17}{4}$

1. 73 : $y = x^2 - 3x + 7$

1. 74 : $y = -\frac{1}{3}(x-3)^2 + 3$

1. 75 : $y = x \qquad y = -2x$

1. 76 : $a \in]-12;0[$

1. 77 : $n = 4 \qquad V(1;10)$

1. 78 : $m_1 = 3 \quad x = \frac{-3}{2} \qquad m_2 = 1 \quad x = \frac{-1}{2}$

1. 79 : $V(-1;8) \qquad y = -2(x+1)^2 + 8$

1. 80 : $g : x \longmapsto y = |\, f(x)\,| = \begin{cases} x^2 - x - 6 & if \;\; x \leq -2 \\ -x^2 + x + 6 & if \;\; -2 < x < 3 \\ x^2 - x - 6 & if \;\; x \geq 3 \end{cases}$

1. 81 : $I_1(-1;0) \qquad I_2(0;0) \qquad I_3(2;0)$

1. 82 : $I_{x_1}(-3;0) \quad I_{x_2}(3;0) \qquad I_y(0;-\frac{9}{4}) \qquad f$ even $\qquad D = \mathbb{R}$
$R = f(D) = [-6,25;+\infty[$

1. 83 : 1) $x_1 = 3 \quad x_{2,3} = \frac{-3 \pm \sqrt{5}}{2} \qquad$ 2) $x \in \left[\frac{-3-\sqrt{5}}{2}; \frac{-3+\sqrt{5}}{2}\right] \cup [3;+\infty[$

1. 84 :

x		0		1	
$f(x)$	+	0	−	0	+

1. 85 : $I_1\left(-1;-\frac{1}{4}\right) \qquad I_2\,(4;1)$

1. 86 : —

1. 87 : —

1. 88 : $k_{1,2} = \frac{7 \pm 2\sqrt{10}}{9}$

1. 89 : $D = [-9; +\infty[$ $f(D) = [0; +\infty[$ $D = [-4; +\infty[$
$g(D) = [-6; +\infty[$ $D = [-3; 3]$ $h(D) = [0; 3]$

1. 90 : $b = -2$ $c = \frac{1}{2}$

1. 91 : $t_1 : y = -x + 1$ $t_2 : y = -x - 3$ $P_1(-2; 3)$ $P_2(-4; 1)$

1. 92 : —

1. 93 : —

1. 94 : $(f \circ g)(x) = x^2 + 2x + 1$ $(g \circ f)(x) = x^2 + 1$

1. 95 : $(h \circ g)(x) = x^2 - 6x + 9$ $((h \circ g) \circ f)(x) = 4x^2 - 12x + 9$
$(g \circ f)(x) = 2x - 3$ $(h \circ (g \circ f))(x) = 4x^2 - 12x + 9$

1. 96 : $(f \circ g)(x) = (g \circ f)(x) = x$ $f =^i g$

1. 97 : ${}^i f(x) = \frac{3x+1}{2x-1}$

1. 98 : $f \circ g(x) = \frac{-7x-8}{16x+9}$ ${}^i f(x) = \frac{5x+3}{2-x}$
${}^i g(x) = (x) = \frac{-1-2x}{3x-1}$ $({}^i g \circ^i f)(x) =^i (f \circ g(x))^{-1} = \frac{-9x-8}{16x+7}$

1. 99 : $D =]-\infty; 1]$ $f(D) = [0; \infty[$ ${}^i f(x) = -x^2 + 1$

1. 100 : $a = -d$

1. 101 : $D = [1; +\infty[$ $f(D) = [-4; +\infty[$ ${}^i f(x) = \sqrt{x+4} + 1$

1. 102 : —

1. 103 : —

1. 104 :

1) 4
2) -2
3) 0
4) Impossible
5) 1
6) $-\frac{1}{3}$
7) $\frac{3}{4}$
8) 4

1. 105 :

1) $\log(p) + \log(q) + \log(r)$
2) $\log(p) + 2\log(q) + 3\log(r)$
3) $2 + \log(p) + 5\log(r)$
4) $\frac{1}{2}\log(p) - \log(q) - \frac{1}{2}\log(r)$
5) $\log(q) + 7\log(r) + \log(p) - 1$
6) $\frac{1}{2} + 5\log(p) + \frac{1}{2}\log(r) - 0,5\log(q)$

1. 106 :

1) $5\log(a) + \frac{1}{3}\log(b) - \log(c)$
2) $\log\left(\frac{x+1}{x-1}\right)$
3) $\log\left(\frac{a^2c^3}{\sqrt{b}}\right)$
4) $\frac{1}{2}\log(a) + \frac{1}{6}\log(b)$

1. 107 : 189 13594

1. 108 :

1) $x = \log(9,56)$
2) Impossible
3) $x = 10^{2,5}$
4) $x = \frac{101}{400}$
5) $x = 10^{-2}$
6) $x_{1,2} = \pm 11$
7) $x = -0.5$
8) $x = 10^{10}$
9) $x_1 = 2 \quad x_2 = -1$
10) $x = \frac{1}{\log(2)}$
11) $x = \frac{\log(2)}{\log(1,03)}$
12) $x = \frac{31}{5}$
13) $x = \frac{\log(5)}{\log(2,8)}$
14) $x = 500$
15) $x = -1$
16) $x_1 = 100 \quad x_2 = 10^{-1}$
17) $x_{1,2} = \pm 2$
18) $x_{1,2} = \pm 2$
19) $x = 101$
20) $x = 7$
21) $x_1 = 7 \quad x_2 = -13$
22) $x = \sqrt{\frac{25}{10000}}$