

EXERCICE S.1

$$\begin{pmatrix} 3 \\ -8 \\ m \end{pmatrix} \parallel \begin{pmatrix} n \\ 9,6 \\ p \end{pmatrix} \parallel \begin{pmatrix} -9 \\ 9 \\ -15 \end{pmatrix}$$

$$\begin{matrix} \vec{a} & \vec{b} & \vec{c} \end{matrix}$$

$$\vec{a} \parallel \vec{b} \Rightarrow \exists t \in \mathbb{R}: \vec{a} = t \cdot \vec{b} \Rightarrow \begin{pmatrix} 3 \\ -8 \\ m \end{pmatrix} = t \begin{pmatrix} n \\ 9,6 \\ p \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{matrix} 3 = t \cdot n & \Rightarrow 3 = -0,833 \cdot n \Leftrightarrow n = -3,6 \checkmark \\ -8 = t \cdot 9,6 & \Rightarrow t = -0,833 \end{matrix}$$

$$m = t \cdot p \Rightarrow m = -0,833 \cdot p \Rightarrow m = -0,833(6) + 15 \checkmark$$

$$\vec{b} \parallel \vec{c} \Rightarrow n = k \cdot (-9) \Rightarrow -3,6 = -9k \Rightarrow k = +0,4$$

$$9,6 = k \cdot 9 \Rightarrow 9,6 = +0,4 \cdot 9 = +3,6 \checkmark$$

$$p = k \cdot (-15) \Rightarrow p = +0,4(-15) \Rightarrow p = -6 \checkmark$$

EXERCICES S.2

$$a) \vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

On suppose que $\vec{c} = \lambda \vec{a} + \mu \vec{b} \Rightarrow$

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \Leftrightarrow \begin{cases} 2 = \lambda + 2\mu & (1) \\ 1 = \lambda + 3\mu & (2) \\ 3 = \lambda + 4\mu & (3) \end{cases}$$

$$(1) - (2) \text{ donne } 1 = -\mu \Leftrightarrow \mu = -1$$

$$\text{D'où } (2) \quad 1 = \lambda - 3 \Leftrightarrow \lambda = 4$$

On remplace en (3) $3 = 4 + (-4)$ impossible

Donc $\vec{c}$ n'est pas une combinaison linéaire de $\vec{a}$ et $\vec{b}$. C'est que $\vec{c}$ n'est pas lin. dépend de $\vec{a}$ et $\vec{b}$.

On doit aussi justifier que $\vec{a} \times \vec{b} \neq \vec{0}$.

$$\text{Soit } \vec{a} \parallel \vec{b} \Rightarrow \vec{a} = t \cdot \vec{b} \Leftrightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = t \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \Rightarrow \begin{matrix} t = 1/2 \\ t = 1/3 \\ t = 1/4 \end{matrix}$$

Donc $\vec{a}, \vec{b}, \vec{c}$ sont lin. ind.

et ils forment une base de $\mathbb{R}^3$

impossible

2-

$$b) \vec{a} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 7 \\ -2 \\ -11 \end{pmatrix}$$

$$\text{Soit } \vec{c} = \lambda \vec{a} + \mu \vec{b} \Rightarrow \begin{pmatrix} 7 \\ -2 \\ -11 \end{pmatrix} = \lambda \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 7 = -\lambda + 2\mu & \textcircled{1} \\ -2 = 4\lambda + 5\mu & \textcircled{2} \\ -11 = 3\lambda - \mu & \textcircled{3} \end{cases}$$

$$\textcircled{1} + 2\textcircled{3}$$

$$\begin{array}{r} 7 = -\lambda + 2\mu \\ + -22 = 6\lambda - 2\mu \\ \hline -15 = 5\lambda \end{array}$$

$$\Leftrightarrow \lambda = -3$$

$$\textcircled{1}: 7 = 3 + 2\mu \Leftrightarrow 4 = 2\mu \Leftrightarrow \mu = 2$$

$$\textcircled{2}: -2 = 4(-3) + 5 \cdot 2$$

$$-2 = -12 + 10 \quad \checkmark$$

Donc $\vec{c}, \vec{a}, \vec{b}$ sont linéairement dépendants $\Rightarrow$
ils ne forment pas une base de $\mathbb{R}^3$

$$b. \quad \vec{a} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 2 \\ -4 \\ 15 \end{pmatrix}$$

$$\vec{c} = \lambda \vec{a} + \mu \vec{b} \Rightarrow \begin{pmatrix} 2 \\ -4 \\ 15 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 2 = 2\lambda + 4\mu \\ -4 = -3\lambda - 5\mu \\ 15 = 5\lambda + 2\mu \end{cases} \Leftrightarrow \begin{cases} 1 = \lambda + 2\mu & *(-3) \textcircled{1} \\ -4 = -3\lambda - 5\mu & \textcircled{2} \\ 15 = 5\lambda + 2\mu & \textcircled{3} \end{cases}$$

$$\textcircled{1}, \textcircled{2} \quad -3 = -3\lambda - 6\mu$$

$$+ 4 = 3\lambda + 5\mu$$

$$1 = -\mu \Leftrightarrow \mu = -1$$

$$\textcircled{1} \quad 1 = \lambda - 2 \Leftrightarrow \lambda = 3$$

On remplace en $\textcircled{3}$

$$15 = 5 \cdot 3 + 2 \cdot (-1) \Leftrightarrow 15 = 15 - 2 \Leftrightarrow \underline{\underline{2 = 0}}$$

$$d) \vec{a} = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{c} = \frac{1}{2}\vec{a} - \frac{1}{3}\vec{b} \Rightarrow c = \frac{1}{2} \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \\ 2 \end{pmatrix} - \begin{pmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{pmatrix} =$$

$$\Rightarrow \vec{c} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{3} \\ \frac{5}{2} & +\frac{1}{3} \\ 2 & -\frac{2}{3} \end{pmatrix} \Rightarrow \vec{c} = \begin{pmatrix} -\frac{1}{6} \\ \frac{17}{6} \\ \frac{4}{3} \end{pmatrix}$$

$$\vec{d} = -5\vec{a} + 7\vec{b} \Rightarrow \vec{d} = \begin{pmatrix} -5 \\ -25 \\ -20 \end{pmatrix} + \begin{pmatrix} 21 \\ -7 \\ 14 \end{pmatrix} \Rightarrow \vec{d} = \begin{pmatrix} 16 \\ -32 \\ -6 \end{pmatrix}$$

$$2\vec{a} - 3\vec{b} - 5\vec{c} = 0 \Leftrightarrow 2\vec{a} - 3\vec{b} = 5\vec{c} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{5}(2\vec{a} - 3\vec{b}) = \vec{c} \Leftrightarrow \vec{c} = \frac{1}{5} \left[\begin{pmatrix} 2 \\ 10 \\ 8 \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix} \right] = \frac{1}{5} \begin{pmatrix} -7 \\ 13 \\ 2 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \vec{c} = \begin{pmatrix} -\frac{7}{5} \\ \frac{13}{5} \\ \frac{2}{5} \end{pmatrix}$$

EXERCISE 5.3

A	(1; -2; 5)	(-5; $\frac{1}{2}$; 6)	(4; -5; -13) ^③	(0; -1; 6)	(4; ...; $\frac{1}{4}$)
B	(5; ...; 1; 4) ^①	(4; 1; $\frac{1}{3}$)	(6; 2; -4)	(a; 3; b) ^⑦	(...; $\frac{1}{2}$; $\frac{1}{2}$) ^⑥
$\vec{AB}$	$\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$	$\begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$ ^②	$\begin{pmatrix} 2 \\ 7 \\ 9 \end{pmatrix}$	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} // \begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix}$ ^⑤	$\begin{pmatrix} 0 \\ 5 \\ \dots \end{pmatrix}$

$$\textcircled{1} \quad \vec{AB} = \vec{OB} - \vec{OA} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \Leftrightarrow \begin{cases} 4 = x - 1 \\ 3 = y + 2 \\ -1 = z - 5 \end{cases} \Leftrightarrow \begin{cases} x = 5 \\ y = 1 \\ z = 4 \end{cases}$$

$$\textcircled{2} \quad \vec{AB} = \begin{pmatrix} 4 \\ 1 \\ \frac{1}{3} \end{pmatrix} - \begin{pmatrix} -5 \\ \frac{1}{2} \\ 6 \end{pmatrix} \Rightarrow \vec{AB} = \begin{pmatrix} 9 \\ \frac{1}{2} \\ -\frac{17}{3} \end{pmatrix}$$

$$\textcircled{3} \quad \vec{AB} = \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix} - \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2 \\ 7 \\ 9 \end{pmatrix} = \begin{pmatrix} 6-x \\ 2-y \\ -4-z \end{pmatrix} \Leftrightarrow \begin{cases} 2 = 6-x \\ 7 = 2-y \\ 9 = -4-z \end{cases} \Leftrightarrow \begin{cases} x = 4 \\ y = -5 \\ z = -13 \end{cases}$$

$$\textcircled{4}, \textcircled{5} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = t \begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix} \Rightarrow \begin{cases} x = 5t \\ y = -8t \\ z = 2t \end{cases} \quad \text{et} \quad \begin{pmatrix} 5t \\ -8t \\ 2t \end{pmatrix} = \begin{pmatrix} a \\ 3 \\ b \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix} \Rightarrow \begin{cases} 5t = a \\ -8t = 3 + 1 \\ 2t = b - 6 \end{cases}$$

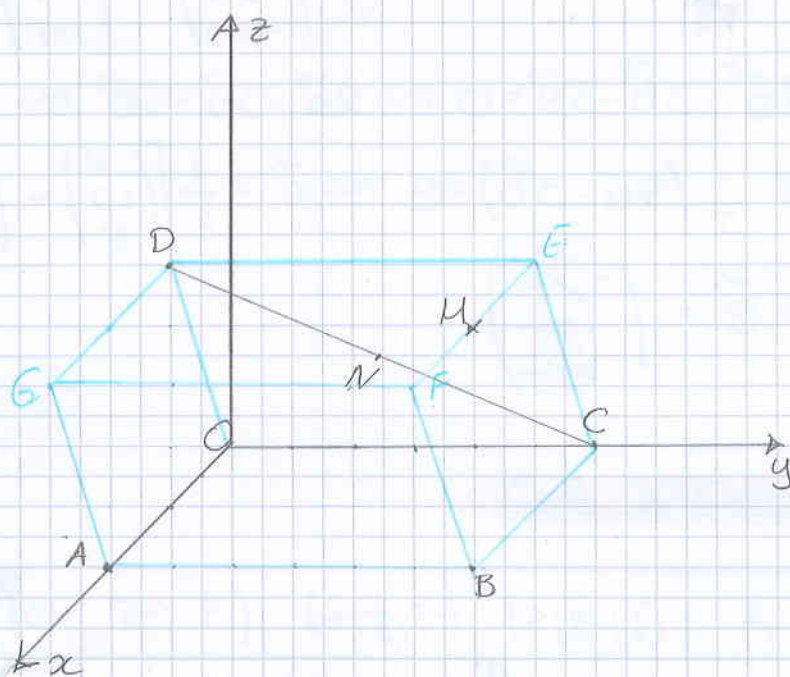
$$-8t = 4 \Leftrightarrow t = -\frac{1}{2} \Rightarrow 5 \cdot \frac{1}{2} = a \Rightarrow a = \frac{5}{2} \quad \frac{-2}{2} = b - 6 \Leftrightarrow b = 5$$

$$⑥ \quad \underset{\text{A}}{\underset{\parallel}{(4; a; \frac{1}{4})}} \quad \underset{\text{B}}{\underset{\parallel}{(b; \frac{1}{2}; \frac{1}{2})}} \quad \underset{\text{AB}}{\underset{\parallel}{(0; 5; c)}}$$

$$\begin{pmatrix} 0 \\ 5 \\ c \end{pmatrix} = \begin{pmatrix} b \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 4 \\ a \\ \frac{1}{4} \end{pmatrix} \Leftrightarrow \begin{cases} 0 = b - 4 \\ 5 = \frac{1}{2} - a \\ c = \frac{1}{2} - \frac{1}{4} \end{cases} \Leftrightarrow \begin{cases} b = 4 \\ a = -\frac{9}{2} \\ c = \frac{1}{4} \end{cases}$$

EXERCICE 5.4

$$O(0;0;0) \quad A(4;0;0) \quad B(4;6;0) \quad C(0;6;0) \quad D(2;0;4)$$



$$\vec{OC} = \vec{DE} \Rightarrow \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} x_E - 2 \\ y_E - 0 \\ z_E - 4 \end{pmatrix} \Leftrightarrow \begin{cases} x_E = 2 \\ y_E = 6 \\ z_E = 4 \end{cases} \quad \underline{E(2;6;4)}$$

$$\vec{EF} = \vec{CB} \Rightarrow \begin{pmatrix} x_F - 2 \\ y_F - 6 \\ z_F - 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} x_F = 6 \\ y_F = 6 \\ z_F = 4 \end{cases} \quad \underline{F(6;6;4)}$$

$$\vec{DG} = \vec{OA} \Rightarrow \begin{pmatrix} x_G - 2 \\ y_G - 0 \\ z_G - 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} x_G = 6 \\ y_G = 0 \\ z_G = 4 \end{cases} \quad \underline{G(6;0;4)}$$

$$M: x_M = \frac{x_E + x_F}{2} \Rightarrow x_M = \frac{2+6}{2} \Rightarrow x_M = 4$$

$$y_M = \frac{y_E + y_F}{2} \Rightarrow y_M = \frac{6+6}{2} \Rightarrow y_M = 6 \quad \underline{M(4;6;4)}$$

$$z_M = \frac{z_E + z_F}{2} \Rightarrow z_M = \frac{4+4}{2} \Rightarrow z_M = 4$$

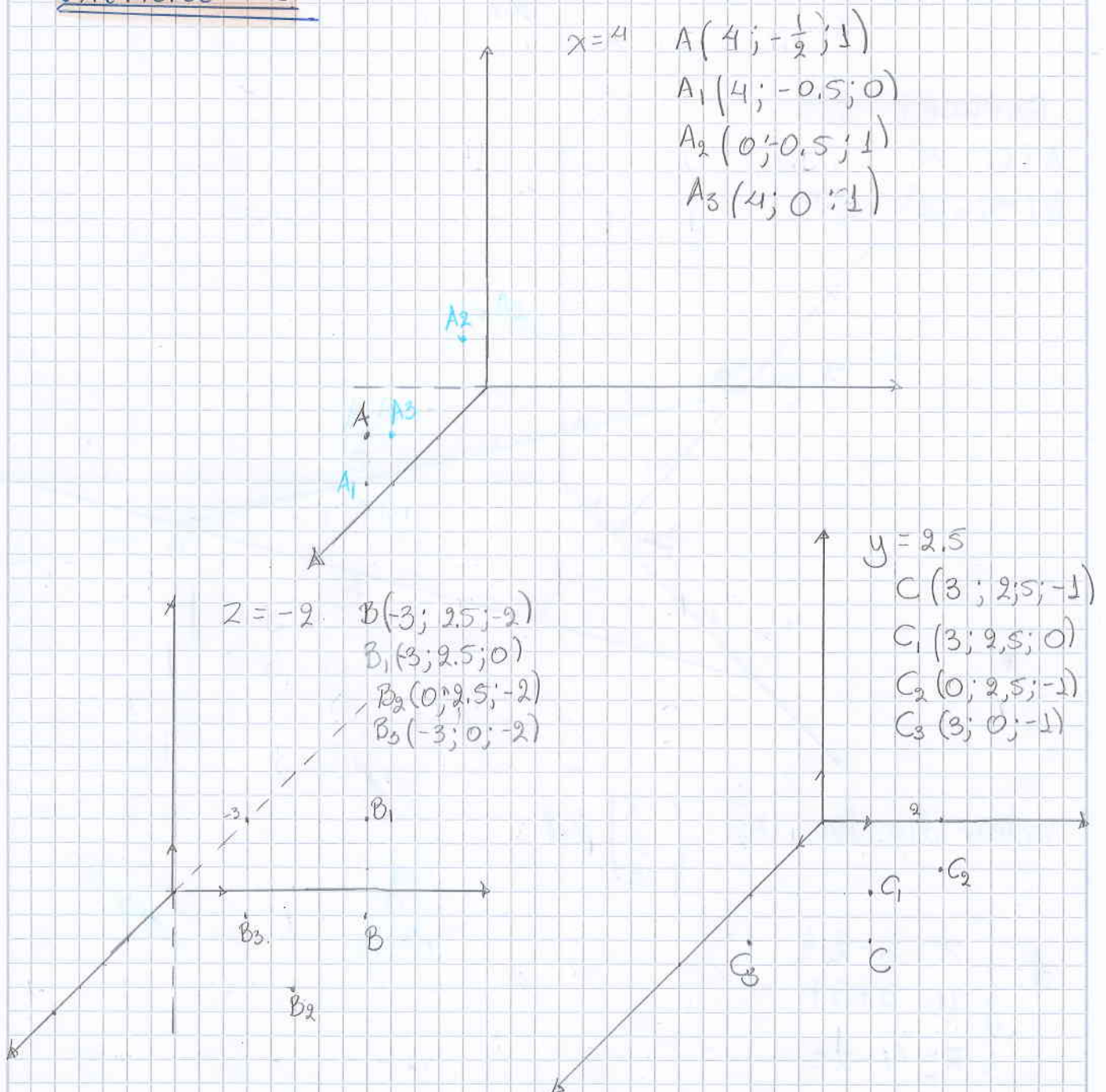
$$\underline{N}: x_N = \frac{x_D + x_C}{2} \Rightarrow x_N = \frac{0+2}{2} \Rightarrow x_N = 1$$

$$y_N = \frac{y_D + y_C}{2} \Rightarrow y_N = \frac{6+0}{2} \Rightarrow y_N = 3. \quad \underline{N=(1;3;2)}$$

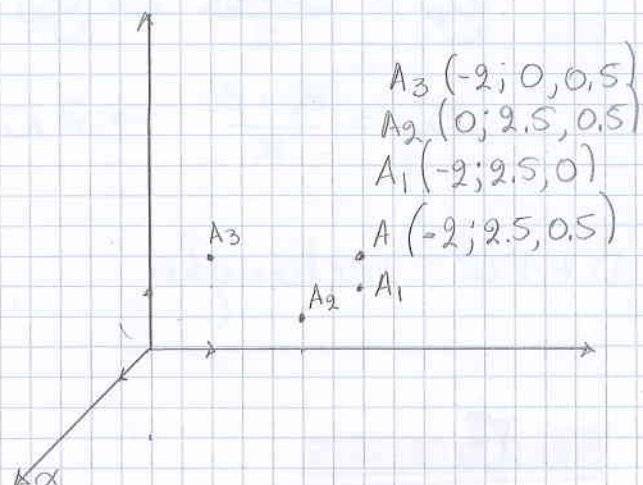
$$z_N = \frac{z_D + z_C}{2} \Rightarrow z_N = \frac{4+0}{2} \Rightarrow z_N = 2$$

DEFG rectangle

EXERCISE 5.5



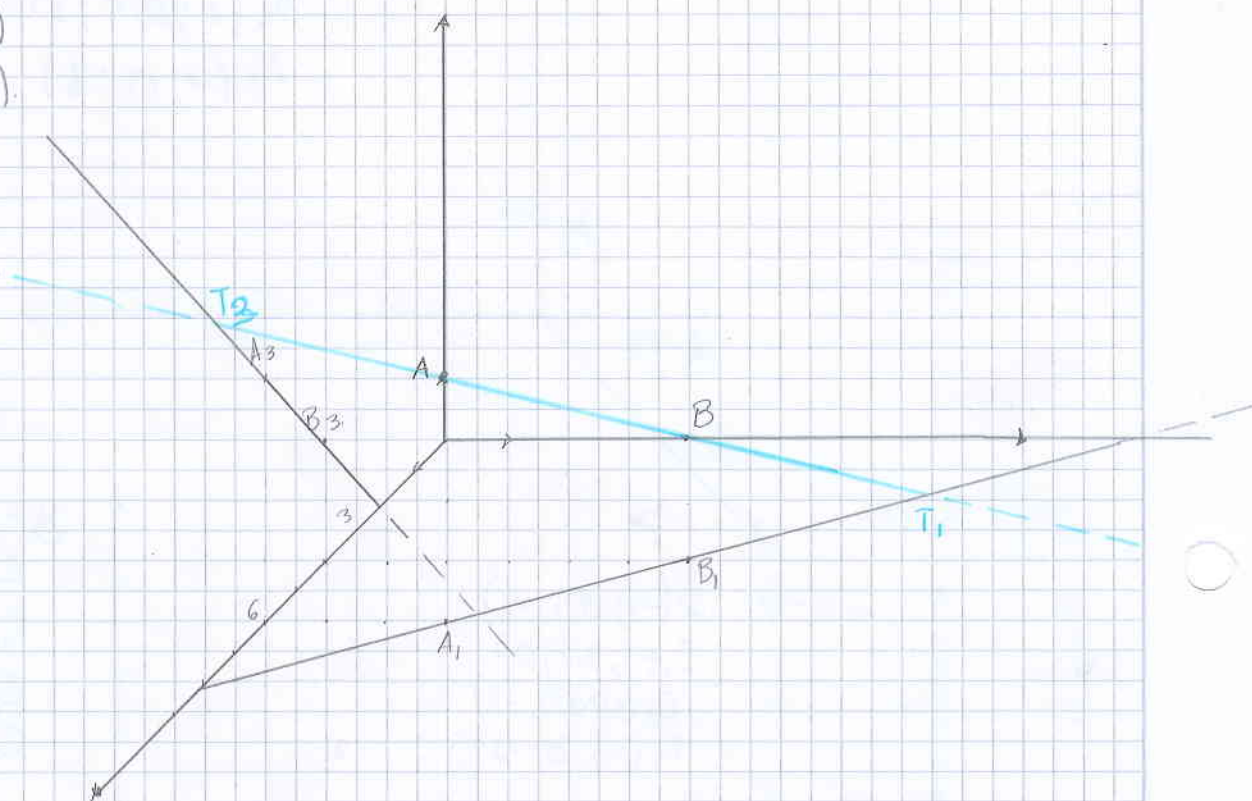
EXERCICE 5.6



EXERCICE 5.7.

$$A(6; 3; 4)$$

$$B(4; 6; 2)$$

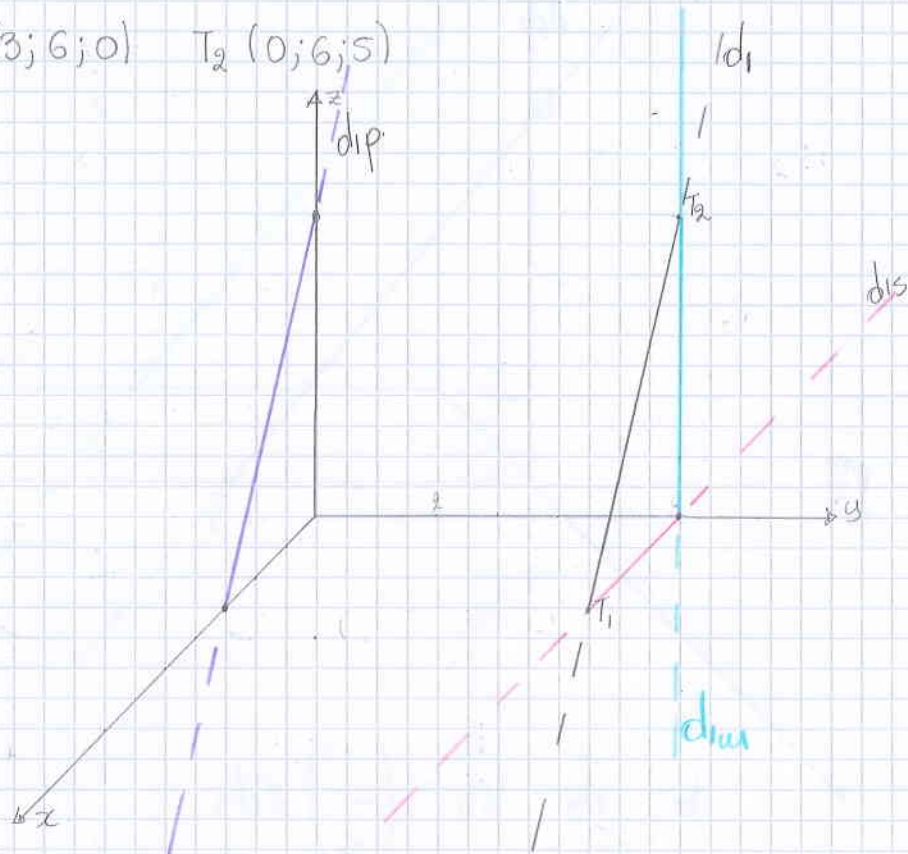


vecteur directeur $\vec{AB} = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix} // d$

$$d: \begin{cases} x = 6 - 2t \\ y = 3 + 3t \\ z = 4 - 2t \end{cases}$$

EXERCICE 5.8

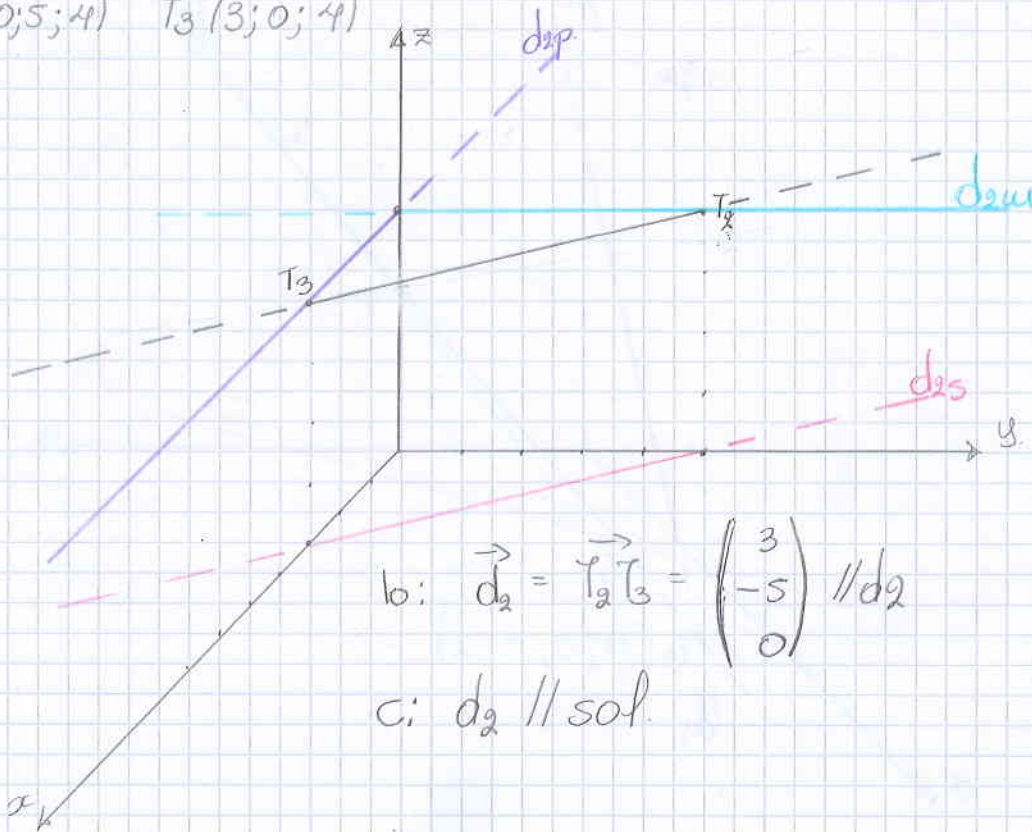
► a) $d_1: T_1(3;6;0) \quad T_2(0;6;5)$



b): vecteur $\vec{d_1} = \overrightarrow{T_1 T_2} = \begin{pmatrix} -3 \\ 0 \\ 5 \end{pmatrix}$

c) $d_1 \parallel Oz$ (paroi)

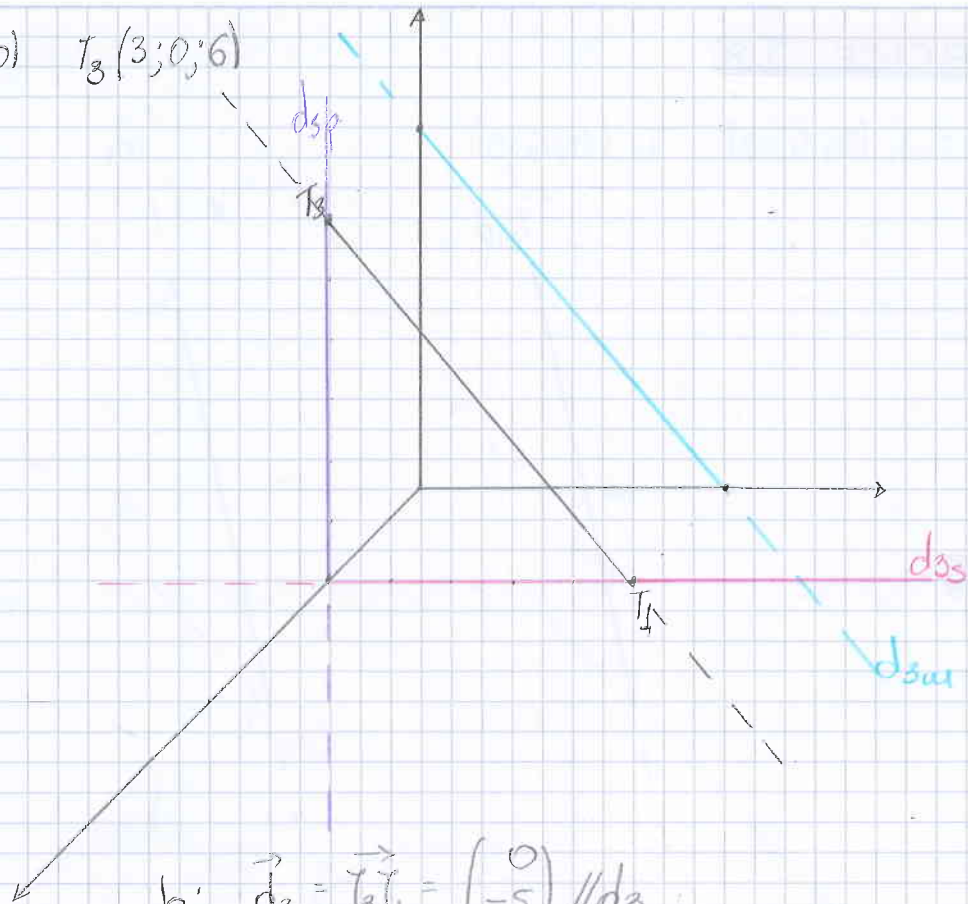
► $T_2(0;5;4) \quad T_3(3;0;4)$



b): $\vec{d_2} = \overrightarrow{T_2 T_3} = \begin{pmatrix} 3 \\ -5 \\ 0 \end{pmatrix} \parallel d_2$

c): $d_2 \parallel sol.$

► $T_1(3;5;0)$ $T_3(3;0;6)$



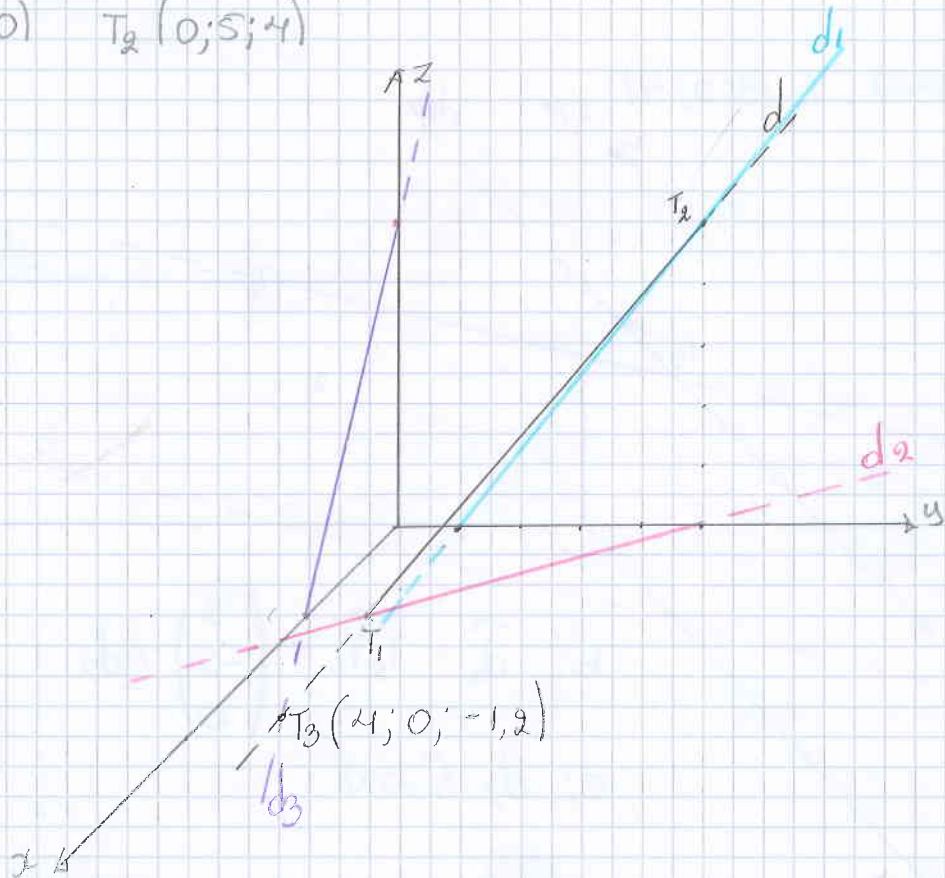
b: $\vec{d}_3 = \vec{T_3T_1} = \begin{pmatrix} 0 \\ -5 \\ 6 \end{pmatrix} \parallel d_3$

c: $d_3 \parallel \text{mur.}$

EXERCICE 5.9

$T_1(3;1;0)$ $T_2(0;5;4)$

a.



c. $\vec{T_2 T_3} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix} // d.$

$$d = \begin{cases} x = 3 + 3t \\ y = 1 - 4t \\ z = 0 - 4t \end{cases}$$

$T_3?$ $y=0 \Leftrightarrow 1-4t=0 \Leftrightarrow t=1/4$

$$x = 3 + 3 \cdot \frac{1}{4} \Rightarrow x = \frac{15}{4} = 3.75$$

$$y = 0$$

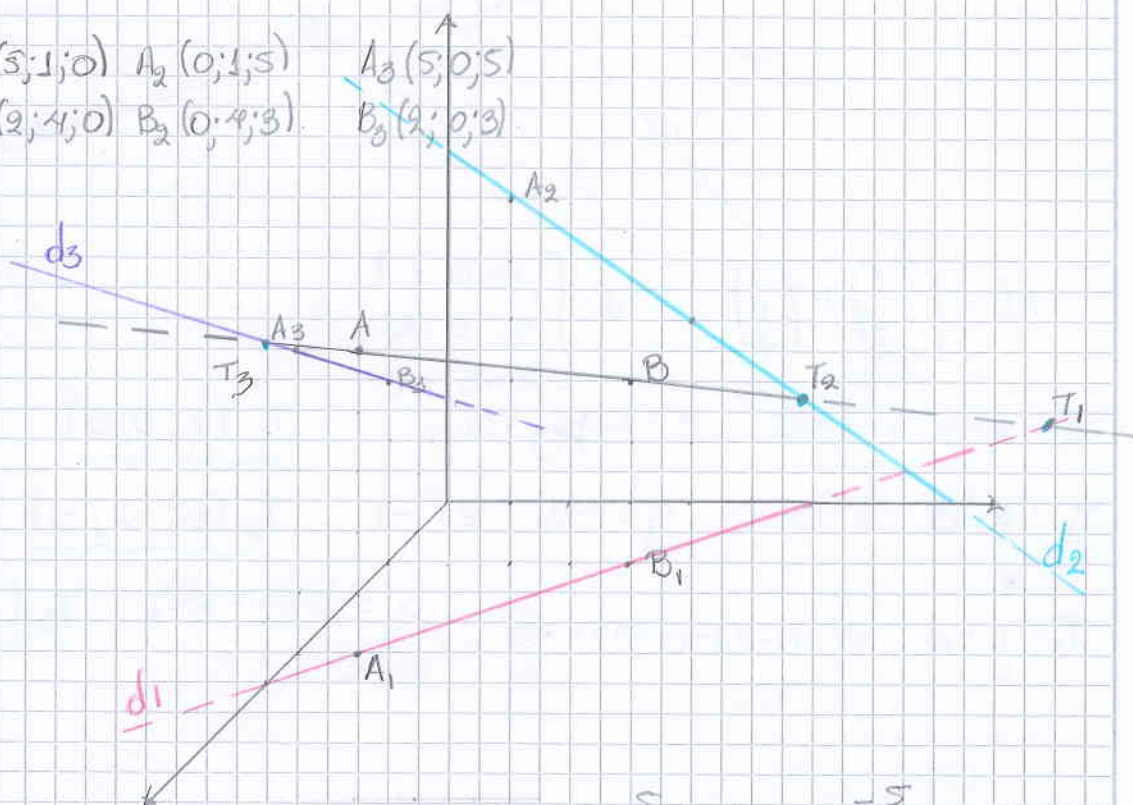
$$z = -4 \cdot \frac{1}{4} = -1$$

Donc $T_3 = (3.75; 0; -1)$

EXERCICE 5.10

Cas A:

$A(5; 1; 5) \quad A_1(5; 1; 0) \quad A_2(0; 1; 5) \quad A_3(5; 0; 5)$
 $B(2; 4; 3) \quad B_1(2; 4; 0) \quad B_2(0; 4; 3) \quad B_3(2; 0; 3)$



$$\vec{AB} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$$

$$d. \begin{cases} x = 5 - 3t \\ y = 1 + 3t \\ z = 5 - 2t \end{cases}$$

$T_1: z=0 \Leftrightarrow t = \frac{5}{2}$

$T_3: y=0 \Leftrightarrow t = -1/3$

$T_2: x=0 \Leftrightarrow t = 5/3$

$$x = 5 - \frac{5}{2} \cdot 3 \Rightarrow x = -\frac{5}{2}$$

$$y = 1 + 3 \cdot \frac{5}{2} \Rightarrow y = \frac{17}{2}$$

$$x = 5 + 3 \cdot \frac{1}{3} \Rightarrow x = 6$$

$$z = 5 + 2 \cdot \frac{1}{3} \Rightarrow z = \frac{17}{3}$$

$$y = 1 + 3 \cdot \frac{5}{3} \Rightarrow y = 6$$

$$z = 5 - 2 \cdot \frac{5}{3} \Rightarrow z = \frac{5}{3}$$

$T_1(-\frac{5}{2}; \frac{17}{2}; 0)$

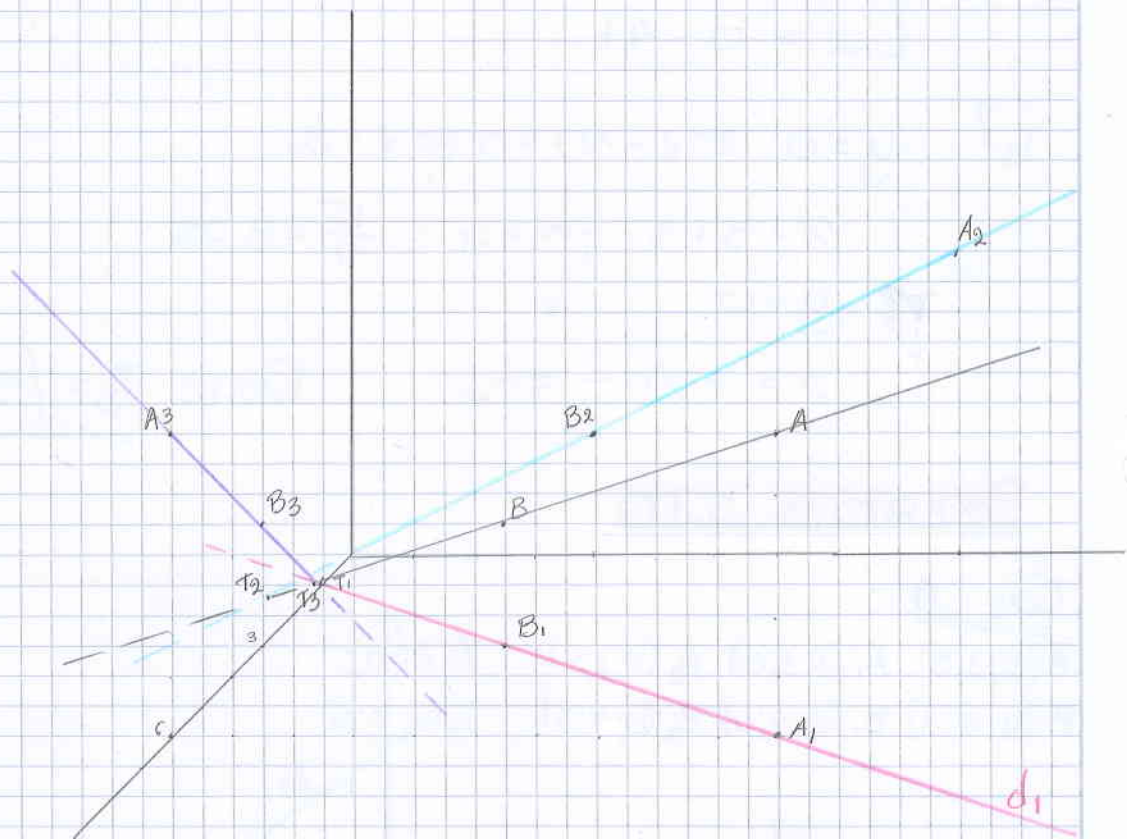
$T_3(6; 0; \frac{17}{3})$

$T_2(0; 6; \frac{5}{3})$

► Coex 6

$$A(6; 10; 5) \quad A_1(6; 10; 0) \quad A_2(0; 10; 5) \quad A_3(6; 0; 5)$$

$$B(3; 4; 2) \quad B_1(3; 4; 0) \quad B_2(0; 4; 2) \quad B_3(3; 0; 2)$$



$$\vec{AB} = \begin{pmatrix} -3 \\ -6 \\ -3 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad d: \begin{cases} x = 6 + t \\ y = 10 + 2t \\ z = 5 + t \end{cases}$$

$$r: z=0 \Rightarrow t=-5 \quad x=1 \quad y=0$$

$$T_1(1; 0; 0)$$

$$T_2: x=0 \Rightarrow t=-6 \quad y=-2 \quad z=-1$$

$$T_2(0; -2; -1)$$

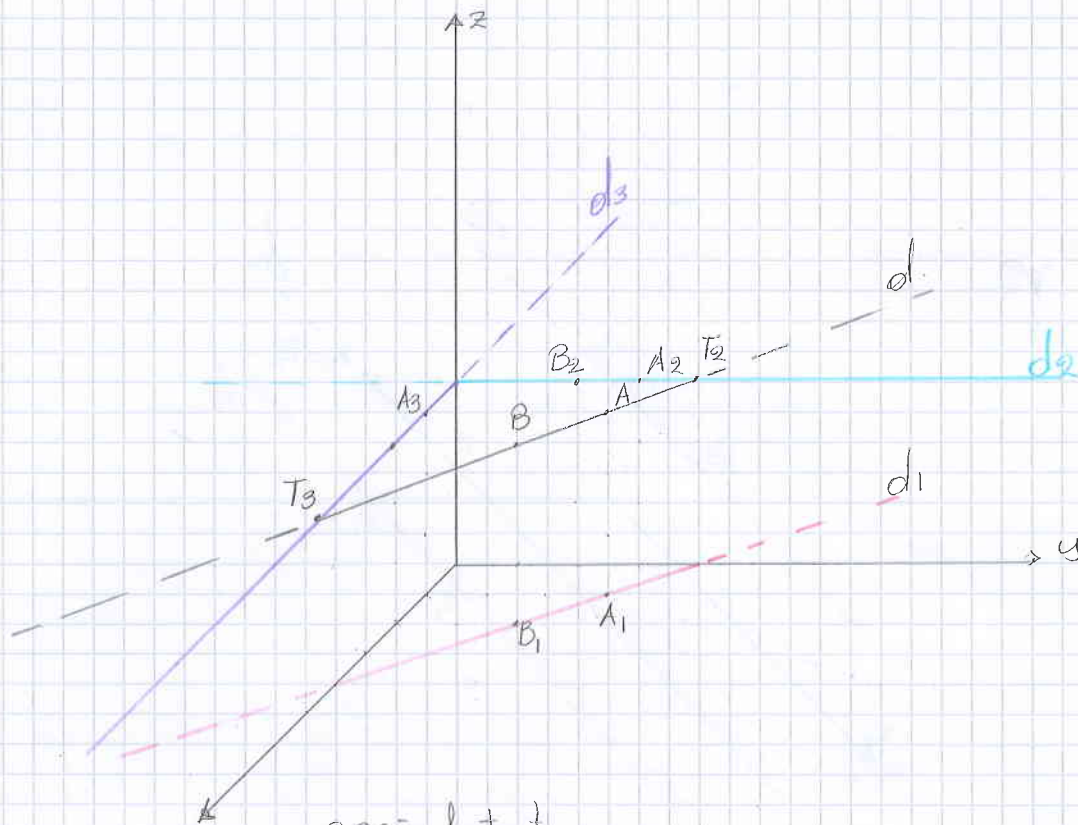
$$T_3: y=0 \Rightarrow 10+2t=0 \Rightarrow t=-5$$

$$x = 6-5 \Rightarrow x=1 \quad z=0 \quad T_3(1; 0; 0)$$

► Exercice C.

$$A(1; 3; 3) \quad A_1(1; 3; 0) \quad A_2(0; 3; 3) \quad A_3(1; 0; 3)$$

$$B(2; 2; 3) \quad B_1(2; 2; 0) \quad B_2(0; 2; 3) \quad B_3(2; 0; 3)$$



$$\vec{AB} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$d: \begin{cases} x = 1 + t \\ y = 3 - t \\ z = 3 \end{cases}$$

$$T_1: z = 0 \quad \text{impossible} \Rightarrow \nexists T_1$$

$$T_2: x = 0 \Rightarrow t = -1 \quad \begin{matrix} y = 4 \\ z = 3 \end{matrix}$$

$$\underline{T_2(0; 4; 3)}$$

$$T_3: y = 0 \Rightarrow t = 3 \quad \begin{matrix} x = 4 \\ z = 3 \end{matrix}$$

$$\underline{T_3(4; 0; 3)}$$

Cas d:

$$A(1; 4; 2)$$

$$A_1(1; 4; 0)$$

$$A_2(0; 4; 2)$$

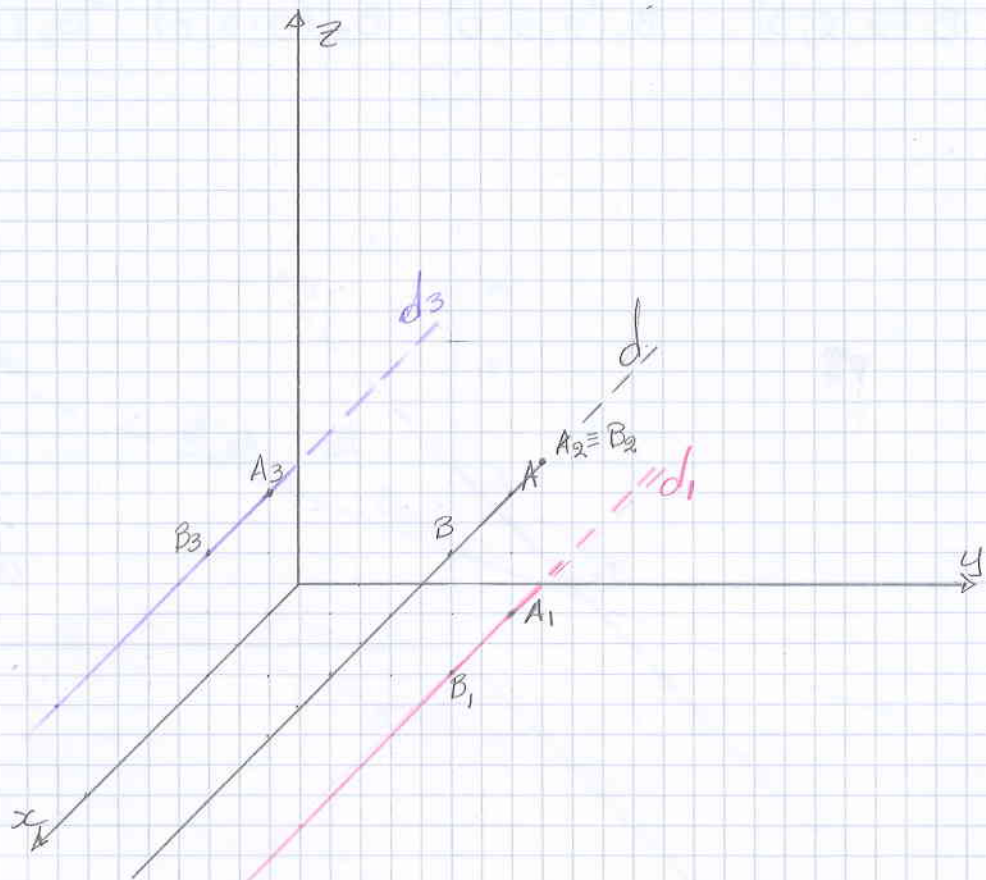
$$A_3(1; 0; 2)$$

$$B(3; 4; 2)$$

$$B_1(3; 4; 0)$$

$$B_2(0; 4; 2)$$

$$B_3(3; 0; 2)$$



$$\vec{AB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$$d: \begin{cases} x = 1 + 2t \\ y = 4 \\ z = 2 \end{cases}$$

$T_1: z=0$ impossible

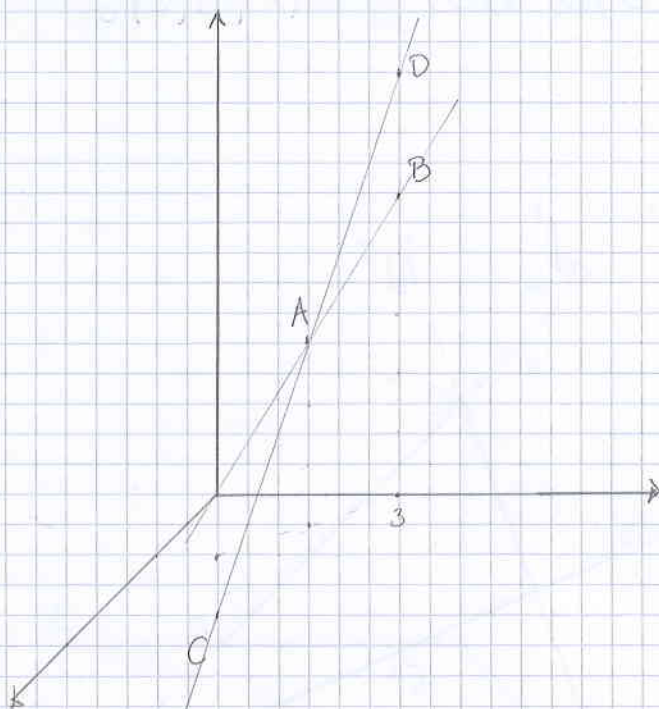
$T_2: x=0 \Leftrightarrow t = -1/2$

$$T_2(0; 4; 2) = A_2 \equiv B_2$$

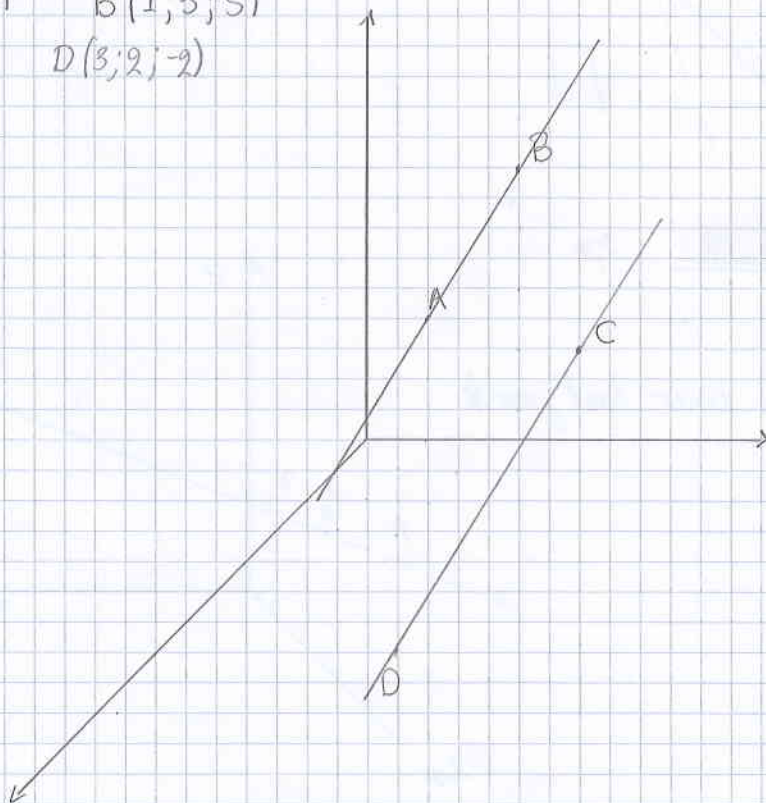
$T_3: y=0$ impossible

EXERCICE 5.14

a. $d_1 : A(1; 2; 3) \quad B(2; 4; 6)$
 $d_2 : C(2; 1; -1) \quad D(0; 3; 7)$



b. $d_1 : A(2; 2; 3) \quad B(1; 3; 5)$
 $d_2 : C(1; 4; 2) \quad D(3; 2; -2)$



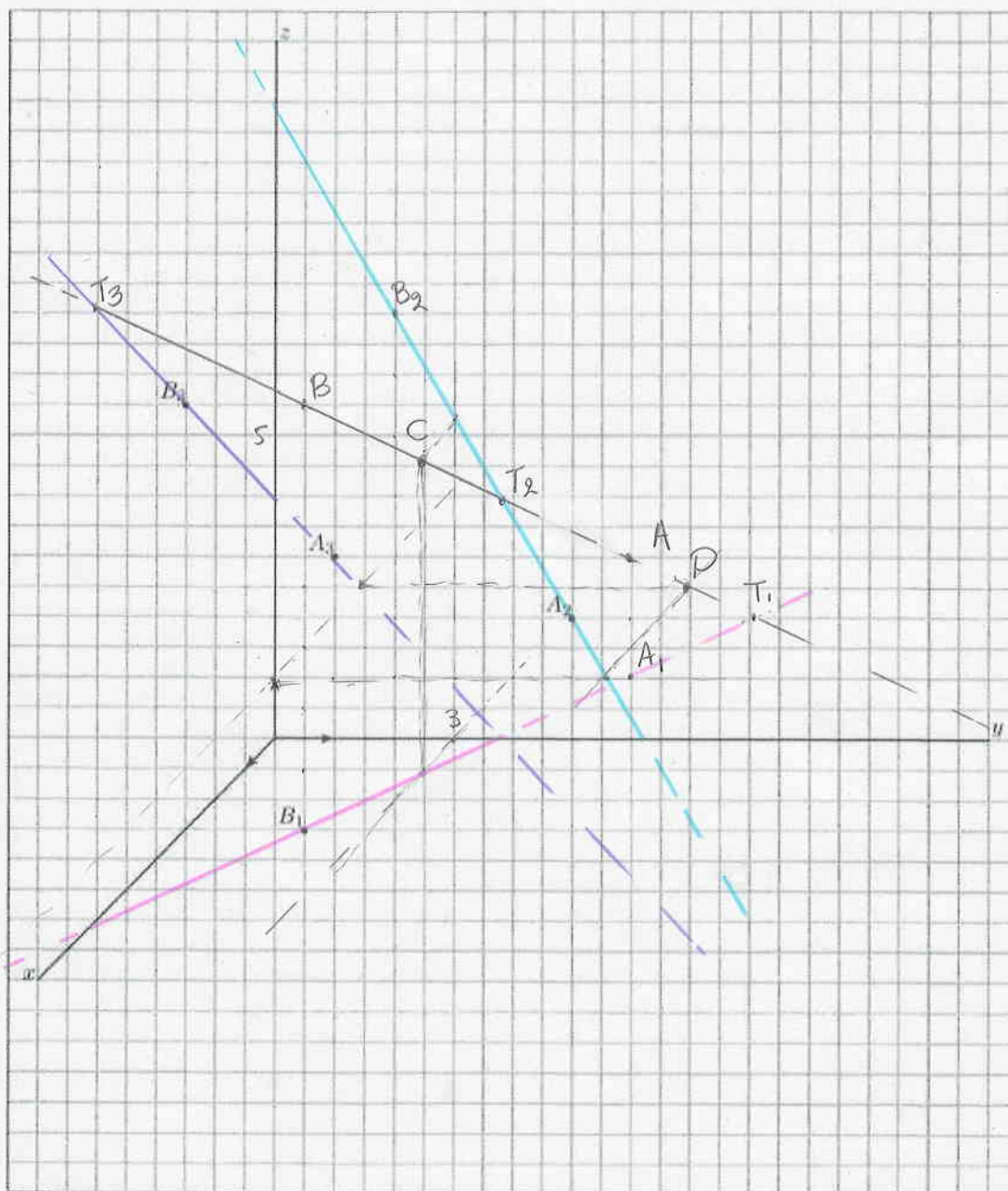
$$A_2(0;5;2) \quad A_3(-2;0;2) \Rightarrow A_1(-2;5;0) \text{ et } A(-2;5;2)$$

$$B_1(3;2;0) \quad B_3(3;0;7) \Rightarrow B_2(0;2;7) \Rightarrow B(3;2;7)$$

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Exercice 5.13

Construire A_1, B_2, A et B . Dessiner la droite d passant par A et B et ses projections.
Construire les deux points de d suivants : C d'ordonnée 3 et D de cote 1.



Exercice 5.14

On donne deux droites d_1 et d_2 par deux points.

Déterminer géométriquement la position relative de ces deux droites :

- $d_1 : A(1; 2; 3)$ et $B(2; 4; 6)$ $d_2 : C(2; 1; -1)$ et $D(0; 3; 7)$
- $d_1 : A(2; 2; 3)$ et $B(1; 3; 5)$ $d_2 : C(1; 4; 2)$ et $D(3; 2; -2)$

EXERCICE 5.15

$$a) d_1: A(6; 3; 0) \quad \vec{a} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \quad d_1: \begin{cases} x = 6 - t \\ y = 3 + t \\ z = t \end{cases}$$

$$d_2: B(0; 0; 4) \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad d_2: \begin{cases} x = k \\ y = k \\ z = 4 - k \end{cases}$$

$$\begin{cases} 6 - t = k & ① \\ 3 + t = k & ② \\ t = 4 - k & ③ \end{cases}$$

$$① + ② \Rightarrow 9 = 2k \Rightarrow k = 4.5$$

$$① \Rightarrow 6 - t = 4.5 \Leftrightarrow t = 1.5$$

On remplace en ③

$$1.5 = 4 - 4.5 \quad \nabla$$

Donc il n'y a pas de points communs
 $\Rightarrow$ $\begin{cases} d_1 \parallel d_2 \\ \text{gauches} \end{cases}$

$\vec{a} \nparallel \vec{b} \Rightarrow d_1, d_2$ gauches

$$b) d_1: A(-3; -1; 2) \quad \vec{a} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad d_1: \begin{cases} x = -3 + 2t \\ y = -1 + t \\ z = 2 - t \end{cases}$$

$$d_2: B(4; -1; 0) \quad \vec{b} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \parallel \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad d_2: \begin{cases} x = 4 + 2k \\ y = -1 + k \\ z = -k \end{cases}$$

$$\begin{cases} -3 + 2t = 4 + 2k \\ -1 + t = -1 + k \\ 2 - t = k \end{cases} \Leftrightarrow \begin{cases} 2t = 2k + 7 \\ t = k \\ 2 - t = k \end{cases} \xrightarrow{①} \begin{cases} 2t = 2t + 7 \\ 0 = 7 \end{cases} \nabla$$

$\Rightarrow$ $\begin{cases} \text{gauches} \\ d_1 \parallel d_2 \end{cases}$

Mais $\vec{a} \parallel \vec{b}$ en particulier $\vec{b} = 2\vec{a} \Rightarrow d_1 \parallel d_2$

$$c) d_1: A(1; 4; 1) \quad \vec{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \quad d_1: \begin{cases} x = 1 + 2t \\ y = 4 \\ z = 1 + t \end{cases}$$

$$d_2: B(5; -1; 0) \quad \vec{b} = \begin{pmatrix} -2 \\ 5 \\ 2 \end{pmatrix} \quad d_2: \begin{cases} x = 5 - 2k \\ y = -1 + 5k \\ z = 2k \end{cases}$$

$$\begin{cases} 1+2t=5-2K \\ 4=-1+5K \\ 1+t=2K \end{cases} \Leftrightarrow \begin{cases} 2t=4-2K \\ 5=5K \\ t=2K-1 \end{cases} \Leftrightarrow \begin{cases} t=2-K \\ K=1 \\ t=2K-1 \end{cases} \Rightarrow$$

$$\Leftrightarrow \begin{cases} t=1 \\ K=1 \\ t=1 \end{cases}$$

✓ Donc le point
(3; 4; 2) est le point commun
 $\Rightarrow$ sécantes

$$d_1: A(2; -1; -4) \quad \vec{a} = \begin{pmatrix} -4 \\ -2 \\ 2 \end{pmatrix} \quad d_1 \begin{cases} x=2-4t \\ y=-1-2t \\ z=-4+2t \end{cases}$$

$$d_2: B(4; 0; -5) \quad \vec{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad d_2 = \begin{cases} x=4+2K \\ y=K \\ z=-5-K \end{cases}$$

$$\begin{cases} 2-4t=4+2K \\ -1-2t=K \\ -4+2t=-5-K \end{cases} \Leftrightarrow \begin{cases} -4t=2+2K \\ K=-1-2t \\ K=-1-2t \end{cases} \Leftrightarrow \begin{cases} -2K=-1-2t \\ K=-1-2t \\ K=-1-2t \end{cases}$$

Donc on a une infinité de solutions.

$\Rightarrow$ d_1, d_2 confondues

EXERCICE S.16

$$d_1: A(1;1;1) \quad \vec{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$d_2: B(1;4;3) \quad \vec{b} = \begin{pmatrix} 3 \\ 3 \\ p \end{pmatrix}$$

$$d_1: \begin{cases} x = 1 + 2t \\ y = 1 - t \\ z = 1 + t \end{cases}$$

$$d_2: \begin{cases} x = 1 + 3k \\ y = 4 + 3k \\ z = 3 + pk \end{cases}$$

$$\begin{cases} 1 + 2t = 1 + 3k & \textcircled{1} \\ 1 - t = 4 + 3k & \textcircled{2} \\ 1 + t = 3 + pk & \textcircled{3} \end{cases}$$

$$\textcircled{1} - \textcircled{2} \quad 3t = -3 \Rightarrow t = -1$$

$$\textcircled{2}: 1 + 1 = 4 + 3k \Rightarrow 3k = -2 \Rightarrow k = -\frac{2}{3}$$

On remplace en $\textcircled{3}$: $1 - 1 = 3 + p(-\frac{2}{3}) \Leftrightarrow$

$$\Leftrightarrow \frac{2}{3}p = 3 \Rightarrow p = \frac{9}{2} \Rightarrow p = 4.5$$

EXERCICE S.17

$$\pi: A(1;2;2) \quad \vec{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad \vec{s} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

$$\pi: \begin{cases} x = 1 + t + 2k & \textcircled{1} \\ y = 2 - t + 2k & \textcircled{2} \\ z = 2 + 2t - k \end{cases}$$

On trouve l'équation cartésienne

$$\textcircled{1} + \textcircled{2} \quad x + y = 3 + 4k$$

$$2 \times \textcircled{2} \quad 2y = 4 - 2t + 4k$$

$$\textcircled{3} \quad + \quad z = 2 + 2t - k$$

$$2y + z = 6 + 3k \quad * (-4) \quad \begin{cases} -8y - 4z = -24 - 12k \\ x + y = 3 + 4k \quad * (3) \end{cases}$$

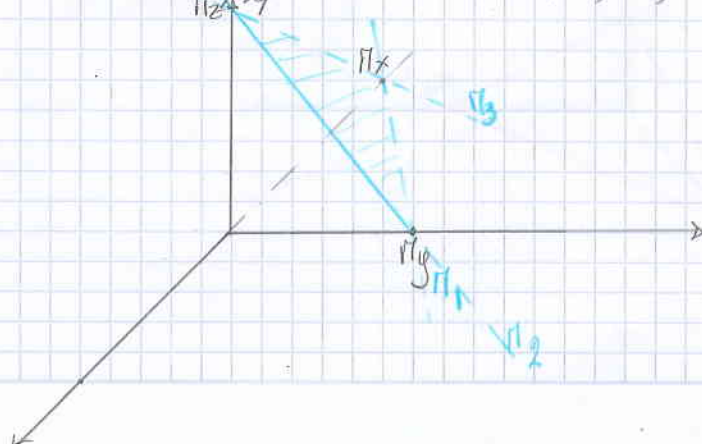
$$+ \quad \begin{cases} 3x + 3y = 9 + 12k \\ -8y - 4z = -24 - 12k \end{cases}$$

$$3x - 5y - 4z = -15 : 11$$

$$\pi_x: y = z = 0 \Rightarrow x = -5 \quad \pi_x(-5; 0; 0)$$

$$\pi_y: x = z = 0 \Rightarrow y = 3 \quad \pi_y(0; 3; 0)$$

$$\pi_z: x = y = 0 \Rightarrow z = \frac{15}{3} = 5 \quad \pi_z(0; 0; 5)$$



EXERCISE 5.18

$$\Pi: A(4; -8; 4) \quad B(0; 1; 1) \quad C(-2; 4.5; 0)$$

$$a) \quad \vec{AB} = \begin{pmatrix} -4 \\ +9 \\ -3 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} -6 \\ 12.5 \\ -4 \end{pmatrix}$$

$$\Pi: \begin{cases} x = 0 - 4t - 6k & (1) \\ y = 1 + 9t + 12.5k & (2) \\ z = 1 - 3t - 4k & (3) \end{cases}$$

$$\begin{array}{rcl} 9 \times (1) & 9x = -36t - 54k & -3 \times (1): -3x = 12t + 18k \\ + 4 \times (2) & 4y = 4 + 36t + 50k & + 4 \times (3): 4z = 4 - 12t - 16k \\ \hline 9x + 4y & = 4 - 4k & -3x + 4z = 4 + 2k \end{array}$$

$$\begin{cases} 9x + 4y = 4 - 4k \\ -3x + 4z = 4 + 2k \end{cases} \quad \begin{array}{l} 9x + 4y = 4 - 4k \\ \times 2: +6x + 8z = 8 + 4k \\ \hline 13x + 4y + 8z = 12 \quad (m) \end{array}$$

$$b) \quad D(5; -2; 0.5) \in \Pi?$$

On remplace:

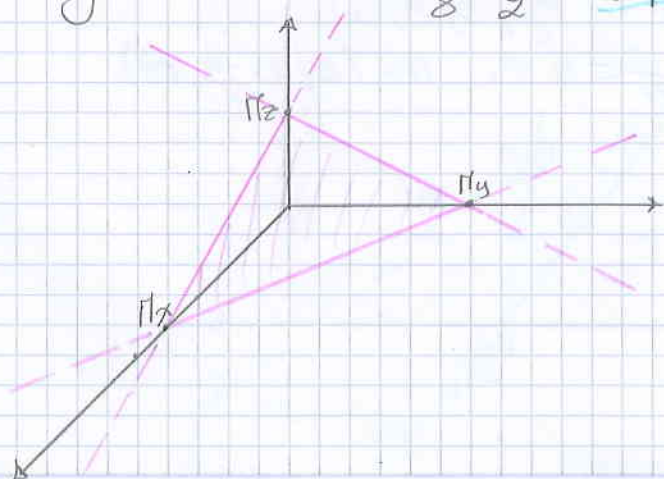
$$3 \cdot 5 + 4(-2) + 8 \cdot 0.5 = 15 - 8 + 4 = 11 \neq 12 \Rightarrow D \notin \Pi$$

$$c) \quad k? \quad E(8; k; -1) \in \Pi$$

$$3 \cdot 8 + 4 \cdot k + 8(-1) = 12 \Rightarrow 24 + 4k - 8 = 12 \Leftrightarrow$$

$$\Leftrightarrow 4k = -4 \Leftrightarrow k = -1$$

$$\begin{array}{lll} d) \quad \Pi_x: y = z = 0 & \Rightarrow x = 4 & \Pi_x(4; 0; 0) \\ \Pi_y: x = z = 0 & \Rightarrow y = 3 & \Pi_y(0; 3; 0) \\ \Pi_z: x = y = 0 & \Rightarrow z = \frac{12 - 3}{8} = \frac{9}{8} & \Pi_z(0; 0; 1.5) \end{array}$$



EXERCICE 5.19

a) $A(6;0;0)$ $B(0;4;0)$ $C(0;0;3)$

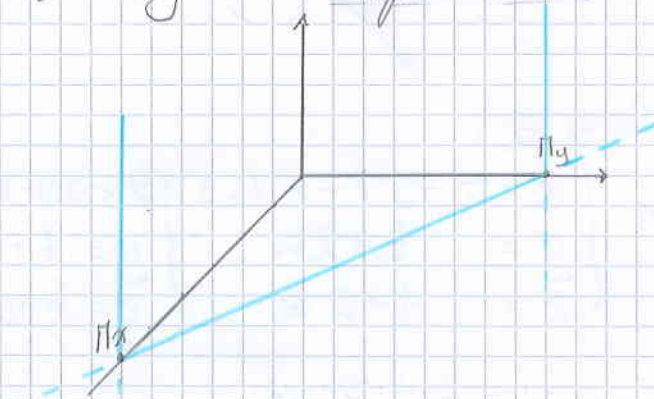
$$\vec{AB} = \begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} \parallel \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} -6 \\ 0 \\ 3 \end{pmatrix} \parallel \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \quad \Pi = \begin{cases} x = 6 - 3t & (1) \\ y = 2t & (2) \\ z = t & (3) \end{cases}$$

$$\begin{array}{rcl} 2 \times (1) & 2x = 12 - 6t \\ 3 \times (2) & + 3y = 6t \\ \hline & 2x + 3y = 12 & (II) \end{array}$$

$\Pi_x: y = z = 0 \quad \Pi_x(6; 0; 0)$

$\Pi_y: x = z = 0 \quad \Pi_y(0; 4; 0)$

$\Pi_z: x = y = 0 \quad \text{Impossible} \quad \Pi \parallel Oz$



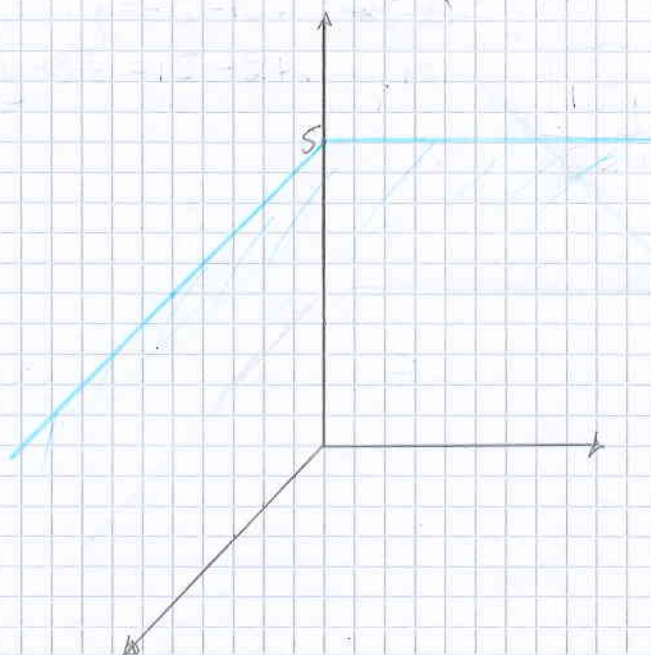
b) $A(2;3;5)$ $B(1;0;5)$ $C(6;-2;5)$

$$\vec{AB} = \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 4 \\ -5 \\ 0 \end{pmatrix} \quad \Pi = \begin{cases} x = 1 - t + 4k \\ y = -3t - 5k \\ z = 5 \end{cases} \quad \Pi: \underline{z - 5 = 0}$$

$\Pi_y, \Pi_x \quad \text{Impossible}$

$\Pi_z: (0; 0; 5)$

$\Pi \parallel Sol$



g) $A(2;1;3) \quad B(5;-1;6) \quad C(1;4;2)$

$$\vec{AB} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$$

$$\Pi: \begin{cases} x = 2 + 3t - k. & \textcircled{1} \times (3) \\ y = 1 - 2t + 3k & \textcircled{2} \\ z = 3 + 3t - k. & \textcircled{3} \times (3) \end{cases}$$

$$\Leftrightarrow \begin{cases} -3x = 6 + 9t - 3k. & \textcircled{1} \\ y = 1 - 2t + 3k & \textcircled{2} \\ -3z = 9 + 9t - 3k. & \textcircled{3} \end{cases}$$

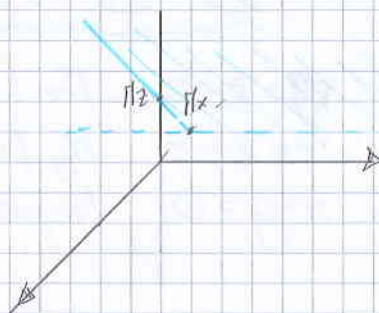
$$\begin{aligned} \textcircled{1} + \textcircled{2} \quad & 3x + y = 7 + 7t \\ \textcircled{2} + \textcircled{3} \quad & y + 3z = 10 + 7t. \end{aligned}$$

$$3x - 3z = -3 \Leftrightarrow x - z = -1 \quad \textcolor{blue}{(IV)}$$

$\Pi_x: y=z=0 \quad x=-1 \quad \Pi_x(-1;0;0)$

$\Pi_y: x=z=0$ Impossible.

$\Pi_z: x=y=0 \quad z=1 \quad \Pi_z(0;0;1)$



d) $A(5;0;0) \quad B(0;1;1) \quad C(4;2;2)$

$$\vec{AB} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$\Pi: \begin{cases} x = 5 - 5t - k. & (\times 2) \quad \begin{cases} 2x = 10 - 10t - 2k & \textcircled{1} \\ y = t + 2k & \textcircled{2} \\ z = t + 2k & \textcircled{3} \end{cases} \end{cases}$$

$$\textcircled{1} + \textcircled{2} \quad 2x + y = 10 - 9t$$

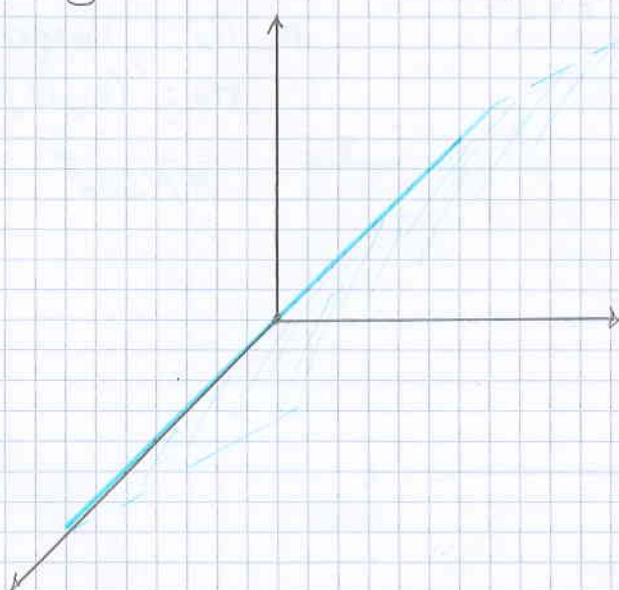
$$\textcircled{2} - \textcircled{3} \quad y - z = 0$$

$$\Rightarrow y - z = 0 \quad \textcolor{blue}{(IV)}$$

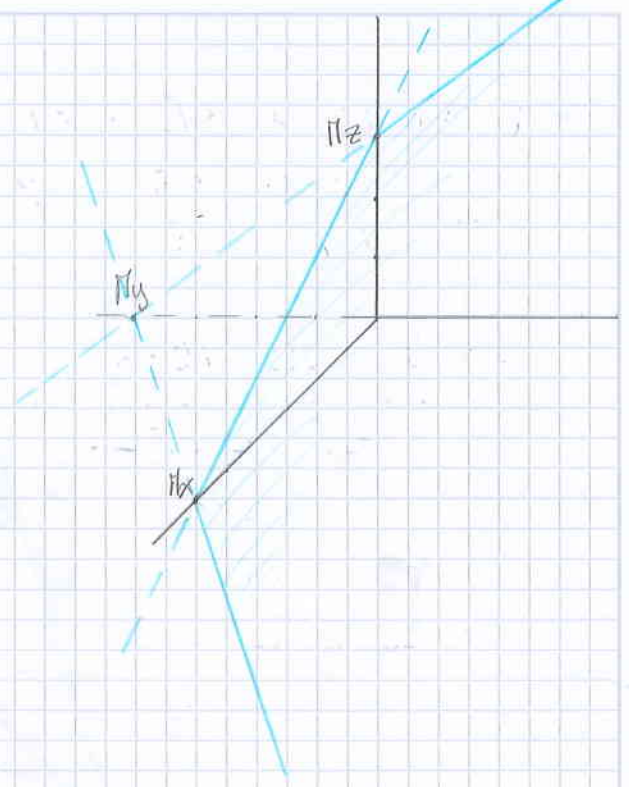
$\Pi_x: y=z=0 \Rightarrow x=1 \in \mathbb{R} \Rightarrow O_x \in \Pi$

$\Pi_y: x=z=0 \Rightarrow y=0 \quad \Pi_y(0;0;0)$

$\Pi_z: x=0=y \Rightarrow z=0 \quad \Pi_z(0;0;0)$



c) $2x - 3y + 4z - 12 = 0$
 $\Pi_x: y = z = 0 \quad \Pi_x(6; 0; 0)$
 $\Pi_y: x = z = 0 \quad \Pi_y(0; -4; 0)$
 $\Pi_z: x = y = 0 \quad \Pi_z(0; 0; 3)$



EXERCICE S.20

d: $A(2; 3; 3) \quad \vec{d} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \quad d: \begin{cases} x = 2 + 2t \\ y = 3 - 2t \\ z = 3 - t \end{cases} \quad p: \begin{cases} x = 4 + 2t \\ y = 4 - 2t \\ z = -t \end{cases}$
 $p \parallel d \quad B(4; 4; 0)$

a) d) $\begin{cases} T_1: z=0 \Rightarrow t=3 \quad x=8 \quad y=-3 \quad T_1(8; -3; 0) \\ T_2: x=0 \Rightarrow t=-1 \quad y=5 \quad z=4 \quad T_2(0; 5; 4) \\ T_3: y=0 \Rightarrow t=\frac{3}{2} \quad x=5 \quad z=3-\frac{3}{2}=\frac{3}{2} \quad T_3(5; 0; 1.5) \end{cases}$

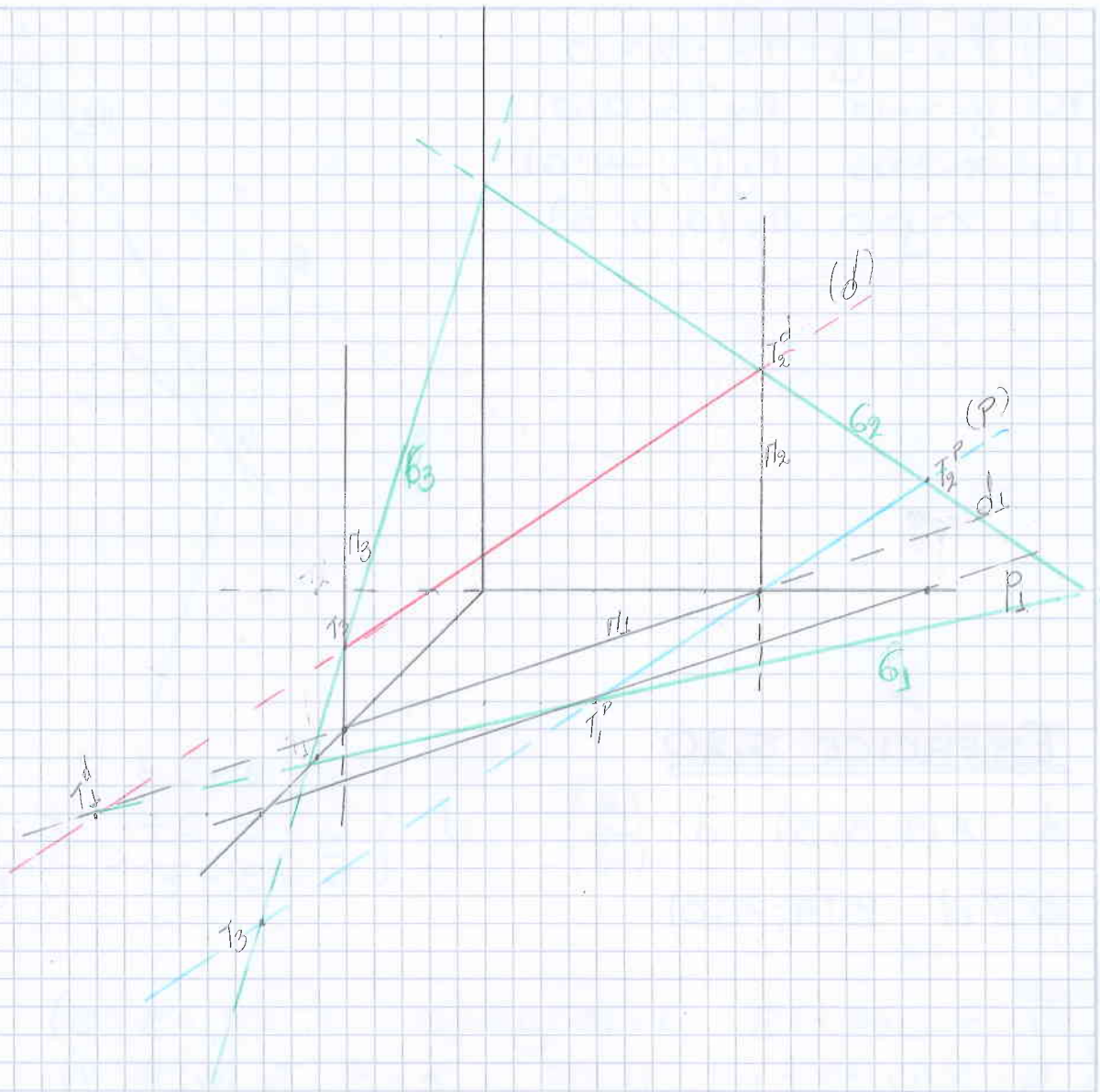
p) $\begin{cases} T_1: z=0 \Rightarrow t=0 \quad x=4 \quad y=4 \quad T_1(4; 4; 0) \\ T_2: x=0 \Rightarrow t=2 \quad y=8 \quad z=2 \quad T_2(0; 8; 2) \\ T_3: y=0 \Rightarrow t=2 \quad x=8 \quad z=-2 \quad T_3(8; 0; -2) \end{cases}$

b) Vecteurs directeurs: $\vec{d}$ et $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ car $\Pi \perp \text{sol}$

$\Pi: \begin{cases} x = 2 + 2t \\ y = 3 - 2t \\ z = 3 - t + k \end{cases} + \underline{x+y=5} \quad (\Pi)$

c) G: $\vec{d} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \quad \vec{AB} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \quad G: \begin{cases} x = 2 + 2t + 2k \quad ① \\ y = 3 - 2t + k \quad ② \\ z = 3 - t - 3k \quad ③ \end{cases}$

①+② $x+y=5+3k$
①+2x③ $x=2+2t+2k$
 $2z=6-2t-6k$
 $x+2z=8-4k$
 $\begin{cases} x+y=5+3k \quad *4 \quad 4x+4y=20+12k \\ x+2z=8-4k \quad *3 \quad 3x+6z=24-12k \\ \hline 7x+4y+6z=44 \end{cases} \quad (6)$



b. $\alpha: 3x + 4y + 2z - 12 = 0$

$\beta: 4x + 2y + z - 8 = 0$

$\alpha_x: (4; 0; 0)$

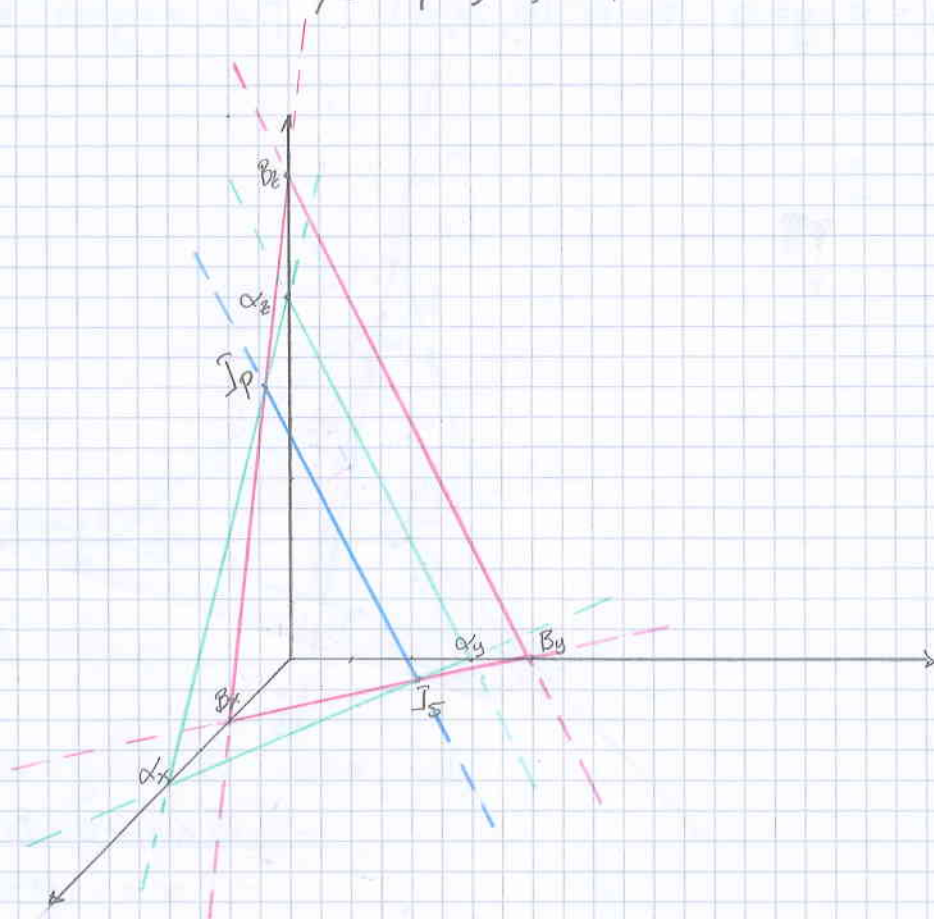
$\beta_x: (2; 0; 0)$

$\alpha_y: (0; 3; 0)$

$\beta_y: (0; 4; 0)$

$\alpha_z: (0; 0; 6)$

$\beta_z: (0; 0; 8)$



$I_s: z=0. \quad 3x + 4y = 12$

$3x + 4y = 12$

$4x + 2y = 8 \quad \times (-2) \quad -8x - 4y = -16$

$-5x = -4 \Rightarrow x = \frac{4}{5}$

$I_s(\frac{4}{5}; 3; 0)$

$3 \cdot \frac{4}{5} + 4y = 12 \Rightarrow$

$\Rightarrow 4y = 12 - \frac{12}{5} \Rightarrow$

$4y = \frac{48}{5} \Rightarrow$

$y = \frac{48}{20} = \frac{12}{5} = 2.4$

$I_p: y=0 \quad \left. \begin{array}{l} 3x + 2z = 12 \\ 4x + z = 8 \end{array} \right\} \begin{array}{l} 3x + 2z = 12 \\ -8x - 2z = -16 \end{array}$

$-5x = -4 \Rightarrow x = \frac{4}{5} \approx 0.8$

$I_p(\frac{4}{5}; 0; 4.8)$

$4 \cdot \frac{4}{5} + z = 8 \Rightarrow z = \frac{40-16}{5} \Rightarrow$

$z = \frac{24}{5} \approx 4.8$

$\vec{I_p I_s} = \begin{pmatrix} 0 \\ 3 \\ -4.8 \end{pmatrix}$

$\vec{l} = \begin{cases} x = \frac{4}{5} + t \\ y = 3 + 3t \\ z = -4.8t \end{cases}$

$$c. \alpha: x+2y+3z-6=0$$

$$\beta: x+3z-4=0$$

$$\alpha_x: (6; 0; 0)$$

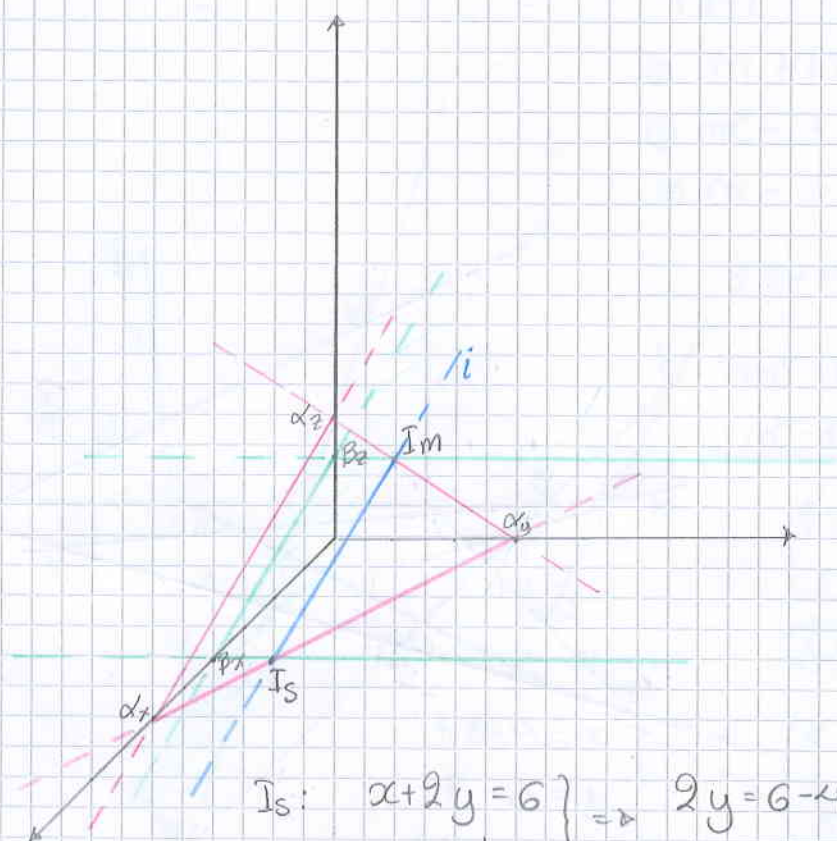
$$\beta_x: (4; 0; 0)$$

$$\alpha_y: (0; 3; 0)$$

$$\beta_y: \text{impossible.}$$

$$\alpha_z: (0; 0; 2)$$

$$\beta_z: (0; 0; \frac{4}{3})$$



$$I_s: \begin{cases} x+2y=6 \\ x=4 \end{cases} \Rightarrow 2y=6-4 \Leftrightarrow y=1$$

$$I_s = (4; 1; 0)$$

$$I_m: \begin{cases} 2y+3z=6 \\ 3z=4 \end{cases} \Rightarrow z=\frac{4}{3}$$

$$I_m = (0; 1; \frac{4}{3})$$

$$\vec{I_m I_s} = \begin{pmatrix} 4 \\ 0 \\ -\frac{4}{3} \end{pmatrix}$$

$$(i) = \begin{cases} x = 4 + 4t \\ y = 1 \\ z = 0 - \frac{4}{3}t \end{cases}$$

EXERCICE 5.22

$$\Pi: A(3;5;1) \quad B(6;1;3) \quad C(4;2;0)$$

$$d: D(4;6;0) \quad E(1;5;3)$$

$$\vec{AB} = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$$

$$\Pi: \begin{cases} x = 3 + 3t + m & ① \\ y = 5 - 4t - 3m & ② \\ z = 1 + 2t - m & ③ \end{cases}$$

$$①+③ \quad x+z = 4+5t$$

$$①: 3x = 9 + 9t + 3m$$

$$②: y = 5 - 4t - 3m$$

$$3x + y = 14 + 5t$$

$$-x + z = 4 + 5t$$

$$\Pi \quad 2x + y - z = 10$$

$$d: \vec{DE} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix} \quad \begin{cases} x = 4 - 3t \\ y = 6 - t \\ z = 0 + 3t \end{cases}$$

$$d \rightarrow \Pi: 2(4-3t) + (6-t) - (3t) = 10 \Leftrightarrow$$

$$\Leftrightarrow 8 - 6t + 6 - t - 3t = 10 \Leftrightarrow -10t = -4 \Leftrightarrow t = \frac{2}{5}$$

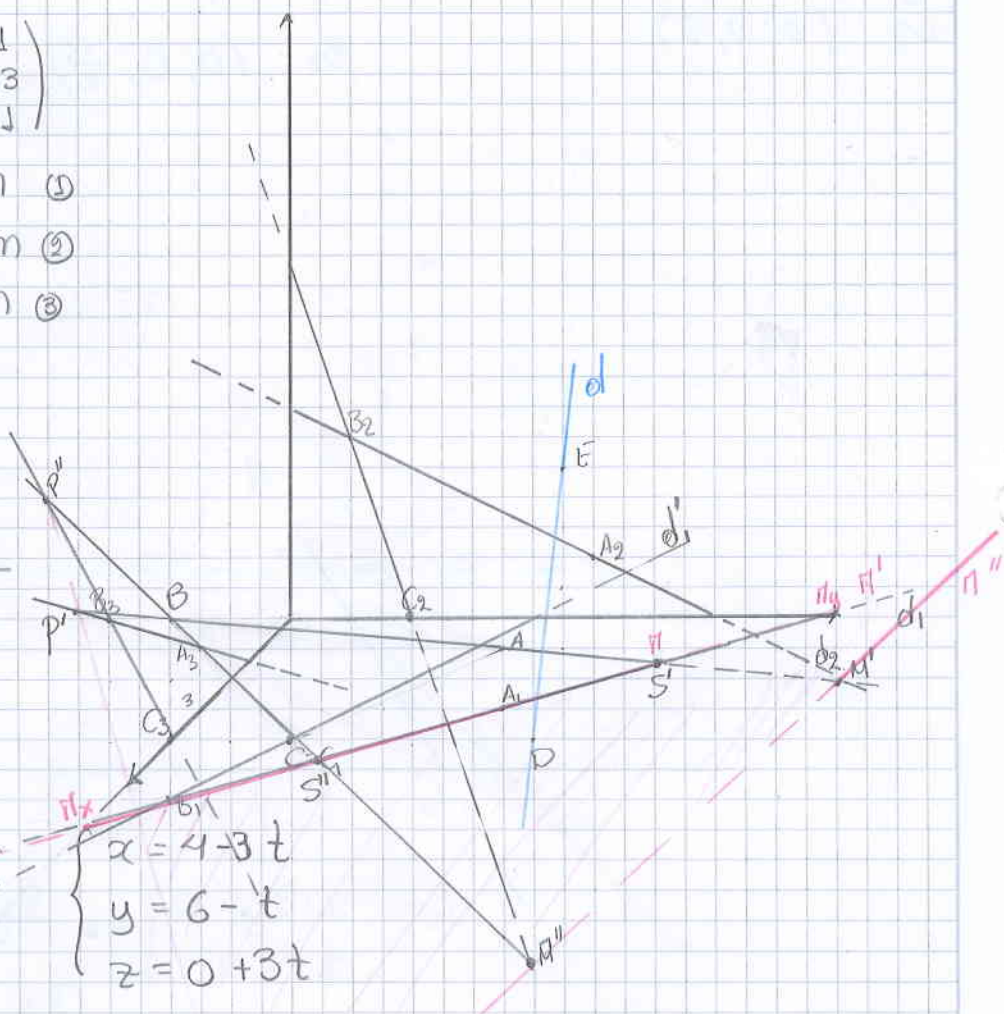
Donc on a un point d'intersection (se'cants)

$$x = 4 - 3 \cdot \frac{2}{5} \Rightarrow x = \frac{20-6}{5} \Rightarrow x = \frac{14}{5}$$

$$y = 6 - \frac{2}{5} \Rightarrow y = \frac{28}{5}$$

$$z = 3 \cdot \frac{2}{5} \Rightarrow z = \frac{6}{5}$$

$$I \left(\frac{14}{5}; \frac{28}{5}; \frac{6}{5} \right)$$



EXERCICE 5.23

$$a: d = \begin{cases} x = 2 - 2\lambda \\ y = 1 + \lambda \\ z = 2\lambda \end{cases}$$

$$\pi: A(6; -1; 1) \quad B(2; 0; 2) \quad C(2; 4; 0)$$

$$\vec{AB} = \begin{pmatrix} -4 \\ 1 \\ 1 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} -4 \\ 5 \\ -1 \end{pmatrix}$$

$$\pi: \begin{cases} x = 6 - 4t - 8m & ① \\ y = -1 + t + 5m & ② \\ z = 1 + t - m & ③ \end{cases}$$

$$\begin{aligned} ① - 8 \times ③ \quad x &= 6 - 4t - 8m \\ &+ -8z = -8 - 8t + 8m \\ \hline x - 8z &= -2 - 12t \\ + 2y + 10z &= 8 + 12t \\ \hline \pi \quad x + 2y + 2z &= 6 \end{aligned}$$

$$\begin{aligned} ② + 5 \times ③ \quad y &= -1 + t + 5m \\ &+ 5z = 5 + 5t - 5m \\ \hline y + 5z &= 4 + 6t \end{aligned}$$

$$d \rightarrow \pi \quad 2 - 2\lambda + 2(1 + \lambda) + 2 \cdot 2\lambda = 6 \Leftrightarrow$$

$$\Leftrightarrow 2 - 2\lambda + 2 + 2\lambda + 4\lambda = 6 \Leftrightarrow 4\lambda = 2 \Leftrightarrow \lambda = \frac{1}{2}$$

$$\text{Se'cours:} \quad I = d \cap \pi \quad x = 2 - 2 \cdot \frac{1}{2} \Rightarrow x = 1$$

$$I(1; \frac{3}{2}; 1)$$

$$y = 1 + \frac{1}{2} \Rightarrow y = \frac{3}{2}$$

$$z = 1$$

$$b) d: A(3; 1; 2) \quad B(-1; 2; 0)$$

$$\pi: x + 2y - z - 3 = 0$$

$$\vec{AB} = \begin{pmatrix} -4 \\ 1 \\ -2 \end{pmatrix} \quad d = \begin{cases} x = 3 - 4t \\ y = 1 + t \\ z = 2 - 2t \end{cases}$$

$$d \rightarrow \pi: 3 - 4t + 2(1 + t) - (2 - 2t) - 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow 3 - 4t + 2 + 2t - 2 + 2t - 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow 0 \cdot t = 0 \Rightarrow d \in \pi$$

$$c) d: \begin{cases} x = 1 + 5\lambda \\ y = 1 - 3\lambda \\ z = 3 \end{cases}$$

$$\pi: \begin{cases} x = 1 - \mu + 2\tau & ① \\ y = 1 + \mu - \tau & ② \\ z = -3 - 2\mu - \tau & ③ \end{cases}$$

$$\begin{array}{r} ②, ③ \quad y = 1 + \mu - \tau \\ -z = 3 + 2\mu + \tau \\ \hline y - z = 4 + 3\mu \\ -3x + 6y = 9 + 3\mu \end{array}$$

$$\begin{array}{r} x = 1 - \mu + 2\tau \\ + 2y = 2 + 2\mu - 2\tau \\ \hline x + 2y = 3 + \mu \end{array}$$

$$-3x - 5y - z = -5 \Leftrightarrow 3x + 5y + z = 5 \quad \pi$$

$$d \rightarrow \pi \quad 3(1 + 5\lambda) + 5(1 - 3\lambda) + 3 = 5 \Leftrightarrow$$

$$\Leftrightarrow 3 + 15\lambda + 5 - 15\lambda + 3 = 5 \Leftrightarrow 0 \cdot \lambda = -6 \quad \text{impossible}$$

$$\Rightarrow d \parallel \pi$$

EXERCICE 5.24

$$a) a. \quad \mathcal{C}: \begin{cases} x = 2 \\ y = 1 + \mu \\ z = 0 \end{cases}$$

$$\pi: x + 2y - z - 3 = 0$$

$$\mathcal{C}: z = 0$$

$$\vec{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \cdot 2 \\ 1 \cdot 0 \\ 0 \cdot 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

pour $x = 0$ $\begin{cases} z = 0 \\ 2y - z = 3 \end{cases} \quad \begin{matrix} z = 0 \\ y = \frac{3}{2} \end{matrix} \quad A(0; \frac{3}{2}; 0)$

$$d = \mathcal{C} \cap \pi \quad \begin{cases} x = -2t \\ y = \frac{3}{2} + t \\ z = 0 \end{cases}$$

$$b) \text{ } \mathcal{C}: -3x - 6y + 3z - 9 = 0 \Leftrightarrow x + 2y - z + 3 = 0$$

$$\Pi: x + 2y - z - 3 = 0$$

$\Pi // \mathcal{C}$

$$b) \text{ } \mathcal{C} // \Pi: x + 2y - z - 3 = 0 \quad A(2; 4; 3)$$

$$\vec{n} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \Rightarrow \mathcal{C}: x + 2y - z + C = 0$$

$$A: 2 + 8 - 3 + C = 0 \Leftrightarrow 7 + C = 0 \Leftrightarrow C = -7$$

Donc $\mathcal{C}: x + 2y - z - 7 = 0$

EXERCICE 5.25

$$\vec{a} = \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix}$$

$$\vec{a} \cdot \vec{b} = 40 + 18 + 3 = 61$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \begin{pmatrix} 6 \\ -2 \\ -5 \end{pmatrix} =$$

$$\vec{b} \cdot \vec{a} = 61$$

$$48 + 18 - 5 = 61$$

$$\vec{a} \cdot \vec{c} = 8 + 0 - 8 = 0$$

$$\vec{c} - \vec{a} = \begin{pmatrix} -7 \\ 9 \\ -9 \end{pmatrix} \quad \vec{b} - \vec{a} = \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix}$$

$$(\vec{c} - \vec{a}) \cdot (\vec{b} - \vec{a}) = 21 + 63 - 18 = 84 - 18 = 66$$

EXERCICE 5.26

$$A(-4; 7; 13) \quad B(8,2; 0,7; -9)$$

$$a. \quad |\vec{OA}| = \sqrt{16 + 49 + 169} = 15,3$$

$$|\vec{OB}| = \sqrt{8,2^2 + 0,7^2 + 81} = 12,2$$

$$\vec{AB} = \begin{pmatrix} 12,2 \\ -6,3 \\ -22 \end{pmatrix} \quad |\vec{AB}| = \sqrt{12,2^2 + 6,3^2 + 22^2} = 25,93$$

$$b. \quad \vec{OA} \cdot \vec{OB} = (-4) \cdot 8,2 + 7 \cdot 0,7 + 13(-9) =$$

$$= -32,8 + 4,9 - 117 = \underline{\underline{-144,9}}$$

$$\vec{u}_{OA} = \begin{pmatrix} -4/15.3 \\ 7/15.3 \\ 13/15.3 \end{pmatrix}$$

$$d. |\vec{OA} \cdot \vec{OB}| = \|\vec{a}\| \cdot \|\vec{b}\| \Rightarrow \|\vec{b}\| = \frac{144.9}{15.3} \Rightarrow \|\vec{b}\| = 9.47$$

$$|\vec{OA} \cdot \vec{OB}| = \|\vec{a}\| \cdot \|\vec{b}\| \Rightarrow \|\vec{a}\| = \frac{144.9}{12.2} \Rightarrow \|\vec{a}\| = 11.877$$

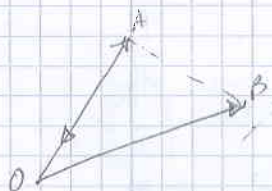
$$e) \cos(\widehat{AOB}) = \frac{|\vec{OA} \cdot \vec{OB}|}{\|\vec{OA}\| \cdot \|\vec{OB}\|} \Rightarrow \cos(\widehat{AOB}) = \frac{-144.9}{15.3 \times 12.2} \Rightarrow \cos \alpha_1 = -0.776 \Rightarrow \alpha_1 = 140.92^\circ$$

$$\cos(\widehat{OAB}) = \frac{|\vec{AO} \cdot \vec{AB}|}{\|\vec{AO}\| \cdot \|\vec{AB}\|} = \frac{378.9}{15.3 \times 25.93} = 0.955 \Rightarrow \alpha_2 = 17.24^\circ$$

$$\vec{AO} = \begin{pmatrix} 4 \\ -7 \\ -13 \end{pmatrix}$$

$$\vec{AB} = \begin{pmatrix} 12.2 \\ -6.3 \\ -2.2 \end{pmatrix}$$

$$\vec{AO} \cdot \vec{AB} = 4 \cdot 12.2 + (-7)(-6.3) + (-13)(-2.2) = 378.9$$



$$f) \text{Aire}(OAB) = \frac{1}{2} \|\vec{a} \wedge \vec{b}\| = \frac{1}{2} \left\| \begin{pmatrix} -4 \\ 7 \\ 13 \\ -4 \\ 7 \end{pmatrix} \wedge \begin{pmatrix} 8.2 \\ 0.7 \\ -9 \\ 8.2 \\ 0.7 \end{pmatrix} \right\| = \frac{1}{2} \left\| \begin{pmatrix} -63 - 9.1 \\ 106.6 - 36 \\ -2.8 - 57.4 \end{pmatrix} \right\| =$$

$$= \frac{1}{2} \left\| \begin{pmatrix} -72.1 \\ 70.6 \\ -60.2 \end{pmatrix} \right\| = \frac{1}{2} \sqrt{72.1^2 + 70.6^2 + 60.2^2} = 58.75 = \text{Aire}(OAB)$$

EXERCICE 5.27

$$a) \vec{a} \cdot \vec{b} = 0 \Rightarrow -3 + (2+k)(-6k) + (-0.5) \cdot 5 = 0 \Leftrightarrow$$

$$\Leftrightarrow -3 - 12k - 6k^2 - 2.5 = 0 \Leftrightarrow -6k^2 - 12k - 5.5 = 0 \Leftrightarrow$$

$$\Leftrightarrow 6k^2 + 12k + 5.5 = 0$$

$$\Delta = 12^2 - 4 \cdot 6 \cdot 5.5 = 12$$

$$k_{1,2} = \frac{-12 \pm \sqrt{12}}{12} = \begin{cases} -0.71 = k_1 \\ -1.289 = k_2 \end{cases}$$

$$b) \begin{pmatrix} 4 \\ 3 \\ 8 \\ 4 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} -5 \\ 20 \\ 9 \\ -5 \\ 20 \end{pmatrix} = \begin{pmatrix} 27 - 160 \\ -40 - 36 \\ 80 + 15 \end{pmatrix} = \begin{pmatrix} -133 \\ -76 \\ 95 \end{pmatrix} = \vec{w}$$

$\vec{w}$ doit être $\parallel \vec{\alpha} \begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix}$

$$-133 = \lambda \cdot 7 \Rightarrow \lambda = -19$$

$$-76 = \lambda \cdot a \Rightarrow a = -76 / -19 \Rightarrow a = 4$$

$$95 = \lambda b \Rightarrow b = 95 / -19 \Rightarrow b = -5$$

EXERCICE 5.28

$$a. \vec{a} = \begin{pmatrix} 16 \\ -2 \\ 8 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -1 \\ 0 \\ m \end{pmatrix} \quad \hat{\alpha} = 60^\circ \Rightarrow \cos \hat{\alpha} = \frac{1}{2}$$

$$\cos \hat{\alpha} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} \quad (1)$$

$$\vec{a} \cdot \vec{b} = -16 + 0 + 8m = -16 + 8m$$

$$\|\vec{a}\| = \sqrt{16^2 + 4 + 64} = 18 \quad \|\vec{b}\| = \sqrt{1 + m^2}$$

$$(1) \Leftrightarrow \frac{1}{2} = \frac{-16 + 8m}{18 \cdot \sqrt{1 + m^2}} \Leftrightarrow 18 \sqrt{1 + m^2} = 2(-16 + 8m) \Leftrightarrow$$

$$\Leftrightarrow 9 \sqrt{1 + m^2} = (-16 + 8m) \Rightarrow 81(1 + m^2) = (-16 + 8m)^2 \Leftrightarrow$$

$$\Leftrightarrow 81(1 + m^2) = 256 - 256m + 64m^2 \Leftrightarrow$$

$$17m^2 + 256m - 175 = 0$$

$$\Delta = 77436$$

$$m_{1,2} = \frac{-256 \pm 278.3}{34} = \begin{cases} -15.72 = m_1 \\ 0.656 = m_2 \end{cases}$$

$$b) \vec{a} = \begin{pmatrix} 6 \\ -3 \\ 4 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ n \\ -2 \end{pmatrix} \quad \vec{a} \cdot \vec{b} = 6 - 3n - 8 = -3n - 2$$

$$\|\vec{a}\| = \sqrt{36 + 9 + 16} = \sqrt{61}$$

$$\|\vec{b}\| = \sqrt{1 + n^2 + 4} = \sqrt{5 + n^2}$$

$$\cos 45 = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} \Rightarrow \frac{\sqrt{2}}{2} = \frac{-3n - 2}{\sqrt{61} \cdot \sqrt{5 + n^2}} \Leftrightarrow \sqrt{2} \sqrt{61} \sqrt{5 + n^2} = 2(-3n - 2) \Leftrightarrow$$

$$\Leftrightarrow 2 \cdot 61 \cdot (5 + n^2) = 4(-3n - 2)^2 \Leftrightarrow 122(5 + n^2) = 4(9n^2 + 12n + 4) \Leftrightarrow$$

$$61(5 + n^2) = 2(9n^2 + 12n + 4) \Leftrightarrow 61n^2 + 305 = 18n^2 + 24n + 8 \Leftrightarrow$$

$$43n^2 - 24n + 297 = 0$$

$$43n^2 - 24n + 297 = 0$$

$$\Delta = -50508 < 0 \quad \text{Pas de solutions}$$

EXERCICE 5.29

$$d: A(2; 6; 0) \quad B(-1; 2; 5)$$

$$a. \quad \vec{AB} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$$

$$d = \begin{cases} x = 2 - 3t \\ y = 6 - 4t \\ z = 5t \end{cases}$$

$$T_1: z = 0 \Rightarrow t = 0 \quad \underline{T_1(2; 6; 0)}$$

$$T_2: x = 0 \Leftrightarrow 2 - 3t = 0 \Leftrightarrow t = \frac{2}{3}$$

$$y = 6 - 4 \cdot \frac{2}{3} = \frac{10}{3} \Rightarrow y = \frac{10}{3}$$

$$z = 5 \cdot \frac{2}{3} \Leftrightarrow z = \frac{10}{3}$$

$$\underline{T_2 = \left(0; \frac{10}{3}; \frac{10}{3}\right)}$$

$$T_3: y = 0 \Leftrightarrow 6 - 4t = 0 \Leftrightarrow t = \frac{3}{2}$$

$$x = 2 - 3 \cdot \frac{3}{2} = \frac{4-9}{2} = -\frac{5}{2}$$

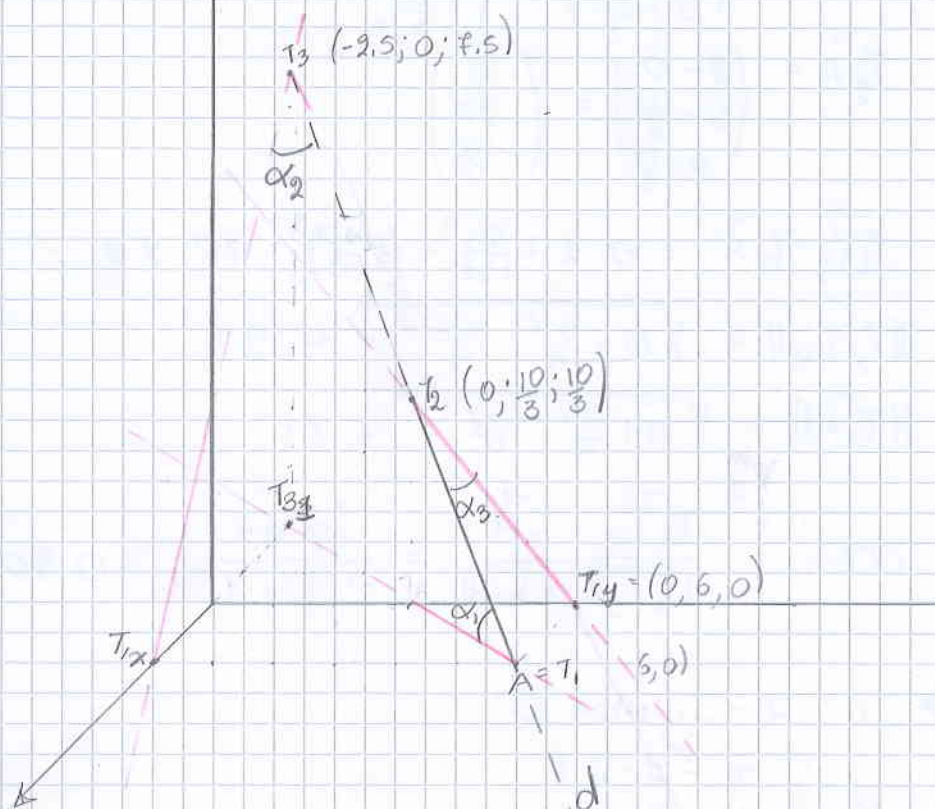
$$z = 5 \cdot \frac{3}{2} \Rightarrow z = \frac{15}{2}$$

$$\underline{T_3 = \left(-\frac{5}{2}; 0; \frac{15}{2}\right)}$$

$$b) \quad T_{3x} \left(-\frac{5}{2}; 0; 0\right)$$

$$T_{1x}(2; 0; 0)$$

$$T_{1y}(0; 6; 0)$$



$$\vec{AT_3} = \begin{pmatrix} -\frac{5}{2} - 2 \\ 0 - 6 \\ \frac{15}{2} - 0 \end{pmatrix} = \begin{pmatrix} -4.5 \\ -6 \\ 7.5 \end{pmatrix}$$

$$\vec{AT_{3s}} = \begin{pmatrix} -\frac{5}{2} - 2 \\ 0 - 6 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} -4.5 \\ -6 \\ 0 \end{pmatrix}$$

$$\cos \alpha_1 = \frac{\vec{AT_3} \cdot \vec{AT_{3s}}}{\|\vec{AT_3}\| \cdot \|\vec{AT_{3s}}\|} = \frac{56.25}{10.61 \cdot 7.5} = 0.7068 \Rightarrow \alpha_1 \approx 45^\circ$$

$$\vec{AT_3} \cdot \vec{AT_{3s}} = (-4.5)^2 + 36 = 56.25$$

$$\|\vec{AT_3}\| = \sqrt{4.5^2 + 36 + 7.5^2} = 10.61$$

$$\|\vec{AT_{3s}}\| = \sqrt{4.5^2 + 36} = 7.5$$

$$\vec{T_3 T_{1x}} = \begin{pmatrix} 2 + \frac{5}{2} \\ 0 - 0 \\ 0 - \frac{15}{2} \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ 0 \\ -\frac{15}{2} \end{pmatrix} \quad \vec{T_3 A} = \begin{pmatrix} 4.5 \\ 6 \\ -7.5 \end{pmatrix}$$

$$\|\vec{T_3 T_{1x}}\| = \sqrt{4.5^2 + 7.5^2} = 8.75$$

$$\|\vec{T_3 A}\| = 10.61$$

$$\vec{T_3 T_{1x}} \cdot \vec{T_3 A} = 4.5^2 + 7.5^2 = 76.5$$

$$\cos \alpha_2 = \frac{76.5}{10.61 \cdot 8.75} = 0.824 \Rightarrow \alpha_2 = 34.51^\circ$$

$$\vec{T_2 T_{1y}} = \begin{pmatrix} 0+0 \\ 6-\frac{10}{3} \\ 0-\frac{10}{3} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{8}{3} \\ -\frac{10}{3} \end{pmatrix}$$

$$\vec{T_2 A} = \begin{pmatrix} 2-0 \\ 6-\frac{10}{3} \\ 0-\frac{10}{3} \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{8}{3} \\ -\frac{10}{3} \end{pmatrix}$$

$$\vec{T_2 T_{1y}} \cdot \vec{T_2 A} = 0 \cdot 2 + \left(\frac{8}{3}\right)^2 + \left(-\frac{10}{3}\right)^2 = 18.22$$

$$\|\vec{T_2 T_{1y}}\| = \sqrt{0 + \frac{64}{9} + \frac{100}{9}} = 4.27$$

$$\|\vec{T_2 A}\| = \sqrt{4 + \frac{64}{9} + \frac{100}{9}} = 4.71$$

$$\cos \alpha_3 = \frac{\vec{T_2 T_{1y}} \cdot \vec{T_2 A}}{\|\vec{T_2 T_{1y}}\| \cdot \|\vec{T_2 A}\|} = \frac{18.22}{4.27 \cdot 4.71} = 0.905 \Rightarrow \alpha_3 = 25.15^\circ$$

$$\triangleright \Pi: x - 5y + 11 = 0$$

$$d: \begin{cases} x = 2 - 3t \\ y = 6 - 4t \\ z = 5t \end{cases}$$

$$d \rightarrow \Pi: 2 - 3t - 5(6 - 4t) + 11 = 0 \Leftrightarrow 2 - 3t - 30 + 20t + 11 = 0 \Leftrightarrow$$

$$\Leftrightarrow 17t - 17 = 0 \Leftrightarrow t = 1$$

Donc I : point d'intersection.

$$\begin{aligned} x &= -1 \\ y &= 2 \\ z &= 5 \end{aligned}$$

$$\underline{I(-1; 2; 5)}$$

$$\vec{d} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$$

$\vec{n}$ = vecteur normal du plan Π .

$\vec{d}$ = vecteur directeur de (d)

$$\vec{n} = \begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix}$$

On trouve l'angle entre $\vec{d}, \vec{n}$

$$\vec{d} \cdot \vec{n} = -3 + 20 = 17$$

$$\|\vec{d}\| = \sqrt{9 + 16 + 25} = \sqrt{50} \quad \|\vec{n}\| = \sqrt{1 + 25} = \sqrt{26}$$

$$\text{donc } \cos(\vec{d}, \vec{n}) = \frac{\vec{d} \cdot \vec{n}}{\|\vec{d}\| \cdot \|\vec{n}\|} = \frac{17}{\sqrt{50} \cdot \sqrt{26}} = 0.47 \Rightarrow (\vec{d}, \vec{n}) = 61.87^\circ$$

$$\Rightarrow (\Pi, d) = 90 - 61.87 = \underline{28.13^\circ} = (\Pi, d)$$

EXERCICE 5.30

d: $A(4; -1; -2) \quad B(1; 3; -3)$

$$\vec{AB} = \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix}$$

le vecteur $\vec{AB}$ est le vecteur normal $\vec{n}$ du plan

Donc $\Pi: -3x + 4y - z + C = 0$

On remplace les coordonnées de A:

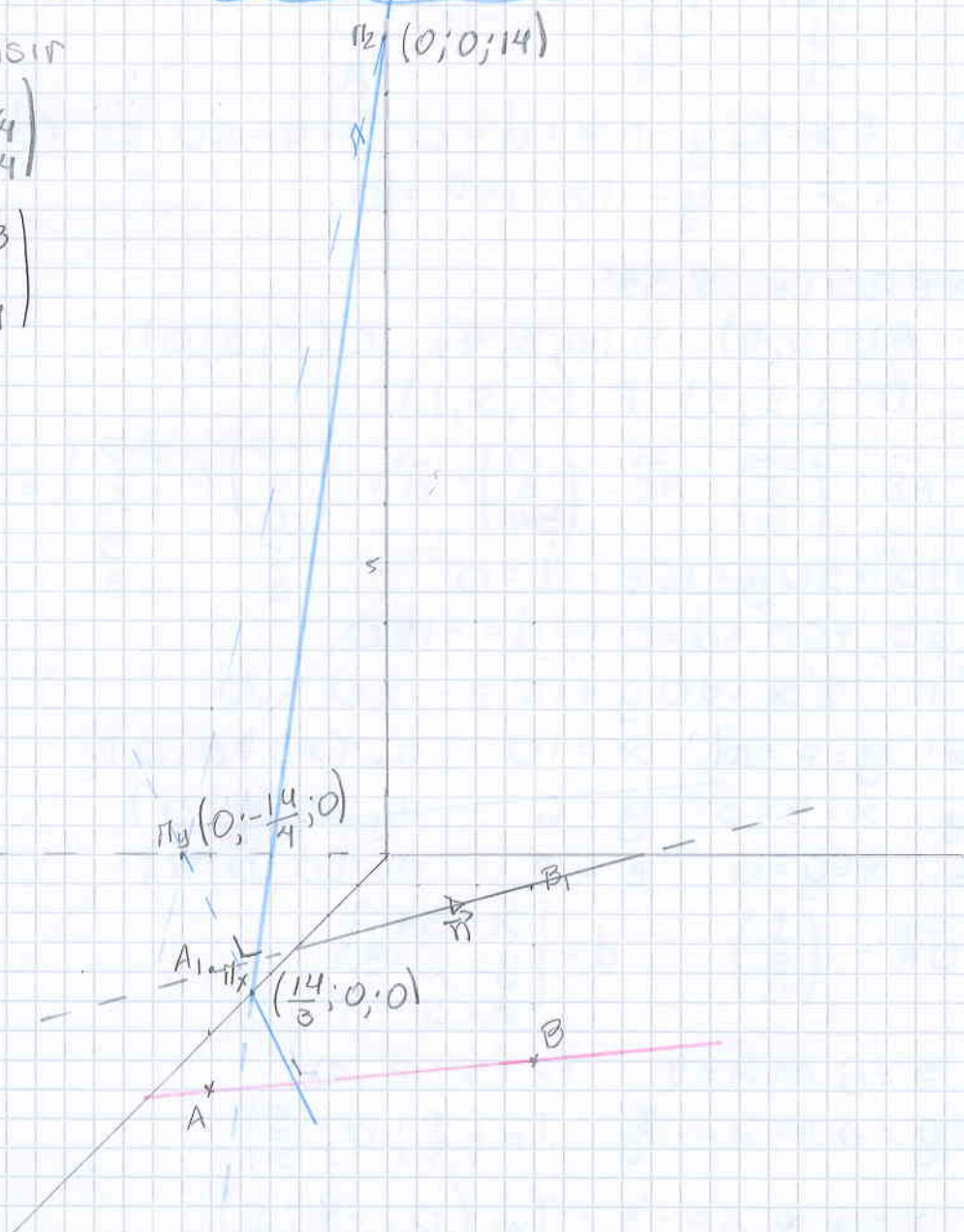
$$-3 \cdot 4 - 4 + 2 + C = 0 \Leftrightarrow -14 + C = 0 \Leftrightarrow C = 14$$

Alors $\Pi: -3x + 4y - z + 14 = 0$

On peut choisir

$$\Pi_z \Pi_y = \begin{pmatrix} 0 \\ -14/4 \\ -14 \end{pmatrix}$$

$$\Pi_z \Pi_x = \begin{pmatrix} 14/3 \\ 0 \\ -14 \end{pmatrix}$$



EXERCICE 5.31

a) $\vec{n} = \begin{pmatrix} 1 \\ -3 \\ 6 \end{pmatrix}$ A $\left(\frac{16}{5}; 9; -4\right)$

$$x - 3y + 6z + d = 0 \quad \downarrow \quad \frac{16}{5} - 27 - 24 + d = 0 \Leftrightarrow d = \frac{239}{5}$$

$$\Pi: x - 3y + 6z + \frac{239}{5} = 0$$

b) $d = \begin{cases} x = 2 + 13\lambda \\ y = -3 - 2\lambda \\ z = 11 + 8\lambda \end{cases}$ A $(-21; 17; 82)$

$$\vec{n} = \begin{pmatrix} 13 \\ -2 \\ 8 \end{pmatrix} \quad \Pi: 13x - 2y + 8z + d = 0$$

$$A: -273 - 34 + 656 + d = 0 \Leftrightarrow d = -349$$

$$\Pi: 13x - 2y + 8z - 349 = 0$$

c) $\vec{n} = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} -3 \\ -8 \\ 9 \\ -3 \\ -8 \end{pmatrix} = \begin{pmatrix} 7 \\ -15 \\ -11 \end{pmatrix}$ A $(-2; 4; 5)$

$$\Pi: 7x - 15y - 11z + d = 0 \quad \downarrow \quad -14 - 60 - 55 + d = 0 \Leftrightarrow d = 129$$

$$7x - 15y - 11z + 129 = 0$$

EXERCICE 5.32

$$\Pi: A(5; 0; 4) \quad B(0; 3; 4) \quad C(5; 3; 0)$$

$$D(3; 2; 2) \quad E(4; 5; 1)$$

a. $\vec{AB} = \begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix}$ $\vec{n} = \begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -12 \\ -20 \\ -15 \end{pmatrix} \parallel \begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix}$

$$12x + 20y + 15z + d = 0$$

$$A: 60 + 60 + d = 0 \Leftrightarrow d = -120$$

$$\Pi: 12x + 20y + 15z - 120 = 0$$

$$\bullet O_x: y = z = 0 \quad x = 10 \quad \Pi_x(10; 0; 0) \quad \Gamma = C$$

$$\bullet O_y: x = z = 0 \quad y = 6 \quad \Pi_y(0; 6; 0)$$

$$\bullet O_z: x = y = 0 \quad z = 8 \quad \Pi_z(0; 0; 8)$$

b. $\vec{DE} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ $d = \begin{cases} x = 3 + \lambda \\ y = 2 + 3\lambda \\ z = 2 - \lambda \end{cases}$

$$T_s: z = 0 \Leftrightarrow \lambda = 2 \quad T_s(5; 8; 0)$$

$$T_p: y = 0 \Leftrightarrow \lambda = -\frac{2}{3} \quad T_p\left(\frac{7}{3}; 0; \frac{8}{3}\right)$$

$$T_m: x = 0 \Leftrightarrow \lambda = -3 \quad T_m(0; -7; 5)$$

$$c. \hat{A}: \cos \hat{A} = \frac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\| \cdot \|\vec{AC}\|} = \frac{9}{\sqrt{34} \cdot 5} = 0.309 \Rightarrow \hat{A} = 72^\circ$$

$$\vec{AB} \cdot \vec{AC} = 9 \quad \|\vec{AB}\| = \sqrt{25+9} = \sqrt{34} \quad \|\vec{AC}\| = \sqrt{9+16} = 5$$

$$\vec{BA} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} \quad \vec{BC} = \begin{pmatrix} 5 \\ 0 \\ -4 \end{pmatrix} \quad \vec{BA} \cdot \vec{BC} = 25$$

$$\|\vec{BA}\| = \sqrt{34} \quad \|\vec{BC}\| = \sqrt{25+16} = \sqrt{41}$$

$$\cos \hat{B} = \frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \cdot \|\vec{BC}\|} = \frac{25}{\sqrt{34} \cdot \sqrt{41}} = 0.67 \Rightarrow \hat{B} = 47.96^\circ$$

$$\text{Donc } \hat{C} = 180^\circ - 72^\circ - 47.96^\circ \Rightarrow \hat{C} = 60^\circ$$

$$A_{ABC} = \frac{1}{2} \|\vec{AB} \wedge \vec{AC}\| = \frac{1}{2} \sqrt{25+9} = \frac{1}{2} \sqrt{34}$$

$$\vec{AB} \wedge \vec{AC} = \begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix}$$

$$d. d = \begin{cases} x = 3 + \lambda \\ y = 2 + 3\lambda \\ z = 2 - \lambda \end{cases} \rightarrow \Pi: 12x + 20y + 15z - 120 = 0$$

$$12(3+\lambda) + 20(2+3\lambda) + 15(2-\lambda) - 120 = 0 \Leftrightarrow$$

$$\Leftrightarrow 36 + 12\lambda + 40 + 60\lambda + 15 - 15\lambda - 120 = 0 \Leftrightarrow$$

$$57\lambda = 29 \Leftrightarrow \lambda = \frac{29}{57} \Rightarrow \text{se'conds}$$

$$\text{Le point d'intersection: } I \left(\frac{200}{57}; \frac{201}{57}; \frac{85}{57} \right)$$

$$e. \cdot \cos \alpha = \frac{\begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\left\| \begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix} \right\| \cdot 1} = \frac{12}{\sqrt{769}} = 0.433 \Rightarrow \alpha = 64.36^\circ$$

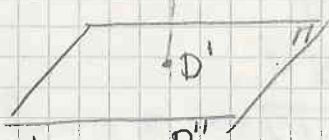
$$\cdot \cos \beta_1 = \frac{\begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}}{\sqrt{769} \cdot \sqrt{11}} = \frac{57}{\sqrt{769} \cdot \sqrt{11}} = 0.62 \Rightarrow \beta_1 = 51.7^\circ$$

$$\Rightarrow \beta_2 = 90 - \beta_1 \Rightarrow \beta_2 = 38.3^\circ$$

$$\cdot \cos \gamma_1 = \frac{\begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{11} \cdot \sqrt{1}} = \frac{15}{\sqrt{11}} = 0.3 \Rightarrow \gamma_1 = 72.45^\circ$$

$$\text{t.p. } \gamma_2 = 90 - \gamma_1 \Rightarrow \gamma_2 = 17.55^\circ$$

f.



$$t: \begin{cases} x = 3 + 12\lambda \\ y = 2 + 20\lambda \\ z = 2 + 15\lambda \end{cases}$$

$$D' = t \cap \Pi: 12(3+12\lambda) + 20(2+20\lambda) + 15(2+15\lambda) - 120 = 0$$

$$\Leftrightarrow 36 + 144\lambda + 40 + 400\lambda + 30 + 225\lambda - 120 = 0 \Leftrightarrow$$

$$\Leftrightarrow 769\lambda = 14 \Rightarrow \lambda = \frac{14}{769}$$

$$D' (3.22; 2.36; 2.3)$$

$$\vec{DD'} = \begin{pmatrix} 0.22 \\ 0.36 \\ 0.3 \end{pmatrix} \quad \vec{DD''} = 2\vec{DD'} \Rightarrow \begin{pmatrix} x-3 \\ y-2 \\ z-2 \end{pmatrix} = \begin{pmatrix} 0.44 \\ 0.72 \\ 0.6 \end{pmatrix} \Leftrightarrow$$

$$\Rightarrow x = 3.44 \quad y = 2.72 \quad z = 2.6$$

$$D''(3.44; 2.72; 2.6)$$

EXERCISE 5.33

$$\vec{a} \wedge \vec{b} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -12 \\ -4 \\ 8 \end{pmatrix}$$

$$\vec{b} \wedge \vec{a} = -\vec{a} \wedge \vec{b} = \begin{pmatrix} 12 \\ 4 \\ -8 \end{pmatrix}$$

$$\vec{a} \wedge \vec{c} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ -13 \\ 4 \end{pmatrix}$$

$$\vec{b} \wedge \vec{c} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} \wedge \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 16 \\ -2 \\ 4 \end{pmatrix}$$

EXERCISE 5.34

$$\Pi: 6x + 2y + 3z - 12 = 0 \quad G: x + 2y - 2z - 4 = 0$$

$$\alpha: \cos \alpha = \frac{|\vec{n}_\Pi \cdot \vec{n}_G|}{\|\vec{n}_\Pi\| \cdot \|\vec{n}_G\|} = \frac{4}{7 \cdot 3} = 0.19 \Rightarrow \alpha \approx 79^\circ$$

$$\vec{n}_\Pi \cdot \vec{n}_G = 6 - 4 - 6 = -4 \quad \|\vec{n}_\Pi\| = \sqrt{49} = 7 \quad \|\vec{n}_G\| = \sqrt{9} = 3$$

$$b. \text{ sol: } 6x + 2y - 12 = 0 \Rightarrow 3x + y - 6 = 0 \quad \Leftrightarrow$$

$$\begin{cases} 3x + y = 6 \\ -3x - 6y = -12 \end{cases} \quad (+) \quad -5y = -6 \Rightarrow y = \frac{6}{5}$$

$$x = -\frac{12}{5} + 4 = \frac{8}{5}$$

$$A\left(\frac{8}{5}; \frac{6}{5}; 0\right)$$

$$\text{paroi: } \begin{cases} 6x + 3z = 12 \\ x - 2z = 4 \end{cases} \quad (-6) \quad \begin{cases} 6x + 3z = 12 \\ -6x + 12z = -24 \end{cases}$$

$$x = 4 - \frac{8}{5} \Rightarrow x = \frac{12}{5}$$

$$15z = -12 \Rightarrow z = -\frac{4}{5}$$

$$B\left(\frac{12}{5}; 0; -\frac{4}{5}\right)$$

$$\vec{AB} = \begin{pmatrix} \frac{4}{5} \\ -\frac{6}{5} \\ -\frac{4}{5} \end{pmatrix} \parallel \begin{pmatrix} 4 \\ -6 \\ -4 \end{pmatrix} \parallel \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} \quad i = \begin{cases} x = \frac{8}{5} + 2\lambda \\ y = \frac{6}{5} - 3\lambda \\ z = -2\lambda \end{cases}$$

EXERCICE 5.35

$$\pi: 3x - 6y + 22z - 66 = 0$$

$$\text{dis}(A; \pi) = \frac{|3(-\frac{1}{3}) - 6(-2) + 22 \cdot 2.5 - 66|}{\sqrt{3^2 + 6^2 + 22^2}} = 0 \Rightarrow A \in \pi$$

$$\text{dis}(B; \pi) = \frac{|16 + 48 + 88 - 66|}{\sqrt{3^2 + 6^2 + 22^2}} = \frac{76}{23} \approx 3.3$$

$$\text{dist}(C; \pi) = \frac{|11.2 + 12 + 125.4 - 66|}{23} = \frac{72.6}{23} \approx 3.16$$

EXERCICE 5.36

$$A(0; 4; 1)$$

$$B(-3; 1; 7)$$

$$\pi: 7x - 4y + 4z + 11 = 0$$

$$\alpha. \vec{AB} = \begin{pmatrix} -3 \\ -3 \\ 6 \end{pmatrix} // \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \quad d = \begin{cases} x = \lambda \\ y = 4 + \lambda \\ z = 1 - 2\lambda \end{cases}$$

$$\text{Soit } P \in d: P(\lambda; 4 + \lambda; 1 - 2\lambda)$$

$$\text{dis}(P; \pi) = 5 \Rightarrow \frac{|7\lambda - 4(4 + \lambda) + 4(1 - 2\lambda) + 11|}{\sqrt{49 + 16 + 16}} = 5 \Leftrightarrow$$

$$\Leftrightarrow |7\lambda - 16 - 4\lambda + 4 - 8\lambda + 11| = 45 \Leftrightarrow |-5\lambda - 1| = 45 \Leftrightarrow$$

$$\begin{cases} -5\lambda - 1 = 45 \Leftrightarrow 5\lambda = -46 \Rightarrow \lambda_1 = -\frac{46}{5} \\ -5\lambda - 1 = -45 \Leftrightarrow 5\lambda = 44 \Rightarrow \lambda = \frac{44}{5} \end{cases}$$

$$\left(-\frac{46}{5}; -\frac{26}{5}; \frac{97}{5}\right) \text{ et } \left(\frac{44}{5}; \frac{64}{5}; -\frac{83}{5}\right)$$

$$b. \text{ Soit } Q \in Ox. Q(x, 0, 0) \quad \vec{QA} = \begin{pmatrix} -x \\ 4 \\ 1 \end{pmatrix} \quad \vec{QB} = \begin{pmatrix} -3-x \\ 1 \\ 7 \end{pmatrix}$$

$$\|\vec{QA}\| = \|\vec{QB}\| \Leftrightarrow \sqrt{x^2 + 16 + 1} = \sqrt{(3+x)^2 + 1 + 49} \Leftrightarrow$$

$$\Leftrightarrow x^2 + 17 = 9 + 6x + x^2 + 50 \Leftrightarrow 6x = -42 \Leftrightarrow x = -7$$

$$Q(-7; 0; 0)$$

EXERCICE 5.37

$$\alpha. A(2; 3.5; -1)$$

$$B(3; 8; -5)$$

$$\vec{OA} \times \vec{OB} = \begin{pmatrix} 2 \\ 3.5 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} = \begin{pmatrix} -9.5 \\ 7 \\ 5.5 \end{pmatrix}$$

$$\text{Aire} = \|\vec{OA} \times \vec{OB}\| = \sqrt{9.5^2 + 7^2 + 5.5^2} \approx 95.42$$

b. $A_1(1; 3; -2)$ $B_1(-6; y; 12)$

$$\vec{OA_1} \wedge \vec{OB_1} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \wedge \begin{pmatrix} -6 \\ y \\ 12 \end{pmatrix} = \begin{pmatrix} 36+2y \\ 0 \\ y+18 \end{pmatrix}$$

$$A = \sqrt{5} \Rightarrow \sqrt{(36+2y)^2 + (y+18)^2} = \sqrt{5} \Leftrightarrow$$

$$\Leftrightarrow 1296 + 4y^2 + 144y + y^2 + 36y + 324 = 5 \Leftrightarrow$$

$$\Leftrightarrow 5y^2 + 180y + 1615 = 0 \Leftrightarrow y^2 + 36y + 323 = 0$$

$$\Delta = 4 \quad y_{1,2} = \frac{-36 \pm 2}{2} \quad \begin{matrix} y_1 = -17 \\ y_2 = -19 \end{matrix}$$

EXERCICE 5.38

$$\Pi_x(3; 0; 0) \quad \Pi_y(0; 5; 0) \quad \Pi_z(0; 0; 8)$$

a. $\text{ppmc}(3; 5; 8) = 120$

$$\frac{120}{3}x + \frac{120}{5}y + \frac{120}{8}z + 120 = 0 \Rightarrow$$

$$\Pi: 40x + 24y + 15z + 120 = 0$$

b: $\text{dis}(O; \Pi) = \frac{|120|}{\sqrt{40^2 + 24^2 + 15^2}} = \frac{120}{49} \approx 2.45$

c. On calcule l'aire de la base

$$\Pi_x \Pi_y = \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} \quad \Pi_x \Pi_z = \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix} \quad \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix} = \begin{pmatrix} 40 \\ 24 \\ 15 \end{pmatrix}$$

$$Ab = \frac{1}{2} \left\| \begin{pmatrix} 40 \\ 24 \\ 15 \end{pmatrix} \right\| = \frac{49}{2}$$

Aire de la pyramide: $\frac{Ab \times h}{3} = \frac{\frac{49}{2} \times 2.45}{3} \approx 20$

EXERCICE 5.39

$$A(2; 2; -5) \quad B(4; -1; -4) \quad \vec{AB} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad d = \begin{cases} x = 2 + 2\lambda \\ y = 2 - 3\lambda \\ z = -5 + \lambda \end{cases}$$

$$C(2; 1; 1) \quad \text{dis}(C, d) = \frac{\|\vec{AB} \times \vec{AC}\|}{\|\vec{AB}\|} = \frac{\sqrt{437}}{\sqrt{14}}$$

$$\vec{AB} \times \vec{AC} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix} = \begin{pmatrix} -17 \\ -12 \\ -2 \end{pmatrix} \quad \|\vec{AB}\| = \sqrt{14} \\ \|\vec{AB} \times \vec{AC}\| = \sqrt{437}$$

$$D(-6; -3; 5) \quad \vec{AD} = \begin{pmatrix} -8 \\ -5 \\ 10 \end{pmatrix} \quad \vec{AD} \times \vec{AB} = \begin{pmatrix} -8 \\ -5 \\ 10 \end{pmatrix} \times \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 25 \\ 28 \\ 34 \end{pmatrix}$$

$$\|\vec{AD} \times \vec{AB}\| = \sqrt{2565}$$

$$\text{dis}(D, d) = \frac{\|\vec{AD} \times \vec{AB}\|}{\|\vec{AB}\|} = \frac{\sqrt{2565}}{\sqrt{14}} \approx 13.54$$

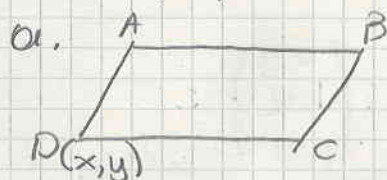
$$E(6; -4; -3) \quad \vec{AE} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix} \quad \vec{AE} \times \vec{AB} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\|\vec{AE} \times \vec{AB}\| = 0$$

$$\text{dis}(E, d) = \frac{\|\vec{AE} \times \vec{AB}\|}{\|\vec{AB}\|} = 0 \Rightarrow E \in d$$

EXERCICE 5.40

$$A(3; 0; 1) \quad B(0; 1; 3) \quad C(5; 4; 3)$$



$$\vec{AB} = \vec{DC} \Leftrightarrow \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5-x \\ 4-y \\ 3-z \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -3 = 5-x \Leftrightarrow x = 8 \\ 1 = 4-y \Leftrightarrow y = 3 \\ 2 = 3-z \Leftrightarrow z = 1 \end{cases} \quad D(8; 3; 1)$$

$$b. \quad \vec{AB} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \quad \vec{AD} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} \quad \vec{AB} \times \vec{AD} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \wedge \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 10 \\ -14 \end{pmatrix}$$

$$A_{\#} = \|\vec{AB} \times \vec{AD}\| = \sqrt{332}$$

$$\vec{AC} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$$

$$\vec{AB} \times \vec{AC} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ 10 \\ -14 \end{pmatrix}$$

$$\text{dist}(C, AB) = \frac{\|\vec{AB} \times \vec{AC}\|}{\|\vec{AB}\|} =$$

$$\|\vec{AB} \times \vec{AC}\| = \frac{\sqrt{332}}{\sqrt{14}} \approx 4.84$$

$$c. \vec{n}_\Pi = \vec{AB} \times \vec{AC} = \begin{pmatrix} -6 \\ 10 \\ -14 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix}$$

$$\Pi: 3x - 5y + 7z + d = 0$$

$$A: 9 + 7 + d = 0 \Leftrightarrow d = -16$$

$$\Pi: 3x - 5y + 7z - 16 = 0$$

$$d. \vec{n}_{\Pi'} = \vec{n}_\Pi \times \vec{AB} = \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix} \times \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -17 \\ -27 \\ -12 \end{pmatrix} \parallel \begin{pmatrix} 17 \\ 27 \\ 12 \end{pmatrix}$$

$$17x + 27y + 12z + d = 0$$

$$A: 51 + 12 + d = 0 \Leftrightarrow d = -63$$

$$\Pi': 17x + 27y + 12z - 63 = 0$$

$$e. \text{dist}(C, \Pi') = \frac{|85 + 108 + 36 - 63|}{\sqrt{17^2 + 27^2 + 12^2}} = \frac{166}{\sqrt{1162}} \approx 4.87 = \text{dis}(C, AB)$$

EXERCICE 5.41

$$d_1 = \begin{cases} x = 3 - 2\lambda \\ y = 1 + \lambda \\ z = -4\lambda \end{cases}$$

$$d_2: A(0; 3; -7) \quad B(-4; 5; -15)$$

$$\vec{AB} = \begin{pmatrix} -4 \\ 2 \\ -8 \end{pmatrix} \parallel \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

$$C(3, 1, 0) \quad \vec{d}_1 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$$

$$\text{c'est d'ailleurs } d_1 \parallel d_2 \rightarrow \text{dist}(d_1, d_2) = \text{dis}(A, d_1)$$

$$\text{dis}(A, d_1) = \frac{\|\vec{CA} \times \vec{d}_1\|}{\|\vec{d}_1\|} = \frac{\sqrt{6}}{\sqrt{21}} \approx 0.53$$

$$\vec{CA} = \begin{pmatrix} -3 \\ 2 \\ -7 \end{pmatrix} \quad \vec{CA} \times \vec{d}_1 = \begin{pmatrix} -3 \\ 2 \\ -7 \end{pmatrix} \times \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

EXERCICE 5.42

$$d_1 = \begin{cases} x = 1 + 2m_1 \\ y = 2 - m_1 \\ z = 3 + 3m_1 \end{cases}$$

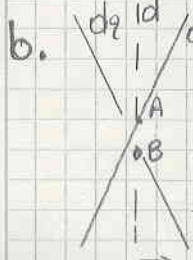
$$d_2 = \begin{cases} x = 3 + 3m_2 \\ y = 1 + m_2 \\ z = -1 + 2m_2 \end{cases}$$

$$a. \vec{d}_1 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \times \vec{d}_2 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \Rightarrow \begin{cases} \text{se croisent} \\ \text{gauches} \end{cases}$$

$$(1) 1 + 2m_1 = 3 + 3m_2 \quad (2) 2 - m_1 = 1 + m_2 \quad (3) 3 + 3m_1 = -1 + 2m_2$$

$$(2) 2 - m_1 = 1 + m_2 \Rightarrow m_2 = 1 - m_1$$

$$(3) 3 + 3m_1 = -1 + 2m_2 \Rightarrow 3 + 3m_1 = -1 + 2(1 - m_1) \Rightarrow 3 + 3m_1 = -1 + 2 - 2m_1 \Rightarrow 3 + 3m_1 = 1 - 2m_1 \Rightarrow 5m_1 = -2 \Rightarrow m_1 = -\frac{2}{5}$$



On cherche le plan $\Pi: d, d_2 \in \Pi$

$$\vec{n}_H = \vec{d}_1 \wedge \vec{d}_2 = \begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} 3 \\ 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 4 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$$

$$(3, 1, -1) \in H: +3 + 5 + 4 + d = 0 \Leftrightarrow d = -12$$

$$H: x + 5y - 4z - 12 = 0$$

$$A = H \cap d_1: (1 + 2m_1) + 5(2 - m_1) - 4(3 + 3m_1) - 12 = 0 \Leftrightarrow$$

$$\Leftrightarrow 1 + 2m_1 + 10 - 5m_1 - 12 - 12m_1 - 12 = 0 \Leftrightarrow -15m_1 = 13 \Rightarrow$$

$$\Leftrightarrow m_1 = -\frac{13}{15} \Rightarrow A\left(1 - \frac{26}{15}; 2 + \frac{13}{15}; 3 - \frac{39}{15}\right) = \left(-\frac{11}{15}; \frac{43}{15}; \frac{6}{15}\right)$$

$$d: \begin{cases} x = -\frac{11}{15} + \lambda \\ y = \frac{43}{15} - \lambda \\ z = \frac{6}{15} - \lambda \end{cases}$$

$$B = d_2 \cap d: \begin{cases} -\frac{11}{15} + \lambda = 3 + 3m_2 \\ \frac{43}{15} - \lambda = 1 + m_2 \\ \frac{6}{15} - \lambda = -1 + 2m_2 \end{cases} \Rightarrow \frac{32}{15} = 4 + 4m_2 \Rightarrow m_2 = -\frac{7}{15}$$

$$B\left(\frac{8}{5}; \frac{8}{15}; -2\right)$$

$$c. \text{dis}(d_1, d_2) = \|\vec{AB}\| = 4.08$$

$$\vec{AB} = \begin{pmatrix} \frac{8}{5} + \frac{11}{15} \\ \frac{8}{15} - \frac{43}{15} \\ -2 - \frac{6}{15} \end{pmatrix} = \begin{pmatrix} \frac{35}{15} \\ -\frac{35}{15} \\ -\frac{36}{15} \end{pmatrix} = \begin{pmatrix} \frac{7}{3} \\ -\frac{7}{3} \\ -\frac{12}{5} \end{pmatrix}$$

d. C' est le point A.

EXERCICE 5.43

$$a. (x-8)^2 + (y+2)^2 + (z-7)^2 = 12^2$$

$$b. AB \text{ diamètre: } C\left(-\frac{1+5}{2}; \frac{11+9}{2}; \frac{25+13}{2}\right) = (2; 10; 6)$$

$$r = \frac{1}{2} \|\vec{AB}\| \quad \vec{AB} = \begin{pmatrix} 6 \\ -2 \\ -38 \end{pmatrix} \quad \|\vec{AB}\| = 38.523 \Rightarrow r =$$

$$(x-2)^2 + (y-10)^2 + (z-6)^2 = 38^2$$

$$c. \text{tangente au sol} \Rightarrow |z_0| = r \Rightarrow r = 10$$

$$(x-1)^2 + (y+7)^2 + (z+10)^2 = 100$$

$$d. r = \text{dis}(H; A) = \frac{|4 + 2 - 10 - 4|}{\sqrt{1+4+4}} = \frac{|-8|}{3} = \frac{8}{3}$$

$$(x-4)^2 + (y-1)^2 + (z+5)^2 = \frac{64}{9}$$

EXERCICE 5.44

$$a. x^2 + y^2 + z^2 - 6x + 4y - 18z + 97 = 0 \Leftrightarrow$$

$$(x-3)^2 - 9 + (y+2)^2 - 4 + (z-9)^2 - 81 + 97 = 0 \Leftrightarrow$$

$$(x-3)^2 + (y+2)^2 + (z-9)^2 = -3 \quad \text{impossible}$$

$$b. x^2 + y^2 + z^2 - 2y + 3z - 5.75 = 0 \Leftrightarrow x^2 + (y-1)^2 - 1 + (z+\frac{3}{2})^2 - \frac{9}{4} - 5.75 = 0$$

$$\Leftrightarrow x^2 + (y-1)^2 + (z+\frac{3}{2})^2 = 9 \quad C(0, 1; -\frac{3}{2}) \quad r=3$$

$$c. 3x^2 + y^2 + z^2 + 66x - 8y + 10z + 378 = 0 \quad \text{pas une sphere.}$$

$$d. 2x^2 + 2y^2 + 2z^2 - 12x + 28y - 48z + 204 = 0 \Leftrightarrow$$

$$\Leftrightarrow x^2 + y^2 + z^2 - 6x + 14y - 24z + 102 = 0 \Leftrightarrow$$

$$\Leftrightarrow (x-3)^2 - 9 + (y+7)^2 - 49 + (z-12)^2 - 144 + 102 = 0$$

$$\Leftrightarrow (x-3)^2 + (y+7)^2 + (z-12)^2 = 100 \quad C(3, -7; 12) \quad r=10$$

EXERCICE 5.45

$$P(2; 3; 2) \quad (x-4)^2 + y^2 + (z+1)^2 = 64 \quad C(4; 0; -1) \quad r=8$$

$$d(P, C) < r \Rightarrow \sqrt{(2-4)^2 + (3)^2 + (2+1)^2} < 8 \Leftrightarrow 4 + 9 + (z+1)^2 < 64 \Leftrightarrow$$

$$(z+1)^2 < 51 \Leftrightarrow |z+1| < \sqrt{51} \Leftrightarrow -\sqrt{51} < z+1 < \sqrt{51} \Leftrightarrow$$

$$-\sqrt{51} - 1 < z < \sqrt{51} - 1 \Rightarrow -8.14 < z < 6.14$$

EXERCICE 5.46

$$a. \mathcal{C}(0; 4; 5) \quad A(-3; 4; 5) \quad \vec{AO} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \quad \|\vec{AO}\| = 3 = r$$

$$x^2 + (y-4)^2 + (z-5)^2 = 9$$

$$b. d_1 = \begin{cases} x = 3 - 2\lambda \\ y = 7 - \lambda \\ z = 2 + \lambda \end{cases} \quad d_2 = \begin{cases} x = 3 + \frac{1}{2}\mu \\ y = 7 - \mu \\ z = 2 + \mu \end{cases}$$

$$d_1 \cap \mathcal{C}: (3-2\lambda)^2 + (7-\lambda-4)^2 + (2+\lambda-5)^2 = 9 \Leftrightarrow$$

$$\Leftrightarrow (3-2\lambda)^2 + (3-\lambda)^2 + (\lambda-3)^2 = 9 \Leftrightarrow 9 - 12\lambda + 4\lambda^2 + 9 - 6\lambda + \lambda^2 + \lambda^2 - 6\lambda + 9 = 9 \Leftrightarrow$$

$$\Leftrightarrow 6\lambda^2 - 24\lambda + 18 = 0 \Leftrightarrow \lambda^2 - 4\lambda + 3 = 0 \Leftrightarrow \lambda_1 = 3 \quad \lambda_2 = 1$$

$$I_1(-3; 4; 5) \quad I_2(1; 6; 3)$$

$$d_2 \cap \mathcal{C}: (3+\frac{1}{2}\mu)^2 + (3-\mu)^2 + (\mu-3)^2 = 9 \Leftrightarrow$$

$$\Leftrightarrow 9 + 3\mu + \frac{1}{4}\mu^2 + 9 - 6\mu + \mu^2 + \mu^2 - 6\mu + 9 = 9 \Leftrightarrow$$

$$\Leftrightarrow \frac{9}{4}\mu^2 - 9\mu + 18 = 0 \Leftrightarrow \frac{1}{4}\mu^2 - \mu + 2 = 0 \Leftrightarrow \mu^2 - 4\mu + 8 = 0$$

$$\Delta = 16 - 32 < 0 \quad \text{disjoints}$$

$$c. d_3 = \begin{cases} x = 3 + k\tau \\ y = 7 - \tau \\ z = 2 + \tau \end{cases} \quad d(\mathcal{C}; d_3) = r \quad \mathcal{C}(3; 7; 2)$$

$$\text{dis}(\mathcal{C}, d_3) = \frac{\|\vec{d_3} \wedge \vec{D_C}\|}{\|\vec{d_3}\|}$$

$$\vec{D_C} = \begin{pmatrix} -3 \\ -7 \\ 3 \end{pmatrix} \quad \vec{d_3} \wedge \vec{D_C} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} -3 \\ -7 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -3-3k \\ -3k-3 \end{pmatrix}$$

$$\|\vec{d_3} \wedge \vec{D_C}\| = \sqrt{(3+3k)^2 + (3k+3)^2} \quad \text{dis}(\mathcal{C}, d_3) = \frac{\sqrt{2(3+3k)^2}}{\sqrt{k^2+2}}$$

$$\|\vec{d_3}\| = \sqrt{k^2+1+1} = \sqrt{2+k^2}$$

$$d(\underline{0}; d_3) = r \Rightarrow \sqrt{\frac{2(3+3k)^2}{k^2+9}} = 3 \Leftrightarrow 2(3+3k)^2 = 9k^2 + 18 \Leftrightarrow$$

$$\Leftrightarrow 2(9 + 18k + 9k^2) = 9k^2 + 18 \Leftrightarrow 18 + 36k + 18k^2 = 9k^2 + 18 \Leftrightarrow$$

$$\Leftrightarrow 9k^2 + 36k = 0 \Leftrightarrow k^2 + 4k = 0 \Leftrightarrow k(k+4) = 0 \Leftrightarrow k = 0 \quad k = -4$$

EXERCISE 5.47

a. $x^2 + y^2 + z^2 - 6x - 8y - 17z + 25 = 0$

$$\Leftrightarrow (x-3)^2 - 9 + (y-4)^2 - 16 + (z - \frac{17}{2})^2 - \frac{17^2}{4} + 25 = 0 \Leftrightarrow$$

$$\Leftrightarrow (x-3)^2 + (y-4)^2 + (z - \frac{17}{2})^2 = (\frac{17}{2})^2 \quad \underline{C}(3; 4; \frac{17}{2}) \quad \rho = \frac{17}{2}$$

b. $\text{dist}(C, \Pi) = \frac{|3+8+17+81|}{\sqrt{1+4+4}} = \frac{36}{3} = 12 > \rho \Rightarrow \text{disjoint}$

Comme $|z_0| = \rho \Rightarrow$ tangents

c. $\Pi_1: x + 2y + 2z + d = 0$

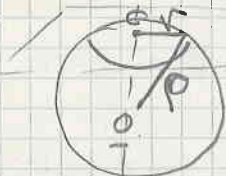
$$\text{dist}(\Pi_1, \underline{0}) = r \Rightarrow \frac{|3+8+17+d|}{3} = \frac{17}{2} \Rightarrow |28+d| = \frac{51}{2} \Rightarrow$$

$$\Rightarrow 28+d = \frac{51}{2} \Rightarrow d_1 = -\frac{5}{2} \quad \text{ou} \quad 28+d_2 = -\frac{51}{2} \Rightarrow d_2 = -\frac{107}{2}$$

$$\Pi_1: x + 2y + 2z - 2.5 = 0$$

$$\Pi_2: x + 2y + 2z - 53.5 = 0$$

d.



$$\Pi': x + 2y + 2z - 10 = 0$$

$$\vec{d} = \vec{n}_{\Pi'} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \Rightarrow d = \begin{cases} x = 3 + \lambda \\ y = 4 + 2\lambda \\ z = \frac{17}{2} + 2\lambda \end{cases}$$

$$d \quad C = \Pi' \cap d: (3+\lambda) + 2(4+2\lambda) + 2(\frac{17}{2} + 2\lambda) - 10 = 0 \Leftrightarrow$$

$$\Leftrightarrow 3 + \lambda + 8 + 4\lambda + 17 + 4\lambda - 10 = 0 \Leftrightarrow 9\lambda = -18 \Rightarrow \lambda = -2$$

$$C(1; 0; 4.5)$$

$$C_0 = \text{dist}(\underline{0}; \Pi') = \frac{|3+8+17-10|}{3} = \frac{18}{3} = 6$$

$$\rho^2 = r^2 + C_0^2 \Rightarrow r^2 = \frac{17^2}{4} - 6^2 \Rightarrow r^2 = \frac{145}{4} \Rightarrow r = \frac{\sqrt{145}}{2}$$

c. $P: \begin{cases} x = 7 + \lambda \\ y = 4 \\ z = \lambda \end{cases} \quad \parallel \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$P \cap G: 4^2 + 0 + (\lambda - \frac{17}{2})^2 = (\frac{17}{2})^2 \Rightarrow (\lambda - \frac{17}{2})^2 = \frac{225}{4} \Rightarrow$$

$$\lambda - \frac{17}{2} = \pm \frac{15}{2} \Rightarrow \lambda_1 = \frac{32}{2} = 16 \quad \lambda_2 = 1$$

$$A(7; 4; 16) \quad B(7; 4; 1)$$

$$\vec{OA} = \begin{pmatrix} 7 \\ 4 \\ 16 \end{pmatrix} = \vec{n}_A \quad \Pi_A: 4x + 7.5z + d = 0$$

$$A: 28 + 120 + d = 0 \Rightarrow d = -148$$

$$\Pi_A: 4x + 7.5z - 148 = 0$$

$$\vec{OB} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix} = \vec{n}_B \quad \Pi_B: 4x + 7.5z + d = 0$$

$$B: 28 - 7.5 + d = 0 \Rightarrow d = -20.5$$

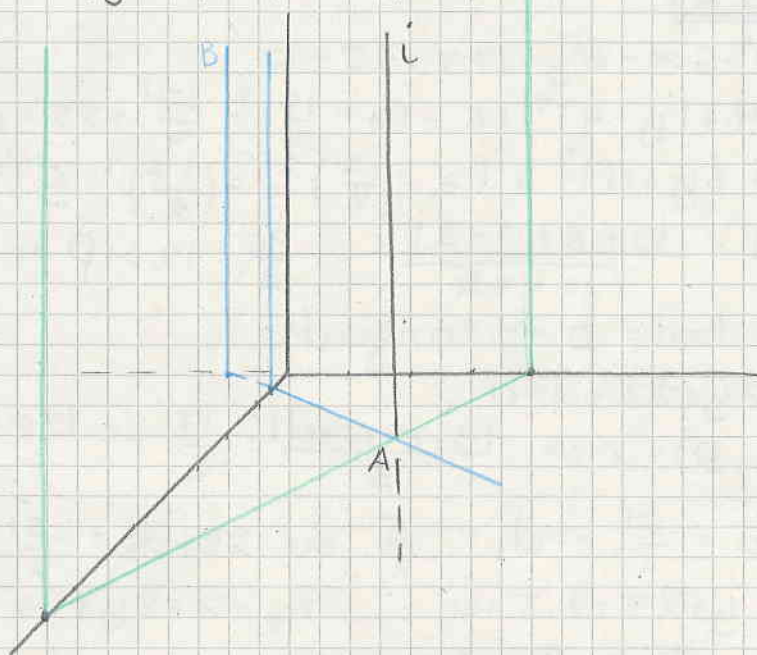
$$\Pi_B: 4x - 7.5z - 20.5 = 0 \Rightarrow 8x - 15z - 41 = 0$$

EXERCICE 5.48

$$\alpha: x+2y-8=0 \quad \beta: 2x-y-1=0$$

$$a) \alpha_x (8;0;0) \quad \alpha_y (0;4;0) \quad \alpha_z \emptyset$$

$$\beta_x (\frac{1}{2};0;0) \quad \beta_y (0;-1;0) \quad \beta_z \emptyset$$



$$b) \vec{n}_\alpha = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \quad \vec{n}_\beta = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \quad \vec{n}_\alpha \cdot \vec{n}_\beta = 0 \Rightarrow \alpha \perp \beta$$

$$c) \begin{cases} x+2y=8 \\ 2x-y=1 \end{cases} \Rightarrow \begin{cases} x+2y=8 \\ 4x-2y=2 \end{cases} \Rightarrow 5x=10 \Rightarrow x=2 \Rightarrow y=3 \quad A(2;3;0)$$

$$i: \begin{cases} x=2 \\ y=3 \\ z=\lambda \end{cases} \quad \text{car } i \parallel O_z$$

$$b. e) O_1(1;1;3) \quad \text{dist}(O_1, \alpha) = r \Rightarrow r = \frac{|1+2-8|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$\text{Donc } \Sigma_1: (x-1)^2 + (y-1)^2 + (z-3)^2 = 5$$

$$f) \text{ On trouve la droite } O_1T: \vec{O_1T} = \vec{n}_\alpha = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \quad O_1T: \begin{cases} x=1+\lambda \\ y=1+2\lambda \\ z=3 \end{cases}$$

$$T = \alpha \cap O_1T: (1+\lambda) + 2(1+2\lambda) - 8 = 0 \Rightarrow 1+\lambda + 2+4\lambda - 8 = 0 \Rightarrow 5\lambda = 5 \Rightarrow \lambda = 1 \Rightarrow T(2;3;3)$$

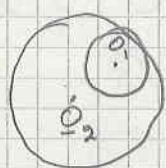
$$h) \vec{O_1R} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \quad O_1R: \begin{cases} x=2+\lambda \\ y=3+2\lambda \\ z=3-3\lambda \end{cases} \quad \vec{OR} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$\text{dis}(O, O_1R) = \frac{\|\vec{OR} \wedge \vec{O_1R}\|}{\|\vec{O_1R}\|} = \frac{\sqrt{118}}{\sqrt{14}} \approx 2.9$$

$$\vec{OR} \wedge \vec{O_1R} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \\ 1 \end{pmatrix}$$

$$i) A_{\text{irc}} = \frac{1}{2} \|\vec{R}\vec{O} \wedge \vec{R}\vec{E}_1\| = \frac{1}{2} \cdot \sqrt{9^2 + 36 + 1} \approx 5.43$$

$$c. \mathcal{C}_2: \mathcal{C}_2(1; 1; z) \quad r=6$$

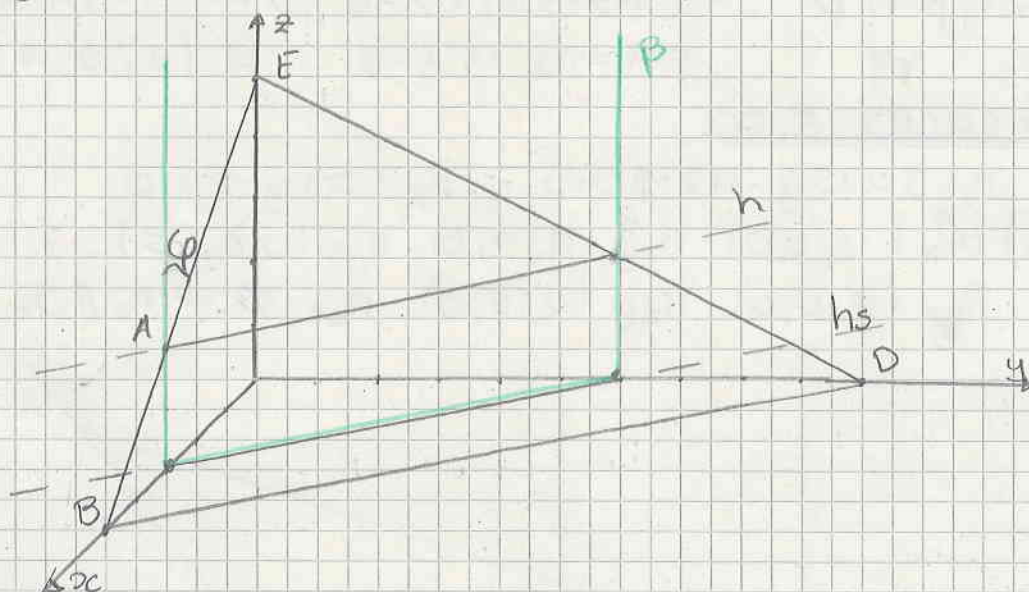


$$\|\vec{O}_1\vec{O}_2\| = r_2 - r_1 \Rightarrow \left\| \begin{pmatrix} 0 \\ 0 \\ z-3 \end{pmatrix} \right\| = 6 - \sqrt{5} \Rightarrow$$

$$\Rightarrow |z-3| = 6 - \sqrt{5} \Rightarrow \begin{cases} z-3 = 6 - \sqrt{5} \Rightarrow z_1 = 9 - \sqrt{5} \\ z-3 = -6 + \sqrt{5} \Rightarrow z_2 = -3 + \sqrt{5} \end{cases}$$

EXERCICE 5.49

$$a. \alpha: 2x + y + 2z - 10 = 0 \quad \alpha_x(5; 0; 0) \quad \alpha_y(0; 10; 0) \quad \alpha_z(0; 0; 5)$$



$$b. h: A(3; 0; 2) \quad h \text{ horizontale} \Rightarrow h \parallel BD$$

$$\vec{BD} = \begin{pmatrix} -5 \\ 1 \\ 0 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \quad h: \begin{cases} x = 3 + \lambda \\ y = -2\lambda \\ z = 2 \end{cases}$$

$$c. S: C(-2; 0; z) \quad r = \|\vec{AC}\| = \sqrt{(-2-3)^2 + (0)^2 + (z-2)^2} = \sqrt{50} = 5\sqrt{2}$$

$$a) (x+2)^2 + y^2 + (z-2)^2 = 50$$

$$b) F = h \cap S. \quad (3+\lambda+2)^2 + (-2\lambda)^2 + (2-2)^2 = 50 \Leftrightarrow (5+\lambda)^2 + 4\lambda^2 + 25 = 50$$

$$\Leftrightarrow 25 + 10\lambda + \lambda^2 + 4\lambda^2 - 25 = 0 \Leftrightarrow 5\lambda^2 + 10\lambda = 0 \Leftrightarrow 5\lambda(\lambda+2) = 0 \Leftrightarrow$$

$$\lambda = 0 \quad \lambda = -2. \quad F(1; 4; 2)$$

$$d. \vec{d} = \vec{h} \wedge \vec{AC} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} -5 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -10 \\ -5 \\ -10 \end{pmatrix} \parallel \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

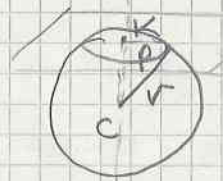
$$d: \begin{cases} x = 3 + 2\mu \\ y = +\mu \\ z = 2 + 2\mu \end{cases}$$

$$e. \beta: 2x + y + k = 0 \Rightarrow \beta: 2x + y - 6 = 0$$

$$a) h \in \beta: 2(3+\lambda) + (-2\lambda) + k = 0 \Leftrightarrow 6 + 2\lambda - 2\lambda + k = 0 \Leftrightarrow k = -6$$

$$b) \beta_x(3; 0; 0) \quad \beta_y(0; 6; 0) \quad \beta_z \emptyset \Rightarrow \beta \parallel Oz$$

$$c) \cos \varphi = \frac{\vec{n}_\alpha \cdot \vec{n}_\beta}{\|\vec{n}_\alpha\| \cdot \|\vec{n}_\beta\|} = \frac{\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{9} \cdot \sqrt{5}} = \frac{5}{3\sqrt{5}} = \frac{\sqrt{5}}{3} \Rightarrow \varphi = 41.8^\circ$$

d.  $r = 5\sqrt{2}$ $\beta: 2x + y + 6 = 0$ $C(-2; 0; 7)$
 $\text{dist}(\beta; C) = \frac{|1 - 4 - 6|}{\sqrt{5}} = \frac{10}{\sqrt{5}} = 2\sqrt{5}$
 $\rho^2 = r^2 - KC^2 \Rightarrow \rho^2 = 50 - 20 \Rightarrow \rho = \sqrt{30}$

$P: \vec{p} = \vec{n}_\beta = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ $P: \begin{cases} x = -2 + 2\lambda \\ y = \lambda \\ z = 7 \end{cases}$

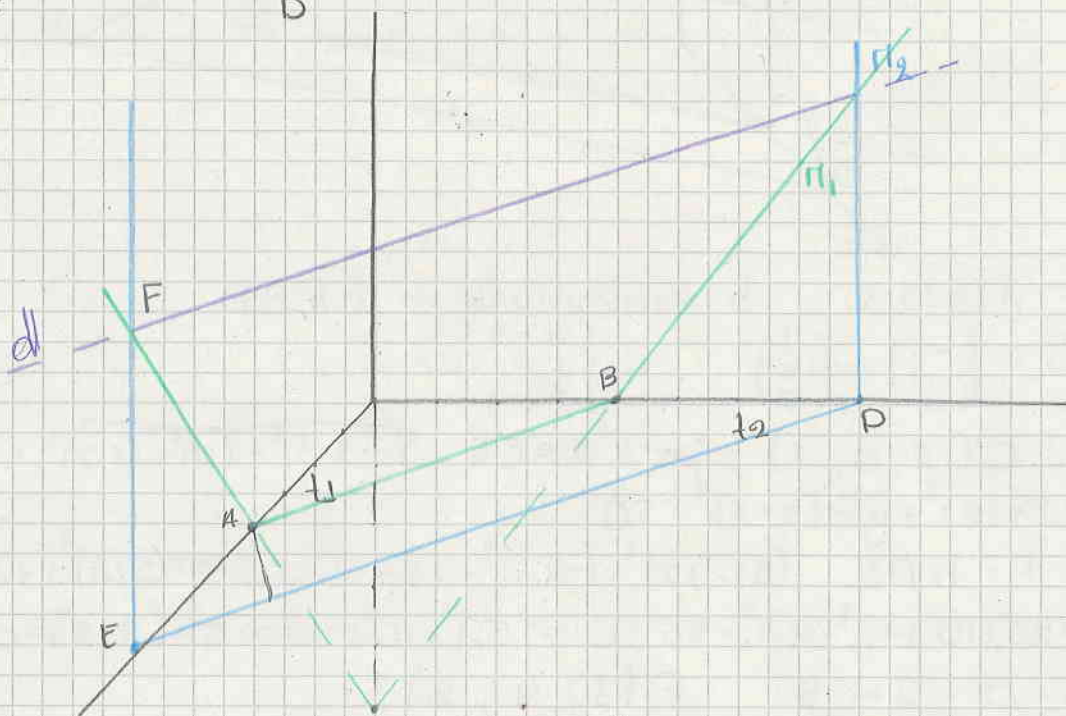
$K = \beta \cap P: 2(-2 + 2\lambda) + \lambda - 6 = 0 \Leftrightarrow -4 + 4\lambda + \lambda - 6 = 0 \Leftrightarrow 5\lambda = 10 \Rightarrow \lambda = 2$ $K(2; 2; 7)$

EXERCISE 5.50

a. $\pi_1: 5x + 5y - 4z - 20 = 0$ $\pi_2: x + y - 8 = 0$

a) $M_{1,x}^A(4; 0; 0)$ $M_{1,y}^B(0; 4; 0)$ $M_{1,z}(0; 0; -5)$

$M_{2,x}(8; 0; 0)$ $M_{2,y}(0; 8; 0)$ $M_{2,z} \emptyset \Rightarrow \pi_2 \parallel Oz$



b) $\vec{t}_2 = \begin{pmatrix} -8 \\ 8 \\ 0 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ $t_2: \begin{cases} x = 8 + \lambda \\ y = -\lambda \\ z = 0 \end{cases}$

$\text{dist}(t_1, t_2) = \text{dist}(A, t_2) = \frac{\|\vec{EA}\| \cdot \|\vec{t}_2\|}{\|\vec{t}_2\|^2} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$\vec{EA} \wedge \vec{t}_2 = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ $\|\vec{t}_2\| = \sqrt{2}$

c) $\cos \varphi = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|} = \frac{\begin{pmatrix} 5 \\ 5 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{66} \cdot \sqrt{2}} = 0.87 \Rightarrow \varphi = 29.5^\circ$

c) $\vec{d} \parallel \vec{t}_2 \Rightarrow \vec{d} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

F: $y=0: \begin{cases} 5x-4z=20 \\ x=8 \end{cases} \Rightarrow 40-20=4z \Leftrightarrow z=5 \Rightarrow F(8;0;5)$

ol: $\begin{cases} x=8+\lambda \\ y=-\lambda \\ z=5 \end{cases}$

b. s: $(x-3)^2 + (y+2)^2 + (z-1)^2 = 100$ K(3;-2;1) r=10

$\pi_3: x+2y-2z+40=0$

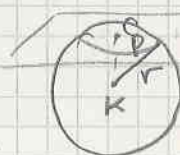
f) $\text{dis}(K, \pi_3) = \frac{|3-4-2+40|}{\sqrt{1+4+4}} = \frac{|37|}{\sqrt{9}} = \frac{37}{3} = 12.\bar{3} > r \Rightarrow \text{disjoints}$



$q = \text{dis}(K, \pi_3) = 12.\bar{3} - 10 \approx 2.3$

g) C(1;-6;5)

$\vec{CK} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \vec{n}_4$



$\pi_4: x+2y-2z+d=0$

$C \in \pi_4: 1-12-10+d=0 \Rightarrow d=-21$

$\pi_4: x+2y-2z-21=0$

$\|\vec{CK}\| = 3$

$\rho^2 = r^2 - CK^2 \Rightarrow \rho^2 = 100 - 9 \Rightarrow \rho = \sqrt{91} \approx 9.54$

h)

$t: \begin{cases} x=13+4\lambda \\ y=3-3\lambda \\ z=3+2\lambda \end{cases} \rightarrow s$

$(13+4\lambda-3)^2 + (3-3\lambda+2)^2 + (3+2\lambda-1)^2 = 100 \Leftrightarrow$

$\Leftrightarrow (10+4\lambda)^2 + (5-3\lambda)^2 + (2+2\lambda)^2 = 100 \Leftrightarrow$

$\Leftrightarrow 100 + 80\lambda + 16\lambda^2 + 25 - 30\lambda + 9\lambda^2 + 4 + 8\lambda + 4\lambda^2 = 100 \Leftrightarrow$

$\Leftrightarrow 29\lambda^2 + 58\lambda + 29 = 0 \Leftrightarrow \lambda^2 + 2\lambda + 1 = 0 \Leftrightarrow (\lambda+1)^2 = 0 \Leftrightarrow \lambda = -1$

$\Rightarrow t$ est tangente à la sphère. T(9;6;1)

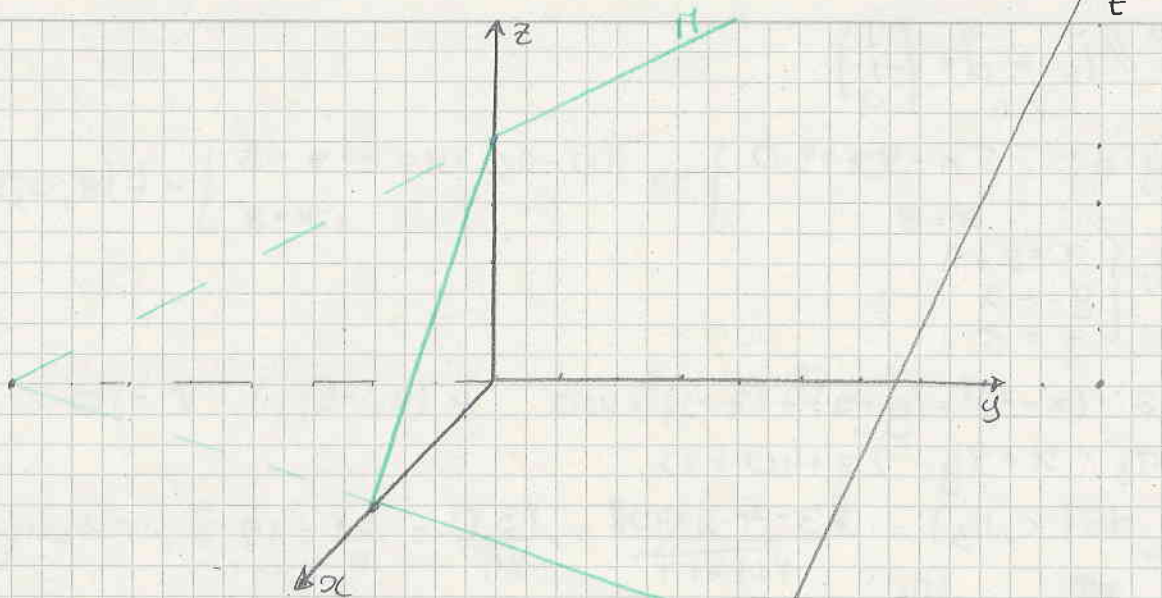
i) T(9;6;1): $(9-3)^2 + (6+2)^2 + (1-1)^2 = 100 \checkmark \Rightarrow T \in s$

j) $\vec{t}' = \vec{CT} \wedge \vec{t} = \begin{pmatrix} 8 \\ 12 \\ -4 \\ 8 \\ 12 \end{pmatrix} \wedge \begin{pmatrix} 4 \\ -3 \\ 2 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 12 \\ -32 \\ -72 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ -8 \\ -18 \end{pmatrix} = \vec{t}'$

EXERCICE S.SI

a. $\pi: 2x-y+2z-8=0$ A(3;2;2) B(7;6;0) C(5;10;4)

$\pi_x(4;0;0)$ $\pi_y(0;-8;0)$ $\pi_z(0;0;4)$



b) $A(3;2;2)$ $B(7;6;0)$ $C(5;10;4)$

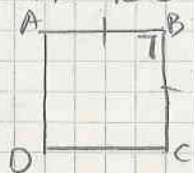
$$\vec{AB} = \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix} \quad \|\vec{AB}\| = 6$$

$$\vec{AC} = \begin{pmatrix} 2 \\ 8 \\ 2 \end{pmatrix} \quad \|\vec{AC}\| = \sqrt{44}$$

$$\vec{BC} = \begin{pmatrix} -2 \\ 4 \\ 4 \end{pmatrix} \quad \|\vec{BC}\| = 6$$

$\|\vec{AB}\| = \|\vec{BC}\|$

$$\vec{AB} \cdot \vec{BC} = -8 + 16 - 8 = 0 \Rightarrow \vec{AB} \perp \vec{BC}$$



Il suffit de trouver D tel que ABCD est un parallélogramme

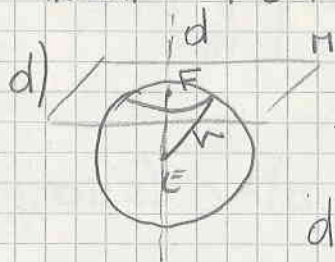
$$\vec{AB} = \vec{DC} \Rightarrow \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 5-x \\ 10-y \\ 4-z \end{pmatrix} \Rightarrow \begin{cases} x=1 \\ y=6 \\ z=2 \end{cases} \quad D(1;6;2)$$

b.c) s: $E(8;4;7)$ $A(3;2;2)$ $\Pi: 2x-y+2z-8=0$

$$r = \|\vec{EA}\| = \sqrt{(-5)^2 + (-2)^2 + (-5)^2} = \sqrt{54}$$

$$\|\vec{EB}\| = \sqrt{1+2^2+(-7)^2} = \sqrt{54} \Rightarrow B \in S$$

$$\|\vec{EC}\| = \sqrt{3^2+6^2+3^2} = \sqrt{54} \Rightarrow C \in S$$



$$EF = \text{dist}(E, \Pi) = \frac{|16-4+14-8|}{\sqrt{4+1+4}} = \frac{18}{3} = 6$$

$$r^2 = R^2 - EF^2 \Rightarrow r = \sqrt{54 - 36} = \sqrt{18}$$

$$d: \vec{d} = \vec{n}_{\Pi} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad d: \begin{cases} x = 8 + 2\lambda \\ y = 4 - \lambda \\ z = 7 + 2\lambda \end{cases}$$

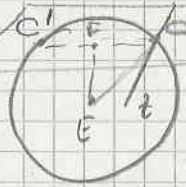
$E \in d$

$$F = d \cap \Pi: 2(8+2\lambda) - (4-\lambda) + 2(7+2\lambda) - 8 = 0 \Rightarrow 16+4\lambda - 4+\lambda + 14+4\lambda - 8 = 0$$

$$\Rightarrow 9\lambda = -18 \Rightarrow \lambda = -2 \quad F(4;6;3)$$

C.

c)



$$\vec{t} = \vec{EF} \wedge \vec{CE} = \vec{n}_n \wedge \vec{CE}$$

$$\vec{n}_n = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \wedge \vec{CE} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \vec{t}$$

$$C \in t: \quad t: \begin{cases} x = 5 + \lambda \\ y = 10 + 2\lambda \\ z = 4 - \lambda \end{cases}$$

$$f) \quad T_s: z=0 \Rightarrow \lambda=4. \quad T_s(9; 10; 0)$$

$$T_m: x=0 \Rightarrow \lambda=-5 \quad T_m(0; 10; 9)$$

$$g) \quad \vec{t}' = \vec{t} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \text{ tangente à } s. \text{ à } C'$$

$$s: (x-8)^2 + (y-4)^2 + (z-7)^2 = 54$$

$$C(5; 10; 4) \quad F(4; 6; 3)$$

$$\vec{CF} = \vec{FC'} \Rightarrow \begin{pmatrix} -1 \\ -4 \\ -1 \end{pmatrix} = \begin{pmatrix} x-4 \\ y-6 \\ z-3 \end{pmatrix} \Rightarrow \begin{cases} x=3 \\ y=2 \\ z=2 \end{cases} \quad C'(3; 2; 2)$$

$$C' \in t': \quad t': \begin{cases} x = 3 + \lambda \\ y = 2 \\ z = 2 - \lambda \end{cases}$$

$$d. \quad d: \begin{cases} x = 6 + \lambda \\ y = 2\lambda \\ z = 0 \end{cases}$$

$$h) \quad (6+\lambda-8)^2 + (2\lambda-4)^2 + (-7)^2 = 54 \Leftrightarrow (\lambda-2)^2 + (2\lambda-4)^2 + 49 = 54$$

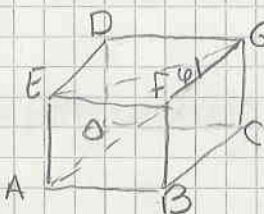
$$\Leftrightarrow \lambda^2 - 4\lambda + 4 + 4\lambda^2 - 16\lambda + 16 + 5 = 0 \Leftrightarrow$$

$$\Leftrightarrow 5\lambda^2 - 20\lambda + 25 = 0 \Leftrightarrow \lambda^2 - 4\lambda + 5 = 0 \Leftrightarrow \begin{matrix} \lambda_1 = 5 \\ \lambda_2 = -1 \end{matrix}$$

$$G(11; 10; 0) \quad H(5; -2; 0)$$

EXERCICE 5.52

$$a=5 \quad O(0,0,0) \quad A(a,0,0) \quad B(0,a,0) \quad D(0,0,a) \quad E(a,0,a) \quad F(a,a,a) \quad G(0,a,a)$$



$$a. \quad \vec{GA} = \begin{pmatrix} a \\ -a \\ -a \end{pmatrix} \quad \vec{GE} = \begin{pmatrix} a \\ -a \\ 0 \end{pmatrix}$$

$$\cos \varphi = \frac{\vec{GA} \cdot \vec{GE}}{\|\vec{GA}\| \|\vec{GE}\|} = \frac{a^2 + a^2}{\sqrt{3a^2} \sqrt{2a^2}} = \frac{a=5}{25\sqrt{6}} = \frac{2}{\sqrt{6}}$$

$$\Rightarrow \varphi = 35.26^\circ$$

$$b. \quad \vec{OD} = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \quad \vec{OF} = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$$

$$\cos \varphi = \frac{\vec{OD} \cdot \vec{OF}}{\|\vec{OD}\| \|\vec{OF}\|} = \frac{25}{5 \cdot 5\sqrt{3}} = \frac{25}{5 \cdot 5\sqrt{3}} \Rightarrow \varphi = 54.73^\circ$$

$$A: \vec{AD} = \begin{pmatrix} -5 \\ 5 \\ 0 \end{pmatrix} \quad \vec{AF} = \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix}$$

$$\cos \hat{A} = \frac{\vec{AD} \cdot \vec{AF}}{\|\vec{AD}\| \cdot \|\vec{AF}\|} = \frac{25}{\sqrt{50} \cdot \sqrt{50}} \Rightarrow \hat{A} = 60^\circ$$

$$C: \vec{AG} = \begin{pmatrix} -5 \\ 5 \\ 5 \end{pmatrix} \quad \vec{DF} = \begin{pmatrix} 5 \\ 5 \\ 0 \end{pmatrix} \quad \vec{AG} \cdot \vec{DF} = -25 + 25 = 0 \Rightarrow \vec{AG} \perp \vec{DF}$$

$$\vec{u} = \vec{AG} \wedge \vec{DF} = \begin{pmatrix} -5 \\ 5 \\ 5 \end{pmatrix} \wedge \begin{pmatrix} 5 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -25 \\ 25 \\ -50 \end{pmatrix} \parallel \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

$$d: H(0, 0, z) \quad \vec{HA} = \begin{pmatrix} 5 \\ 0 \\ -z \end{pmatrix} \quad \vec{HC} = \begin{pmatrix} 0 \\ 5 \\ -z \end{pmatrix}$$

$$\cos \alpha = \cos 45^\circ \Leftrightarrow \frac{\vec{HA} \cdot \vec{HC}}{\|\vec{HA}\| \cdot \|\vec{HC}\|} = \frac{\sqrt{2}}{2} \Leftrightarrow \frac{z^2}{\sqrt{25+z^2} \sqrt{25+z^2}} = \frac{\sqrt{2}}{2} \Leftrightarrow$$

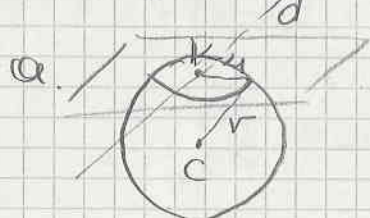
$$\Leftrightarrow \frac{z^2}{25+z^2} = \frac{\sqrt{2}}{2} \Leftrightarrow 2z^2 = \sqrt{2}(25+z^2) \Leftrightarrow 2z^2 = 25\sqrt{2} + z^2\sqrt{2} \Leftrightarrow$$

$$\Leftrightarrow (2-\sqrt{2})z^2 = 25\sqrt{2} \Leftrightarrow z^2 = \frac{25\sqrt{2}}{2-\sqrt{2}} \Leftrightarrow z = \pm \sqrt{\frac{25\sqrt{2}}{2-\sqrt{2}}}$$

EXERCISE 5.53

$$d: \begin{cases} x=3 \\ y=5 \\ z=\lambda \end{cases}$$

$$S: (x-4)^2 + (y-7)^2 + (z-5)^2 = 25 \quad C(4, 7, 5) \quad r=5$$



$$K \in d \Rightarrow K(3; 5; \lambda)$$

$$KC = \sqrt{5^2 + 4^2} = \sqrt{41} = 3 \quad \vec{KC} = \begin{pmatrix} 4-3 \\ 7-5 \\ 5-\lambda \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5-\lambda \end{pmatrix}$$

$$\|\vec{KC}\| = 3 \Rightarrow \sqrt{1+2^2+(5-\lambda)^2} = 3 \Leftrightarrow$$

$$\Leftrightarrow 5 + (5-\lambda)^2 = 9 \Leftrightarrow (5-\lambda)^2 = 4 \Leftrightarrow \begin{cases} 5-\lambda = 2 \Leftrightarrow \lambda = 3 \\ 5-\lambda = -2 \Leftrightarrow \lambda = 7 \end{cases}$$

$$\lambda_1: K_1(3; 5; 3) \quad \lambda_2: K_2(3; 5; 7)$$

$$b. \underline{K_1} \quad \vec{K_1C} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \vec{n}_{\Pi_1} \quad K_1 \in \Pi_1: 3+10+6+d_1=0 \Leftrightarrow d_1=-19$$

$$\Pi_1: x+2y+2z-19=0$$

$$\underline{K_2}: \vec{K_2C} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \vec{n}_{\Pi_2} \quad K_2 \in \Pi_2: 3+10-14+d_2=0 \Leftrightarrow d_2=1$$

$$\Pi_2: x+2y-2z+1=0$$

EXERCISE 5.54

$$d: \begin{cases} x=1+m \\ y=-2m \\ z=5+m \end{cases}$$

$$\Pi_1: 3x+4y-12=0 \quad \Pi_2: 3y-4z+6=0$$

$$c \in d = r C (1+m, -2m, 5+m)$$

$$\text{dist}(\pi_1, c) = \text{dist}(\pi_2, c) \Rightarrow \frac{|3(1+m) + 4(-2m) + 12|}{\sqrt{9+16}} = \frac{|3(-2m) + 4(5+m) + 6|}{\sqrt{9+16}}$$

$$\Leftrightarrow |3+3m+8m-12| = |-6m-20+4m+6| \Leftrightarrow$$

$$\Leftrightarrow |-5m-9| = |-10m-14| = 0 \Leftrightarrow \begin{cases} -5m-9 = -10m-14 \Leftrightarrow m_1 = -1 \\ -5m-9 = 10m+14 \Leftrightarrow m = -\frac{23}{15} \end{cases}$$

$$C_1(0; 2; 4) \quad C_2\left(-\frac{8}{15}; \frac{46}{15}; \frac{52}{15}\right)$$

$$r_1 = \text{dist}(\pi_1, C_1) = \frac{|0+8-12|}{\sqrt{9+16}} = \frac{4}{5}$$

$$r_2 = \text{dist}(\pi_2, C_2) = \frac{4}{5}$$

$$\left. \begin{aligned} S_1: & (x-0)^2 + (y-2)^2 + (z-4)^2 = \frac{16}{25} \\ S_2: & \left(x+\frac{8}{15}\right)^2 + \left(y-\frac{46}{15}\right)^2 + \left(z-\frac{52}{15}\right)^2 = \frac{16}{25} \end{aligned} \right\}$$

EXERCICE 5.55

$$a. \begin{vmatrix} -2 & 5 \\ 3 & 7 \end{vmatrix} = -14 - 15 = -29$$

$$b. \begin{vmatrix} k & 3 \\ k-2 & -k \end{vmatrix} = -k^2 - 3(k-2) = 0 \Leftrightarrow +k^2 + 3k - 6 = 0 \Leftrightarrow k_{1,2} = \frac{3 \pm \sqrt{33}}{-2}$$

$$c. \begin{vmatrix} 2 & -1 & -2 \\ 6 & -1 & 1 \\ 4 & 5 & 3 \end{vmatrix} = 2 \begin{vmatrix} -1 & 1 \\ 5 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 1 \\ 4 & 3 \end{vmatrix} + 2 \begin{vmatrix} 6 & -1 \\ 4 & 5 \end{vmatrix} =$$

$$= 2(-3-5) + (18-4) - 2(30+4) = 2(-8) + 14 - 2 \cdot 34 = -70$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 3 & 2 & 6 \\ 1 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 6 \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 12$$

$$\begin{vmatrix} 3 & 2 & -1 \\ 1 & -2 & 5 \\ 4 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 2 & -1 \\ 4 & 0 & 4 \\ 4 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 2 & -1 \\ 0 & 0 & 4 \\ 2 & 1 & 2 \end{vmatrix} = -4 \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} =$$

$$= -4(4-4) = 0$$

EXERCICE 5.56

$$[a, b, c] = \det(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} 4 & -2 & k \\ 2 & -1 & 1 \\ -3 & 6 & -2 \end{vmatrix} = 0 \Leftrightarrow$$

$$\Leftrightarrow 4 \begin{vmatrix} -1 & 1 \\ 6 & -2 \end{vmatrix} + 2 \begin{vmatrix} 2 & 1 \\ -3 & -2 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ -3 & 6 \end{vmatrix} = 0 \Leftrightarrow$$

$$\Leftrightarrow 4(2-6) + 2(-4+3) + k(12-3) = 0 \Leftrightarrow$$

$$\Leftrightarrow -16-2+k \cdot 9 = 0 \Leftrightarrow 9k-18=0 \Leftrightarrow k=2$$

EXERCICE 5.57

$$A(2; -1; 1) \quad B(5; 5; 4) \quad C(3; 2; -1) \quad D(4; 1; 3)$$

$$\vec{AB} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \quad \vec{AD} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$\det(\vec{AB}, \vec{AC}, \vec{AD}) = \begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 2 \\ 3 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 2 \\ 3 & 2 & 0 \\ 0 & -3 & 0 \end{vmatrix} = 2 \begin{vmatrix} 3 & 2 \\ 0 & -3 \end{vmatrix} = -18$$

$$Vol = \frac{1}{6} |-18| = 3$$

EXERCICE 5.58

$$A(2; 1; -1) \quad B(3; 0; 1) \quad C(2; -1; 3) \quad D(0; y; 0)$$

$$\vec{AB} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix} \quad \vec{AD} = \begin{pmatrix} -2 \\ y-1 \\ 1 \end{pmatrix}$$

$$\det(\vec{AB}, \vec{AC}, \vec{AD}) = \begin{vmatrix} 1 & 0 & -2 \\ -1 & -2 & y-1 \\ 2 & 4 & 1 \end{vmatrix} = \begin{vmatrix} -2 & y-1 \\ 4 & 1 \end{vmatrix} - 2 \begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix} =$$

$$= -2 - 4(y-1) - 2(-4+4) = -2 - 4y + 4 + 8 - 8 = -4y + 2$$

$$V = \frac{1}{6} |-4y + 2| = 5 \Leftrightarrow |-4y + 2| = 30 \Leftrightarrow \begin{cases} -4y + 2 = 30 \Rightarrow y_1 = -7 \\ -4y + 2 = -30 \Rightarrow y_2 = 8 \end{cases}$$

$$D_1(0; -7; 0) \quad D_2(0; 8; 0)$$

EXERCICE 5.59

$$A: \begin{cases} \frac{x}{5} + \frac{y}{6} = 18 \\ \frac{x}{2} - \frac{y}{4} = 21 \end{cases} \Rightarrow \begin{cases} 6x + 5y = 540 \\ 2x - y = 84 \end{cases}$$

$$D = \begin{vmatrix} 6 & 5 \\ 2 & -1 \end{vmatrix} = -6 - 10 = -16 \neq 0 \Rightarrow 1 \text{ seule solution}$$

$$D_x = \begin{vmatrix} 540 & 5 \\ 84 & -1 \end{vmatrix} = -540 - 420 = -960 \Rightarrow x = \frac{-960}{-16} = 60$$

$$D_y = \begin{vmatrix} 6 & 540 \\ 2 & 84 \end{vmatrix} = 504 - 1080 = -576 \Rightarrow y = \frac{-576}{-16} = 36$$

$$D \text{ Done } (x, y) = (60; 36)$$

$$B: \begin{cases} x + 3y + 2z = -13 \\ 2x - 6y + 3z = 32 \\ 3x - 4y - z = 12 \end{cases}$$

$$D = \begin{vmatrix} 1 & 3 & 2 \\ 2 & -6 & 3 \\ 3 & -4 & -1 \end{vmatrix} = 71 \neq 0 \Rightarrow 1 \text{ seule solution}$$

$$D_x = \begin{vmatrix} -13 & 3 & 2 \\ 32 & -6 & 3 \\ 12 & -4 & -1 \end{vmatrix} = -142 \Rightarrow x = \frac{-142}{71} \Rightarrow x = -2$$

$$D_y = \begin{vmatrix} 1 & -13 & 2 \\ 2 & 32 & 3 \\ 3 & 12 & -1 \end{vmatrix} = -355 \Rightarrow y = \frac{-355}{-142} \Rightarrow y = 2.5$$

$$D_z = \begin{vmatrix} 1 & 3 & -13 \\ 2 & -6 & 32 \\ 3 & -4 & 12 \end{vmatrix} = 142 \Rightarrow z = \frac{142}{-142} \Rightarrow z = -1$$

$$(x, y, z) = (-2, 2.5, -1)$$

$$C: \begin{cases} 2x - 3y + z = 0 \\ x + 5y - 3z = 3 \\ 5x + 12y - 8z = 9 \end{cases}$$

$$D = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 5 & -3 \\ 5 & 12 & -8 \end{vmatrix} = 0$$

$$D_x = \begin{vmatrix} 0 & -3 & 1 \\ 3 & 5 & -3 \\ 9 & 12 & -8 \end{vmatrix} = 0 \quad D_y = \begin{vmatrix} 2 & 0 & 1 \\ 1 & 3 & -3 \\ 5 & 9 & -8 \end{vmatrix} = 0 \quad D_z = 0$$

$\Rightarrow$ une infinité de solutions

$$D = \begin{cases} 2x + 3y - 4z = 1 \\ 3x - y + 2z = -2 \\ 5x - 9y + 14z = 3 \end{cases}$$

Impossible

EXERCICE 5.60

$$\begin{cases} \lambda x + y + z = 1 \\ x + \lambda y + z = 1 \\ x + y + \lambda z = 1 \end{cases}$$

$$D = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} \lambda+2 & \lambda+2 & \lambda+2 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} =$$

$$= \begin{vmatrix} \lambda+2 & 0 & 0 \\ 1 & \lambda-1 & 0 \\ 1 & 0 & \lambda-1 \end{vmatrix} = (\lambda+2)(\lambda-1)^2$$

• $\lambda \neq -2$ $\lambda \neq 1$ 1 seule solution

• $\lambda = -2$ $D = 0$ $D_x = 9 \Rightarrow$ impossible (aucune solution)

• $\lambda = 1$ $D = 0$ $D_x = 0$ $D_y = 0$ $D_z = 0 \Rightarrow$ 1 infinité de solutions