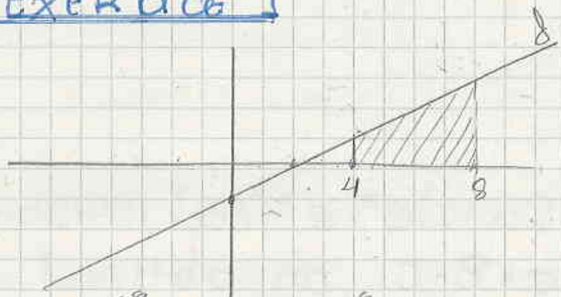
LDDR - Niveau II : Calcul integral

EXERCICE 1



$f(x) = 0 \Rightarrow \frac{1}{2}x - 1 = 0 \Leftrightarrow x = 2$

a. $\int_4^8 f(x)\,dx = \int_4^8 (\frac{1}{2}x - 1)\,dx = \frac{1}{2}\cdot\frac{1}{2}x^2 - x\Big]_4^8 = \frac{1}{4}x^2 - x\Big]_4^8 = 8 - 0 = 8$

b. $\int_0^8 f(x)\,dx = -\int_0^2 f(x)\,dx + \int_2^8 f(x)\,dx = -\left[\frac{1}{4}x^2 - x\right]_0^2 + \left[\frac{1}{4}x^2 - x\right]_2^8 =$

$= 1 + (8 - 1) = 8$

EXERCICE 2

a. $f(x) = 6x^2 - 4x + 5 \Rightarrow F(x) = \frac{6}{3}x^3 - \frac{4}{2}x^2 + 5x = 2x^3 - 2x^2 + 5x$

b. $f(x) = ax^2 + bx + c \Rightarrow F(x) = \frac{1}{3}ax^3 + \frac{b}{2}x^2 + cx$

c. $f(x) = x^n,\ n \neq -1 \Rightarrow F(x) = \frac{1}{n+1}x^{n+1}$

d. $f(x) = (x+3)^2 = x^2 + 6x + 9 \Rightarrow F(x) = \frac{1}{3}x^3 + 3x^2 + 9x$

e. $f(x) = \frac{k}{x} \Rightarrow F(x) = k\ln|x|$

f. $f(x) = e^{2x} \Rightarrow F(x) = \frac{1}{2}e^{2x}$

g. $f(x) = e^{kx} \Rightarrow F(x) = \frac{1}{k}e^{kx}$

h. $f(x) = \sin(x) \Rightarrow F(x) = -\cos(x)$

i. $f(x) = \sin(ax+b) \Rightarrow F(x) = \frac{-1}{a}\cos(ax+b)$

j. $f(x) = 3x^2 \cdot \sin(x^3) \Rightarrow F(x) = -\cos(x^3)$

k. $f(x) = \frac{1}{ax+b} \Rightarrow F(x) = \frac{1}{a}\ln|ax+b|$

l. $f(x) = (ax+b)^n,\ n \neq -1 \quad F(x) = \frac{1}{a(n+1)}(ax+b)^{n+1}$

m. $f(x) = \sqrt{ax+b} \Rightarrow F(x) = \frac{2}{3a}(ax+b)^{3/2}$

n. $f(x) = \frac{ax+b}{x} = a + \frac{b}{x} \Rightarrow F(x) = ax + b\ln|x|$

o. $f(x) = x^2 \cdot \sqrt{x} + \frac{1}{x} = x^{5/2} + \frac{1}{x} \Rightarrow F(x) = \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1} + \ln|x| =$

$= \frac{2}{7}x^{7/2} + \ln|x|$

EXERCICE 3

a) $F'(x) = \frac{1}{ax}\cdot a = \frac{1}{x} \qquad G'(x) = \frac{1}{x}$

b) On a $F'(x) = G'(x) \Rightarrow f(x) = g(x) + c$

ou $\ln(ax) = \ln x + c$ pour $x = 1$

$\ln a = 0 + c \Rightarrow c = \ln a$ et

$\ln(ax) = \ln x + \ln a$

EXERCICE 4

a) $F(x) = (Ax^2 + Bx + C) \cdot e^x$

$F'(x) = (2Ax + B)e^x + (Ax^2 + Bx + C)e^x = e^x(Ax^2 + (2A+B)x + B + C)$

En posant $a = A$ $b = 2A + B$ $c = B + C$ on obtient

$F'(x) = f(x) = (ax^2 + bx + c)e^x$

b) $\left.\begin{array}{l} a = A \\ b = 2A + B \\ c = B + C \end{array}\right\} \Rightarrow \begin{array}{l} a = A \\ b = 2a + B \Rightarrow B = -2a + b \\ C = c - B \Rightarrow C = c + 2a - b \end{array}$

$f(x) = (x^2 + 1)e^x$ $\quad F(x) = (x^2 + 2x + 1)e^x = (x+1)^2 e^x$

$A = 1$ $B = 0$ $C = 1$ $\Rightarrow a = 1$ $b = 2$ $c = 1$

EXERCICE 5

▸ $f(x) = (2x - 5)e^{-x}$ $\quad \int u'v = u \cdot v - \int u \cdot v'$

$u' = e^{-x} \Rightarrow u = -e^{-x}$ $\quad v = 2x - 5 \Rightarrow v' = 2$

$\int (2x-5)e^{-x}dx = -(2x-5)e^{-x} + 2\int e^{-x} = -(2x-5)e^{-x} - 2e^{-x} =$

$= -(2x - 5 + 2)e^{-x} = -(2x-3)e^{-x} + c$

▸ $g(x) = (-x^2 + 3x - 5)e^{x/2}$ $\quad \int u'v = u \cdot v - \int uv'$

$u' = e^{x/2} \Rightarrow u = 2e^{x/2}$ $\quad v = -x^2 + 3x - 5 \Rightarrow v' = -2x + 3$

$\int g(x)dx = 2e^{x/2}(-x^2 + 3x - 5) - 2\int (-2x+3)e^{x/2}dx$

$= 2e^{x/2}(-x^2 + 3x - 5) - 2\left[2e^{x/2}(-2x+3) - 2\int e^{x/2}(-2)dx\right] =$

$= 2(-x^2 + 3x - 5)e^{x/2} - 4(-2x+3)e^{x/2} - 16e^{x/2} =$

$= (-2x^2 + 6x - 10 + 8x - 12 - 16)e^{x/2} =$

$= (-2x^2 + 14x - 38)e^{x/2} = -2e^{x/2}(x^2 - 7x + 19)$

EXERCICE 6

a) $\int_0^4 x(4-x)dx = \int_0^4 (4x - x^2)dx = \left[-\frac{1}{3}x^3 + 2x^2\right]_0^4 = \frac{32}{3} \approx 10.667$

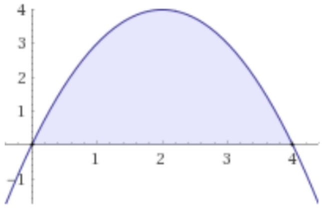
b) $\int_1^4 \frac{dx}{x} = \ln x\Big]_1^4 = \ln 4 \approx 1.3863$

c) $\int_{-1}^{8} \sqrt{x+1}\, dx =$ $\quad \sqrt{x+1} = t \Rightarrow x + 1 = t^2 \Rightarrow x = t^2 - 1 \Rightarrow dx = 2t\,dt.$

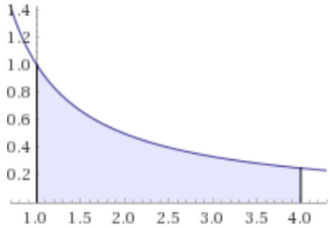
$x = -1$ $t = 0$ $\quad x = 8$ $t = 3$

$= 2\int_0^3 t\,t\,dt = 2\int_0^3 t^2 dt = \frac{2}{3}t^3\Big]_0^3 = \frac{54}{3} = 18$

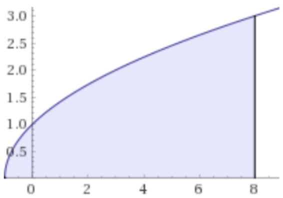
Exercice 6



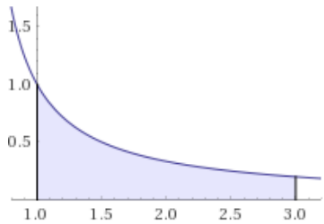
a)



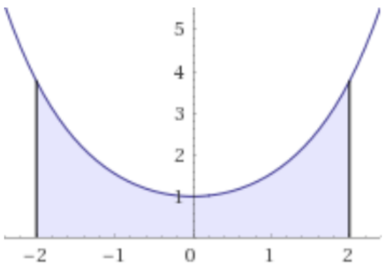
b)



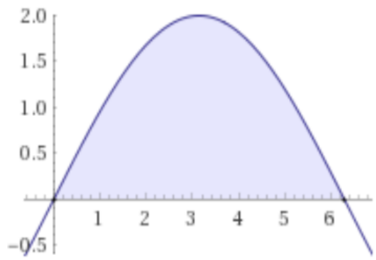
c)



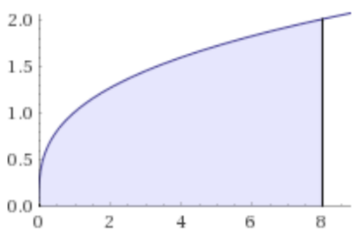
d)



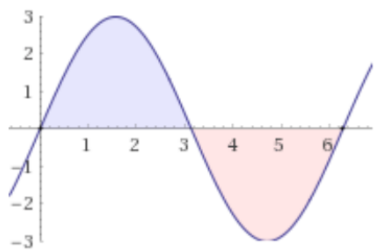
e)



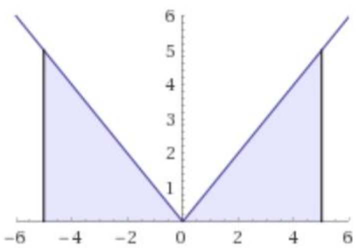
f)



g)



h)



i)

ou $\ln(ax) = \ln x + c$ pour $x = 1$

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$f(x) = (x^2 + 1)e^x$ $F(x) = (x^2 + 2x + 1)e^x = (x+1)^2 e^x$

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▸ $g(x) = (-x^2 + 3x - 5)e^{x/2}$ $\int u'v = u\cdot v - \int uv'$

$u' = e^{x/2} \Rightarrow u = 2e^{x/2}$ $v = -x^2 + 3x - 5 \Rightarrow v' = -2x + 3$

$\int g(x)dx = 2e^{x/2}(-x^2 + 3x - 5) - 2\int(-2x+3)e^{x/2}dx$

$= 2e^{x/2}(-x^2 + 3x - 5) - 2\left[2e^{x/2}(-2x+3) - 2\int e^{x/2}(-2)dx\right] =$

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$= 2\int_0^3 t\,t\,dt = 2\int_0^3 t^2 dt = \frac{2}{3}t^3\Big]_0^3 = \frac{54}{3} = 18$

d) $\int_1^3 \frac{dx}{2x-1} = \frac{1}{2}\ln|2x-1|\Big]_1^3 = \frac{1}{2}\ln 5 \approx 0.80472.$

e) $\int_{-2}^{2} \cosh(x)\,dx = 2\sinh(2) \approx 7.2537$

d) $\int_0^{2\pi} 2\sin\left(\frac{x}{2}\right)dx = -4\cos\left(\frac{x}{2}\right)\Big]_0^{2\pi} = 8$

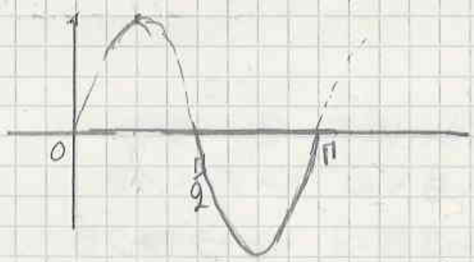
g) $\int_0^{2\pi} 3\sin(x)\,dx = -3\cos x\Big]_0^{2\pi} = 0$

h) $\int_0^8 \sqrt[3]{x}\,dx = \int_0^8 x^{1/3}dx = \frac{3}{4}x^{4/3}\Big]_0^8 = 12$

i) $\int_{-5}^{5} |x|\,dx = 25$

EXERCICE 7

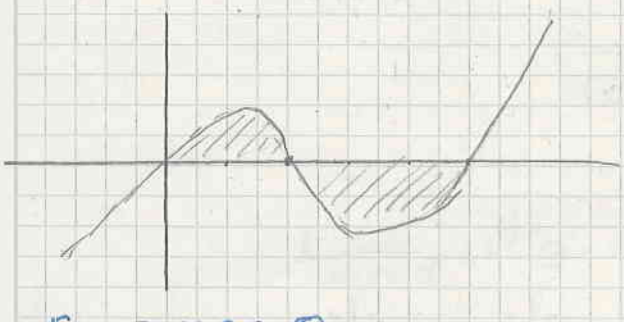
a)



$\int_0^{\pi/2} \sin(2x)\,dx = -\frac{1}{2}\cos(2x)\Big]_0^{\pi/2} = 1$

Donc Aire $= 4 \cdot 1 = 4$

b) $f(x) = x(x-2)(x-5) = (x^2-2x)(x-5) = x^3-2x^2-5x^2+10x =$
$= x^3-7x^2-10x$



Aire $= \int_0^2 f(x)\,dx - \int_2^5 f(x)\,dx =$

$= \left[\frac{1}{4}x^4 - \frac{7}{3}x^3 - 5x^2\right]_0^2 - \left[\frac{1}{4}x^4 - \frac{7}{3}x^3 - 5x^2\right]_2^5$

$= \frac{16}{3} + \frac{63}{4} \approx 21.08$

EXERCICE 8

a) $f(x) = \frac{2x-5}{x+3} = 2 - \frac{11}{x+3}$

$F(x) = 2x - 11\ln|x+3|$

$$\begin{array}{r|l} 2x-5 & x+3 \\ \cline{2-2} -2x-6 & 2 \\ \hline -11 & \end{array}$$

b) $f(x) = \frac{x^2-4x+3}{2x-5} = \frac{1}{2}x - \frac{3}{4} - \frac{\frac{3}{4}}{2x-5} =$

$= \frac{1}{2}x - \frac{3}{4} - \frac{3}{4(2x-5)}$

$F(x) = \frac{1}{4}x^2 - \frac{3}{4}x - \frac{3}{8}\ln|2x-5|$

$$\begin{array}{r|l} x^2-4x+3 & 2x-5 \\ \cline{2-2} -x^2+\frac{5}{2}x & \frac{1}{2}x-\frac{3}{4} \\ \hline -\frac{3}{2}x+3 & \\ +\frac{3}{2}x-\frac{15}{4} & \\ \hline -\frac{3}{4} & \end{array}$$

c) $f(x) = \dfrac{2x^3-5x^2+7x-5}{2x-1} =$

$$\begin{array}{r|l} 2x^3-5x^2+7x-5 & 2x-1 \\ \underline{-2x^3+x^2} & x^2-2x+\frac{5}{2} \\ -4x^2+7x-5 & \\ \underline{+4x^2-2x} & \\ 5x-5 & \\ \underline{-5x+\frac{5}{2}} & \\ -\frac{5}{2} & \end{array}$$

$= x^2-2x+\dfrac{5}{2} - \dfrac{5}{2(2x-1)}$

$F(x) = \frac{1}{3}x^3 - x^2 + \frac{5}{2}x - \frac{5}{4}\ln|2x-1|$

d) $f(x) = \dfrac{1}{x^2} = x^{-2}$ $\quad F(x) = \dfrac{1}{-2+1}x^{-1} = -\dfrac{1}{x}$

e) $f(x) = \dfrac{1}{(x+2)^2}$ $\quad F(x) = -\dfrac{1}{x+2}$

f) $f(x) = \dfrac{1}{(x-3)^2}$ $\quad F(x) = -\dfrac{1}{x-3}$

g) $f(x) = \dfrac{1}{(3x+2)^2}$ $\quad F(x) = -\dfrac{1}{3(3x+2)}$

h) $f(x) = \dfrac{1}{(x-2)(x+2)}$

$\dfrac{1}{(x-2)(x+2)} = \dfrac{A}{x-2} + \dfrac{B}{x+2} \Leftrightarrow 1 = Ax + 2A + Bx - 2B \Rightarrow$

$1 = (A+B)x + 2A - 2B \Rightarrow \begin{cases} A+B = 0 \Rightarrow A = -B \\ 2A-2B = 1 \Rightarrow -2B-2B = 1 \Leftrightarrow \end{cases}$

Alors $f(x) = \dfrac{1}{4(x-2)} - \dfrac{1}{4(x+2)}$ $\quad -4B = 1 \Leftrightarrow B = -1/4 \Rightarrow A = \frac{1}{4}$

$F(x) = \frac{1}{4}\ln|x-2| - \frac{1}{4}|x+2|$

i) $f(x) = \dfrac{1}{x^2+1}$ $\quad F(x) = \mathrm{Arctan}(x)$

EXERCICE 9

a) $\int_0^b e^{-x}dx = -e^{-x}\Big]_0^b = -e^{-b} - (-1) = -e^{-b} + 1 \xrightarrow[b \to +\infty]{} 1$

b) $\int_0^b xe^{-x}dx = -xe^{-x}\Big]_0^b + \int_0^b e^{-x}dx = -xe^{-x} - e^{-x}\Big]_0^b = -e^{-x}(x+1)\Big]_0^b$

$= -e^{-b}(b+1) - (-1) = -e^{-b}(b+1) + 1 \xrightarrow[b \to +\infty]{} 1$

c) $\int_0^b x^2 \cdot e^{-x}dx = -x^2e^{-x} + \int e^{-x}(2x)dx = -x^2e^{-x} + 2\int xe^{-x}dx =$

$= -x^2e^{-x} + 2\left[-xe^{-x} + \int e^{-x}dx\right] = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} =$

$= e^{-x}(-x^2-2x-2)\Big]_0^b = e^{-b}(-b^2-2b-2) + 2 \xrightarrow[b \to +\infty]{} 2$

d) $\int_1^b \frac{dx}{x^2} = -\frac{1}{x}\Big]_1^b = -\frac{1}{b}+1 \xrightarrow[b\to+\infty]{} 1$

EXERCICE 10

a) $\int \ln x\,dx = \int (x)'\ln x\,dx = x\ln x - \int x\frac{1}{x}\,dx = x\ln x - \int dx =$
$= x\ln x - x$

$\int_0^1 \ln x\,dx = [x\ln x - x]_0^1 = -1 - \lim_{x\to 0}(x\ln x - x) = -1 - 0 = -1$

b) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{1}{-\frac{1}{2}+1}x^{-\frac{1}{2}+1} = 2x^{1/2}$

Donc $\int_0^1 \frac{dx}{\sqrt{x}} = [2x^{1/2}]_0^1 = 2 - 0 = 2$

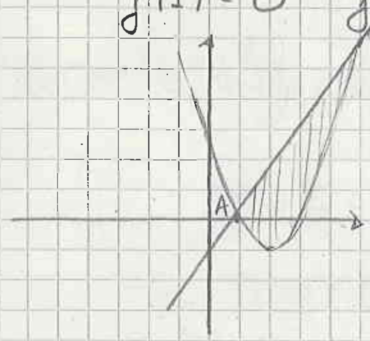
c) $\int_1^\infty \frac{dx}{\sqrt{x}} = [2\sqrt{x}]_1^\infty = \lim_{x\to+\infty}(2\sqrt{x}) - 2 = +\infty$

d) $\int_0^\infty \frac{dx}{x^2+1} = \text{Arctan}(x)\Big]_0^\infty = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

EXERCICE 12.

P: $y = x^2 - 4x + 3$ $\quad A(1;a) \quad B(5;b)$

$f(1) = 0 \quad f(5) = 8$



la droite AB: $m = \frac{8-0}{5-1} = 2$

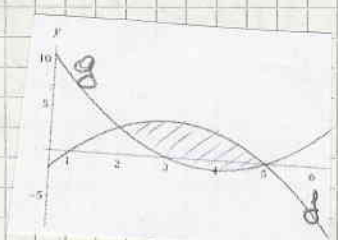
A: $0 = 2\cdot 1 + h \Rightarrow h = -2$

AB: $y = 2x - 2$

Aire $= \int_1^5 (2x-2-(x^2-4x+3))\,dx = \int_1^5 (-x^2+6x-5)\,dx$

$= -\frac{1}{3}x^3 + 3x^2 - 5x\Big]_1^5 = \frac{32}{3} \approx 10.667$

EXERCICE 13



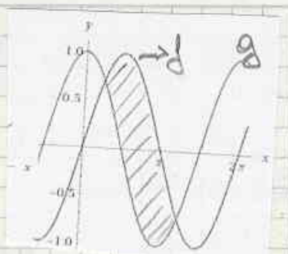
a) $f(x) = -x^2+6x-5 \quad g(x) = x^2-8x+15$

$f(x) = g(x) \Leftrightarrow -x^2+6x-5 = x^2-8x+15 \Leftrightarrow$

$\Leftrightarrow 2x^2-14x+20 = 0 \Leftrightarrow x^2-7x+10 = 0$

$\Leftrightarrow x_1 = 2 \quad x_2 = 5$

Aire $= \int_2^5 (f(x)-g(x))\,dx = \int_2^5 (-2x^2+14x-20)\,dx = \left[-\frac{2}{3}x^3+7x^2-20x\right]_2^5 = 9$



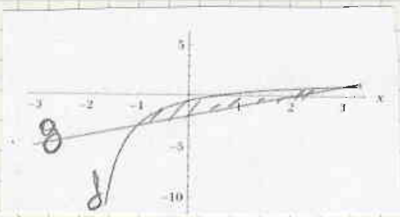
b) $f(x)=\sin x \quad g(x)=\cos x \quad x\in[0,2\pi]$

$f(x)=g(x) \Leftrightarrow \sin x=\cos x \Leftrightarrow \tan x=1 \Rightarrow$

$x=\pi/4$ ou $x=\frac{5\pi}{4}$

Aire $= \displaystyle\int_{\pi/4}^{5\pi/4}(f(x)-g(x))dx = \int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx =$

$= -\cos x-\sin x\Big]_{\pi/4}^{5\pi/4} = 2\sqrt{2}$



c) $f(x)=\frac{2x-1}{x+2} \quad g(x)=x-2$

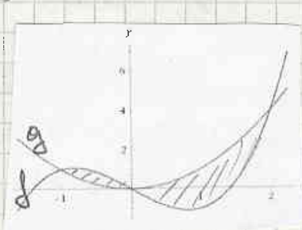
$f(x)=g(x) \Leftrightarrow \frac{2x-1}{x+2}=x-2 \Leftrightarrow$

$\Leftrightarrow 2x-1=x^2-4 \Leftrightarrow x^2-2x-3=0$

$x_1=-1 \quad x_2=3$

Aire $= \displaystyle\int_{-1}^{3}(f(x)-g(x))dx = \int_{-1}^{3}\left(\frac{2x-1}{x+2}-x+2\right)dx = \int_{-1}^{3}\left(2-\frac{5}{x+2}-x+2\right)dx =$

$= \displaystyle\int_{-1}^{3}\left(-\frac{5}{x+2}-x+4\right)dx = -5\ln|x+2|-\frac{1}{2}x^2+4x\Big]_{-1}^{3} = 10-\ln(3125)\simeq 3.9528$



d) $f(x)=x^3-2x \quad g(x)=x^2$

$f(x)=g(x) \Leftrightarrow x^3-2x=x^2 \Leftrightarrow x^3-x^2-2x=0$

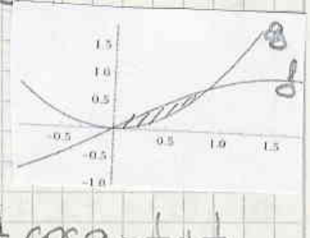
$\Leftrightarrow x(x^2-x-2)=0 \Leftrightarrow x_1=-1 \quad x_2=0 \quad x_3=2$

Aire $= \displaystyle\int_{-1}^{0}(f(x)-g(x))dx + \int_{0}^{2}(g(x)-f(x))dx = \int_{-1}^{0}(x^3-x^2-2x)dx + \int_{0}^{2}(-x^3+x^2+2x)dx$

$= \left[\frac{1}{4}x^4-\frac{1}{3}x^3-x^2\right]_{-1}^{0} + \left[-\frac{1}{4}x^4+\frac{1}{3}x^3+x^2\right]_{0}^{2} = \frac{5}{12}+\frac{8}{3}=\frac{37}{12}$

e) $f(x)=\sin(ax) \quad g(x)=x^2 \quad a\in[0,2\pi] \quad f(1)=g(1)$

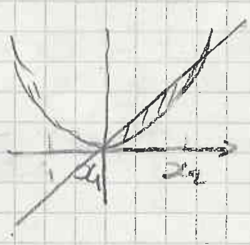
$f(x)=g(x) \Rightarrow \sin(ax)=x^2 \Rightarrow x_1=0 \quad x_2=1$



Aire $= \displaystyle\int_{0}^{1}(f(x)-g(x))dx = \int_{0}^{1}(\sin(ax)-x^2)dx =$

$= -\frac{1}{a}\cos(ax)-\frac{1}{3}x^3\Big]_{0}^{1} = -\frac{1}{a}\cos a-\frac{1}{3}-\left(-\frac{1}{a}\right) = -\frac{1}{a}\cos a-\frac{1}{3}+\frac{1}{a}$

EXERCICE 14



$f(x)=\frac{1}{4}x^2 \quad g(x)=mx$

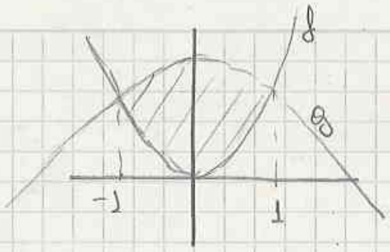
$f(x)=g(x) \Leftrightarrow \frac{1}{4}x^2=mx \Leftrightarrow x^2-4mx=0$

$\Leftrightarrow x(x-4m)=0 \Leftrightarrow x_1=0 \quad x_2=4m$

Aire $= \displaystyle\int_{0}^{4m}\left(mx-\frac{1}{4}x^2\right)dx = \left[\frac{1}{2}mx^2-\frac{1}{12}x^3\right]_{0}^{4m} = 9 \Leftrightarrow 8m^3-\frac{16}{3}m^3=9 \Leftrightarrow$

$\frac{8m^3}{3}=9 \Leftrightarrow m^3=\frac{27}{8} \Rightarrow \boxed{m=\frac{3}{2}}$

EXERCICE 15



$f(x) = x^2 \qquad g(x) = ax^2 + k \quad , k > 0.$

$f'(x) = 2x \quad f'(1) = 2 \quad f'(-1) = -2$

$g'(x) = 2ax \quad g'(1) = -\frac{1}{2} \Rightarrow 2a = -\frac{1}{2} \Leftrightarrow a = -1/4$

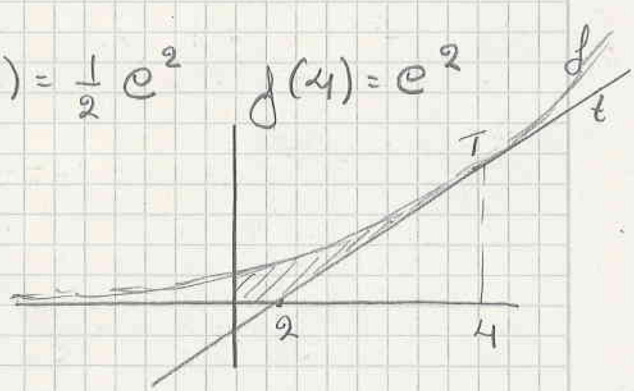
$f(1) = 1 \Rightarrow g(1) = 1 \Rightarrow -\frac{1}{4} \cdot 1 + k = 1 \Leftrightarrow k = \frac{5}{4}$

$g(x) = -\frac{1}{4}x^2 + \frac{5}{4}$

$\text{Aire} = \int_{-1}^{1} (g(x) - f(x))\,dx = \int_{-1}^{1} \left(-\frac{1}{4}x^2 - x^2 + \frac{5}{4}\right) dx = \int_{-1}^{1} -\frac{5}{4}x^2 + \frac{5}{4}\,dx =$

$= -\frac{5}{4}\int_{-1}^{1} (x^2 - 1)\,dx = -\frac{5}{4}\left(\frac{1}{3}x^3 - x\right)\Big]_{-1}^{1} = \frac{5}{3}$

EXERCICE 16



$f(x) = e^{x/2} \qquad x_0 = 4$

tangente: $f'(x) = \frac{1}{2}e^{x/2} \quad f'(4) = \frac{1}{2}e^2 \quad f(4) = e^2$

$e^2 = \frac{1}{2}e^2 \cdot 4 + h \Rightarrow h = -e^2$

$\Rightarrow t: \; y = \frac{1}{2}e^2 x - e^2$

$y = 0 \Rightarrow \frac{1}{2}e^2 x - e^2 = 0 \Leftrightarrow x = 2$

$\text{Aire} = \int_0^2 f(x)\,dx + \int_2^4 (f(x) - y)\,dx =$

$= \int_0^2 e^{x/2}\,dx + \int_2^4 \left(e^{x/2} - \frac{1}{2}e^2 x + e^2\right) dx = \left[2e^{x/2}\right]_0^2 + \left[2e^{x/2} - \frac{1}{4}e^2x^2 + e^2x\right]_2^4 =$

$= 2(e-1) + 4e^2 - 2e = 4e^2 - 2$

EXERCICE 17

a) $\int_0^{\pi/2} \cos^2(x)\sin(x)dx = I$

$t = \cos x \Rightarrow dt = -\sin x\, dx$ et $\cos^2 x = t^2$

$\int \cos^2 x \sin x\, dx = -\int t^2 dt = -\frac{1}{3}t^3 = -\frac{1}{3}\cos^3 x$

$I = -\frac{1}{3}\cos^3 x\Big]_0^{\pi/2} = -\frac{1}{3}\underbrace{\cos^3\frac{\pi}{2}}_{0} + \frac{1}{3}\underbrace{\cos^3 0}_{1} = \frac{1}{3}$

b) $\int \cos x\, e^{\sin x} dx = \int e^t dt = e^t = e^{\sin x}$

$t = \sin x \Leftrightarrow dt = \cos x\, dx$

$I = \left[e^{\sin x}\right]_0^{\pi/2} = e^1 - e^0 = e - 1$

c) $I = \int \frac{\sin x}{\cos x} dx = -\int \frac{1}{t} dt = -\ln|t| = -\ln|\cos x|$

$t = \cos x \Rightarrow dt = -\sin x\, dx$

$I = -\ln|\cos x|\Big]_0^1 = -\ln\cos 1 + \ln\underbrace{\cos 0}_{1} = -\ln\cos 1$

d) $\int x\sin(x^2)dx = \frac{1}{2}\int \sin t\, dt = -\frac{1}{2}\cos t = -\frac{1}{2}\cos x^2$

$x^2 = t \Leftrightarrow 2x\,dx = dt \Leftrightarrow x\,dx = \frac{1}{2}dt$

$I = \left[-\frac{1}{2}\cos(x^2)\right]_0^{\sqrt{\pi}} = -\frac{1}{2}\underbrace{\cos\pi}_{-1} + \frac{1}{2}\cos 0 = \frac{1}{2} + \frac{1}{2} = 1$

e) $\int \sin(\pi x + \pi)dx = \frac{1}{\pi}\int \sin t\, dt = \frac{1}{\pi}(-\cos t) = -\frac{1}{\pi}\cos(\pi x + \pi)$

$\pi x + \pi = t \Leftrightarrow \pi\, dx = dt \Leftrightarrow dx = \frac{1}{\pi}dt$

$I = -\frac{1}{\pi}\cos(\pi x + \pi)\Big]_0^1 = -\frac{1}{\pi}\underbrace{\cos 2\pi}_{1} + \frac{1}{\pi}\underbrace{\cos\pi}_{-1} = -\frac{1}{\pi} - \frac{1}{\pi} = \frac{2}{\pi}$

f) $\int \cos(x)\sin(x)dx = -\int t\, dt = -\frac{1}{2}t^2 = -\frac{1}{2}\cos^2 x$

$t = \cos x \Leftrightarrow dt = -\sin x$

$I = -\frac{1}{2}\cos^2 x\Big]_0^{\pi/2} = -\frac{1}{2}\underbrace{\cos^2\frac{\pi}{2}}_{0} + \frac{1}{2}\underbrace{\cos^2 0}_{1} = \frac{1}{2}$

g) $\int \frac{3e^x}{1+2e^x}dx = \int \frac{3\cancel{t}}{1+2t}\frac{1}{\cancel{t}}dt = 3\int \frac{dt}{1+2t} = 3\frac{1}{2}\ln(1+2t) = ^{*}$

$e^x = t \Rightarrow e^x dx = dt \Rightarrow$

$* = \frac{3}{2}\ln(1+2e^x)$

$I = \frac{3}{2}\ln(1+2e^x)\Big]_0^1 = \frac{3}{2}\ln(1+2e) - \frac{3}{2}\ln(3) =$

$= \frac{3}{2}\ln\left(\frac{1+2e}{3}\right)$

h) $\int \frac{\ln x}{x}dx = \int t\,dt = \frac{1}{2}t^2 = \frac{1}{2}\ln^2 x$

$t = \ln x \Rightarrow dt = \frac{1}{x}dx \Leftrightarrow dx = x\,dt$

$I = \left[\frac{1}{2}\ln^2 x\right]_1^e = \frac{1}{2}\ln^2 e - \frac{1}{2}\ln^2 1 = \frac{1}{2}$

i) $\int \sqrt{e^x-1}\,dx = \int \frac{\sqrt{t}}{t+1}dt \;*$

$t = e^x - 1 \Leftrightarrow dt = e^x dx \Rightarrow dt = (t+1)dx$
$e^x = t+1$

$* \; \sqrt{t} = u \Rightarrow t = u^2 \Rightarrow dt = 2u\,du$

$= \int \frac{u}{u^2+1}2u\,du = 2\int \frac{u^2}{u^2+1}du = 2\int\left(\frac{u^2+1}{u^2+1} - \frac{1}{u^2+1}\right)du =$

$= 2\int du - 2\int \frac{du}{u^2+1} = 2u - 2\tan^{-1}(u) =$

$= 2u - 2\,\mathrm{Arctan}\,u = 2\sqrt{t} - 2\,\mathrm{Arctan}\sqrt{t} =$

$= 2\sqrt{e^x-1} - 2\,\mathrm{Arctan}\sqrt{e^x-1}$

$I = \left[2\sqrt{e^x-1} - 2\,\mathrm{Arctan}\sqrt{e^x-1}\right]_0^{\ln 2} =$

$= \left(2\sqrt{2-1} - 2\,\mathrm{Arctan}\sqrt{2-1}\right) - \left(2\sqrt{1-1} - 2\,\mathrm{Arctan}\sqrt{1-1}\right) =$

$= \left(2 - 2\cdot\frac{\pi}{4}\right) - (0-0) = 2 - \frac{\pi}{2}$

j) $\int \frac{x}{\sqrt{4x+2}}\,dx = \int \frac{(t^2-2)/4}{\cancel{t}}\,\frac{1}{2}\cancel{t}\,dt = \frac{1}{8}\int (t^2-2)\,dt = ^*$

$t = \sqrt{4x+2} \Leftrightarrow t^2 = 4x+2 \Leftrightarrow 2t\,dt = 4\,dx \Leftrightarrow t\,dt = 2\,dx$

$t^2 = 4x+2 \Leftrightarrow \frac{t^2-2}{4}$

$^* = \frac{1}{8}\left(\frac{1}{3}t^3 - 2t\right) = \frac{1}{8}\left(\frac{1}{3}\sqrt{4x+2}^3 - 2\cdot\sqrt{4x+2}\right)$

$I = \frac{1}{8}\left[\frac{1}{3}\sqrt{4x+2}^3 - 2\sqrt{4x+2}\right]_1^4 =$

$= \frac{1}{8}\left[\frac{1}{3}\sqrt{18}^3 - 2\sqrt{18}\right] - \frac{1}{8}\left[\frac{1}{3}\sqrt{6}^3 - 2\sqrt{6}\right] = \ldots$

k) $\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{\cos t\,dt}{\cos t} = \int dt = t = \text{Arcsin}\,x$

$x = \sin t \Rightarrow dx = \cos t \cdot dt$

$x^2 = \sin^2 t \Rightarrow 1 - x^2 = 1 - \sin^2 t \Rightarrow \sqrt{1-x^2} = \sqrt{\cos^2 t} = \cos t$

$I = \left[\text{Arcsin}\,x\right]_0^{1/2} = \text{Arcsin}\frac{1}{2} - \text{Arcsin}\,0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$

l) $\int x^3 e^{-x^2}\,dx = -\frac{1}{2}\int x^2\cdot\cancel{x}\,e^t\,\frac{1}{\cancel{x}}\,dt = -\frac{1}{2}\int(-t)e^t\,dt^*$

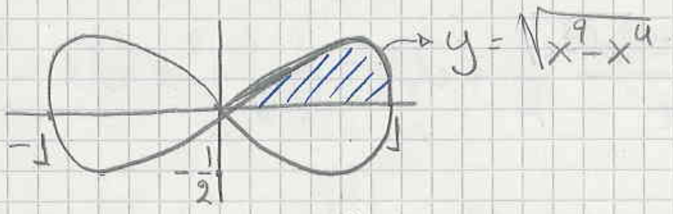
$t = -x^2 \Leftrightarrow dt = -2x\,dx$

$= \frac{1}{2}\int t e^t\,dt = \frac{1}{2}\left(te^t - \int e^t\,dt\right) = \frac{1}{2}\left(te^t - e^t\right)$

$= \frac{1}{2}\left[-x^2 e^{-x^2} - e^{-x^2}\right]$

$I = \frac{1}{2}\left[-e^{-x^2}(x^2+1)\right]_0^1 = \frac{1}{2}\left(-e^{-1}2 + e^0\cdot 1\right) = \frac{1}{2}\left(-\frac{2}{e} + 1\right)$

EXERCICE 18



$$\text{Aire} = 4\int_0^1 \sqrt{x^2 - x^4}\,dx = 4\int_0^1 \sqrt{x^2(1-x^2)}\,dx = 4\int_0^1 x\sqrt{1-x^2}\,dx$$

$$\sqrt{1-x^2} = t \Leftrightarrow 1 - x^2 = t^2 \Leftrightarrow -2x\,dx = 2t\,dt \Leftrightarrow$$
$$-x\,dx = t\,dt$$

$$\int x\sqrt{1-x^2}\,dx = -\int x\,t\cdot\frac{t}{x}\,dt = -\int t^2\,dt =$$

$$= -\frac{1}{3}t^3 = -\frac{1}{3}\sqrt{1-x^2}^{\,3}$$

$$\text{Aire} = 4\left[-\frac{1}{3}\sqrt{1-x^2}^{\,3}\right]_0^1 = 4\left[-\frac{1}{3}\cdot 0 + \frac{1}{3}\sqrt{1}\right] =$$

$$= \frac{4}{3}$$

EXERCICE 19

a) $\int (\overset{u}{3x+7})\overset{v'}{\cos x}\,dx = (\overset{u}{3x+7})\overset{v}{\sin x} - \int \overset{u'}{3}\cdot\overset{v}{\sin x}\,dx =$

$= (3x+7)\sin x + 3\cos x$

b) $\int \overset{v'}{(x-5)}\overset{u}{\ln x}\,dx = \left(\frac{1}{2}x^2 - 5x\right)\ln x\,dx - \int\left(\frac{1}{2}x^2 - 5x\right)\frac{1}{x}\,dx =$

$= \left(\frac{1}{2}x^2 - 5x\right)\ln x - \int\left(\frac{1}{2}x - 5\right)dx =$

$= \left(\frac{1}{2}x^2 - 5x\right)\ln x - \frac{1}{4}x^2 + 5x$

i) $I = \int \overset{u}{\sin x} \cdot \overset{v'}{e^x}\,dx = \overset{u}{\sin x} \cdot \overset{v}{e^x} - \int \overset{u'}{\cos x} \cdot \overset{v}{e^x} =$

$= \sin x \cdot e^x - \cos x \cdot e^x - \int \sin x \cdot e^x \Leftrightarrow$

$\Leftrightarrow 2I = e^x(\sin x - \cos x) \Leftrightarrow$

$I = \frac{1}{2} e^x (\sin x - \cos x)$

c) $\int \ln x = \int (\overset{u'}{x})' \ln x\,dx = x \cdot \ln x - \int x \frac{1}{x}\,dx =$

$= x \ln x - \int dx = x \ln x - x$

d) $\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x} \ln x - \int \left(-\frac{1}{x}\right) \frac{1}{x}\,dx =$

$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x}$

e) $\int \frac{\ln x}{x} = -\frac{1}{x^2} \ln x + \int \frac{1}{x^2} \frac{1}{x}\,dx =$

$= -\frac{1}{x^2} \ln x + \int x^{-3}\,dx = -\frac{1}{x^2} \ln x - \frac{1}{2x^2}$

f) $\int \overset{u}{x} \overset{v'}{e^{-x}}\,dx = -\overset{u}{x} \overset{v}{e^{-x}} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} =$

$= e^{-x}(-x-1)$

g) $\int x^2 e^{-x}\,dx = -x^2 e^{-x} + \int 2x e^{-x}\,dx = -x^2 e^{-x} + 2\int x e^{-x}\,dx$ *

$\int x e^{-x}\,dx = -x e^{-x} + \int e^{-x} = -x e^{-x} - e^{-x}$

* $= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} =$

$= -e^{-x}(x^2 + 2x + 2)$

h) $\int x^3 e^{-x}\,dx = -x^3 e^{-x} + 3\int x^2 e^{-x}\,dx = {}^{**}$

$\int x^2 e^{-x} dx = -x^2 e^{-x} + 2\int x e^{-x} \quad *$

$\int x e^{-x} = -x e^{-x} + \int e^{-x} = -x e^{-x} - e^{-x} = -e^{-x}(x+1)$

$* = -x^2 e^{-x} + 2\left(-e^{-x}(x+1)\right) =$

$= -x^2 e^{-x} - 2e^{-x}\cdot x - 2e^{-x}$

$** = -x^3 e^{-x} - 3x^2 e^{-x} + 6x e^{-x} - 6e^{-x}$

j) $I = \int \cos x \cdot e^x = \cos x \cdot e^x + \int \sin x\, e^x =$

$= \cos x\, e^x + \sin x\, e^x - \int \cos x\, e^x dx \Leftrightarrow$

$\Leftrightarrow 2I = e^x(\cos x + \sin x) \Leftrightarrow$

$I = \frac{1}{2} e^x(\cos x + \sin x)$

k) $\int \overset{u}{\cos x}\, \overset{v'}{\sin x}\, dx = -\overset{u}{\cos x}\, \overset{v}{\cos x} - \int \sin x \cos x\, dx$

$\Leftrightarrow 2I = -\cos^2 x \Leftrightarrow I = -\frac{1}{2}\cos^2 x$

l) $\int \frac{x+3}{(x+1)^3}$

$\frac{x+3}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} \Leftrightarrow$

$\Leftrightarrow x+3 = A(x+1)^2 + B(x+1) + C \Leftrightarrow$

$\Leftrightarrow x+3 = A(x^2+2x+1) + Bx + B + C \Leftrightarrow$

$\Leftrightarrow x+3 = Ax^2 + x(2A+B) + A + B + C$

$A = 0 \quad 2A + B = 1 \Leftrightarrow B = 1 \quad A + B + C = 3$

$1 + C = 3 \Leftrightarrow C = 2$

$$I = \int \frac{1}{(x+1)^2}dx + \int \frac{2}{(x+1)^3}dx = -\frac{1}{x+1} + 2\left(-\frac{1}{2}\right)\frac{1}{(x+1)^2} =$$

$$= -\frac{1}{x+1} - \frac{1}{(x+1)^2}$$

EXERCICE 20

$$\int_0^3 \frac{x}{\sqrt{x+1}}dx$$

a) $x = u-1 \Rightarrow dx = du$

$$\int \frac{x}{\sqrt{x+1}}dx = \int \frac{u-1}{\sqrt{u}}du = \int \frac{u}{\sqrt{u}}du - \int \frac{1}{\sqrt{u}}du =$$

$$= \int u^{1/2}du - \int u^{-1/2}du = \frac{2}{3}u^{3/2} - 2u^{1/2} =$$

$$= \frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2}$$

$$I = \left[\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2}\right]_0^3 = \frac{2}{3}(4)^{3/2} - 2(4)^{1/2} - \frac{2}{3}(1)^{3/2} + 2\cdot 1 =$$

$$= \frac{2}{3}8 - 4 - \frac{2}{3} + 2 = \frac{16}{3} - 2 - \frac{2}{3} = \frac{14}{3} - 2 = \frac{8}{3}$$

b) $x = t^2 - 1 \Rightarrow dx = 2t\,dt \qquad t^2 = x+1 \Rightarrow t = \sqrt{x+1}$

$$\int \frac{x}{\sqrt{x+1}}dx = \int \frac{t^2-1}{\sqrt{t^2}}2t\,dt = 2\int (t^2-1)dt =$$

$$2\frac{1}{3}t^3 - 2t = \frac{2}{3}\sqrt{x+1}^3 - 2\sqrt{x+1}$$

$$I = \left.\frac{2}{3}\sqrt{x+1}^3 - 2\sqrt{x+1}\right]_0^3 = \frac{8}{3}$$

c) $\int \overset{u}{x} \cdot \overset{v'}{\frac{1}{\sqrt{x+1}}}dx = \qquad u = x \quad u' = 1$

$\qquad v = 2\sqrt{x+1} \quad v' = \frac{1}{\sqrt{x+1}}$

$$= 2x\sqrt{x+1} - \int 2\sqrt{x+1}\,dx =$$

$$= 2x\sqrt{x+1} - 2\int (x+1)^{1/2}dx = 2x\sqrt{x+1} - 2\frac{2}{3}(x+1)^{3/2}$$

$$I = \left[2x\sqrt{x+1} - \frac{4}{3}\sqrt{x+1}^3\right]_0^3 = \left[6\cdot 2 - \frac{4}{3}8 - 0 + \frac{4}{3}\right] = 12 - \frac{32}{3} + \frac{4}{3} =$$

$$= 12 - \frac{28}{3} = \frac{8}{3}$$

EXERCICE 21

$u = \sin x \quad v' = \sin x$
$u' = \cos x \quad v = -\cos x$

- $\int \sin^2(x)dx = \int \overset{u}{\sin x} \cdot \overset{v'}{\sin x}\, dx =$

$= -\overset{u}{\sin x}\overset{v}{\cos x} + \int \cos^2 x\, dx =$

$= -\sin x \cos x + \int dx - \int \sin^2 x\, dx =$

$= -\sin x \cos x + x - \int \sin^2 x\, dx \Leftrightarrow$

$2\int \sin^2 x\, dx = -\sin x \cos x + x \Leftrightarrow$

$\Leftrightarrow \int \sin^2 x\, dx = \dfrac{-\sin x \cos x + x}{2}$

- De même $\int \cos^2 x\, dx = \dfrac{\cos x \sin x}{2} + \dfrac{1}{2}\int dx =$

$= \dfrac{\cos x \sin x}{2} + \dfrac{1}{2}x$

Aire : $\sin^2 x = \cos^2 x \Leftrightarrow \left.\begin{array}{l}\sin x = \cos x \\ \sin x = -\cos x\end{array}\right\} x = \dfrac{\pi n}{2} - \dfrac{\pi}{4}$

$n=0 \quad x_1 = -\dfrac{\pi}{4}$

$n=1 \quad x_2 = \dfrac{\pi}{2} - \dfrac{\pi}{4} = \dfrac{\pi}{4}$

$n=2 \quad x_3 = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

Aire $= \displaystyle\int_{\pi/4}^{3\pi/4} (\sin^2 x - \cos^2 x)dx = \left[\dfrac{-\sin x \cos x + x}{2} - \dfrac{\cos x \sin x + x}{2}\right]_{\pi/4}^{3\pi/4}$

$= \left[\dfrac{-2\sin x \cos x}{2}\right] = \left[-\sin x \cos x\right]_{\pi/4}^{3\pi/4} =$

$= \left[-\dfrac{\sqrt{2}}{2}\left(-\dfrac{\sqrt{2}}{2}\right) + \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{2}}{2}\right) = \dfrac{1}{2} + \dfrac{1}{2} = 1$

EXERCICE 22

$$y = \sqrt{r^2 - x^2} \Rightarrow y^2 = r^2 - x^2 \Rightarrow x^2 + y^2 = r^2$$

$$x = r\sin t \Rightarrow dx = r\cos t\,dt \qquad t = \operatorname{Arcsin}\frac{x}{r}$$

$$\int \sqrt{r^2 - x^2}\,dx = \int \sqrt{r^2 - r^2\sin^2 t} \cdot r\cos t\,dt = \int r^2\cos^2 t\,dt =$$

$$= r^2 \int \cos^2 t\,dt = r^2\left(\frac{\cos t\sin t + t}{2}\right) =$$

$$= \frac{r^2}{2}\left(\cos\left(\operatorname{Arcsin}\frac{x}{r}\right) \cdot \frac{x}{r} + \operatorname{Arcsin}\frac{x}{r}\right)\Bigg]_0^r =$$

$$\frac{r^2}{2}\Big(\cos(\operatorname{Arcsin} 1) \cdot 1 + \operatorname{Arcsin} 1 - \cos \operatorname{Arcsin} 0 - \operatorname{Arcsin} 0\Big) =$$

$$= \frac{r^2}{2}\left(\cos\frac{\pi}{2} + \frac{\pi}{2} - \cos 0 - 0\right) = \frac{r^2}{2} \cdot \left(\frac{\pi}{2} - 1\right)$$

EXERCICE 23

a) $y = \arcsin(x)$

$$x = \sin t \Rightarrow dx = \cos t\,dt$$

$$\int \arcsin x\,dx = \int \arcsin(\sin t) \cdot \cos t\,dt =$$

$$= \int \overset{u}{t} \cdot \overset{v'}{\cos t}\,dt = \overset{u}{t} \cdot \overset{v}{\sin t} - \int \sin t\,dt = t\sin t + \cos t$$

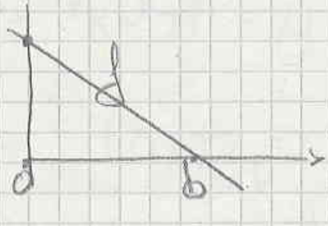
$$= \operatorname{Arcsin} x \cdot x + \cos \underbrace{\arcsin x}_{\to\sqrt{1-x^2}} = x \cdot \arcsin x + \sqrt{1-x^2}$$

b) De même

$$\int \arccos x\,dx = x \arccos x - \sqrt{1-x^2}$$

EXERCICE 24

a) Formule : $V = \pi \int_a^b f^2(x)\,dx$



• $f(x) = 0 \Leftrightarrow -\frac{1}{2}x + 3 = 0 \Leftrightarrow x = 6$

$b = 6$

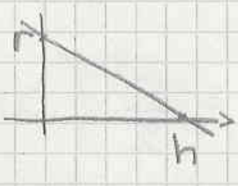
$$V = \pi \int_0^6 \left(-\frac{1}{2}x+3\right)^2 dx = \pi \int_0^6 \left(\frac{1}{4}x^2 - 3x + 9\right) dx =$$

$$= \pi \left[\frac{1}{4}\frac{1}{3}x^3 - \frac{3}{2}x^2 + 9x\right]_0^6 =$$

$$= \pi \left(\frac{1}{12}6^3 - \frac{3}{2}6^2 + 9\cdot 6\right) = \pi(18 - 54 + 54) = \pi 18$$

b) Pour le cône $b = h$

On cherche la fonction f



$m = \frac{r}{-h}$ $\quad y = -\frac{r}{h}x + r = f(x)$

$$V = \pi \int_0^h \left(-\frac{r}{h}x + r\right)^2 dx = \pi \int_0^h \left(\frac{r^2}{h^2}x^2 - 2\frac{r^2}{h}x + r^2\right) dx$$

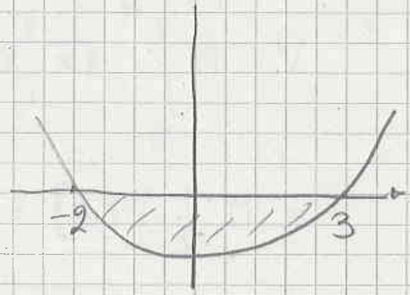
$$= \pi \left[\frac{r^2}{3h^2}x^3 - \frac{2r^2}{h\cdot 2}x^2 + r^2 x\right]_0^h =$$

$$= \pi \left[\frac{r^2}{3\cancel{h^2}}h^{\cancel{3}} - \frac{r^2}{\cancel{h}}\cdot h^{\cancel{2}} + r^2\cdot h\right] = \pi r^2 \left[\frac{h}{3} - \cancel{h} + \cancel{h}\right] =$$

$$= \pi r^2 \left(\frac{h}{3}\right) \Rightarrow V = \frac{\pi r^2 h}{3}$$

EXERCICE 25

$f(x) = x^2 - x - 6 \qquad x_1 = -2 \qquad x_2 = 3$



$$V = \pi \int_{-2}^{3} (x^2 - x - 6)^2 \, dx =$$

$$= \pi \int_{-2}^{3} (x^4 + x^2 + 36 - 2x^3 - 12x^2 + 12x) \, dx =$$

$$= \pi \int_{-2}^{3} (x^4 - 2x^3 - 11x^2 + 12x + 36) \, dx =$$

$$= \pi \left[\frac{1}{5} x^5 - \frac{2}{4} x^4 - \frac{11}{3} x^3 + \frac{12}{2} x^2 + 36x \right]_{-2}^{3} =$$

$$= \pi \left[\frac{1}{5} x^5 - \frac{1}{2} x^4 - \frac{11}{3} x^3 + 6x^2 + 36x \right]_{-2}^{3} =$$

$$= \pi \left[\frac{243}{5} - \frac{1}{2} 81 - \frac{11}{3} 27 + 54 + 108 + \right.$$

$$\left. + \frac{32}{5} + \frac{1}{2} 16 - \frac{11}{3} 8 - 6 \cdot 4 + 72 \right] =$$

$$= \pi \left[\frac{275}{5} - \frac{65}{2} - \frac{35 \cdot 11}{3} + 30 + 180 \right] =$$

$$= \pi \left[55 - \frac{65}{2} - \frac{385}{3} + 210 \right] =$$

$$= \pi \left[265 - \frac{65}{2} - \frac{385}{3} \right] = \pi \left(\frac{1590 - 195 - 770}{6} \right) =$$

$$= \pi \left(\frac{625}{6} \right)$$

EXERCICE 26

$$V = \pi \int_0^4 \sqrt{x}^2 \, dx = \pi \int_0^4 x \, dx = \pi \, \frac{1}{2} x^2 \Big]_0^4 =$$

$$= \pi \, \frac{1}{2} \left[16 \right] = 8 \pi$$