1.1.3 Exercices

1. 1 : Prove the three following formulas :

1) $S_n = 1^2 + 2^2 + 3^2 + 4^2 + \cdots + (n-1)^2 + n^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

2) $S_n = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$

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1. 2 : Prove the following assertions :

1) Any number of the form $2^{3n} - 1$ is divisible by 7.

2) $S = 1 + 2 + 2^2 + 2^3 + \cdots + 2^{n-1} = 2^n - 1 \;\; \forall\, n \in \mathbb{N}^*$.

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1. 3 :

1) Use induction to proof that the sum of the n first powers of a real basis r $(r \neq 1)$ is

$$S_n = 1 + r + r^2 + \ldots + r^{n-1} = \frac{1 - r^n}{1 - r}$$

2) For r such that $|r| < 1$ show that the infinite sum is

$$S = 1 + r + r^2 + r^3 + \ldots$$

3) Use the previous formula to determine the values of the following infinite sums :

a) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$

b) $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots$

c) $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \ldots$

d) $\frac{3}{2} + \frac{3}{8} + \frac{3}{32} + \frac{3}{128} + \ldots$

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1. 4 :

1) Prove that $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \ldots + \frac{n}{2^n} = \frac{2^{n+1} - n - 2}{2^n}$

2) Compute $\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \ldots$

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1. 5 : Given a_n the maximal number of intersection points of n lines of the plane.

1) Find, thanks to a drawing, the values of a_1, a_2, a_3, a_4 and a_5.
2) Explain the equality : $a_{k+1} = a_k + k$
3) Prove that : $a_n = \dfrac{1}{2}n(n-1)$

1.1.4 Solutions

1. 1 : —

1. 2 : —

1. 3 : $S_a = \frac{3}{2}$ $S_b = \frac{4}{5}$ $S_c = \frac{4}{3}$ $S_d = 2$

1. 4 : $S_\infty = 2$

1. 5 : —