LDDR Niveau 1: TE 5 Analyse solutions

Exercice 1

$f(x) = \sqrt{x^2 - x + 3}$ $\quad f'(x) = \dfrac{2x-1}{2\sqrt{x^2-x+3}}$

$f(-2) = \sqrt{9} = 3$ $\quad f'(-2) = \dfrac{-5}{2\cdot 3} = -\dfrac{5}{6}$

$y = mx + h \rightarrow 3 = -\dfrac{5}{6}\cdot(-2) + h \Rightarrow h = 3 - \dfrac{5}{3} \Rightarrow h = \dfrac{4}{3}$

$t:\ y = -\dfrac{5}{6}x + \dfrac{4}{3}$

Exercice 2

$f(x) = \sin(2x)$ $\quad g(x) = -\sin x$ $\quad [\pi; \frac{3\pi}{2}]$

- $f(x) = g(x) \Rightarrow \sin 2x = -\sin x \Leftrightarrow$

$2\sin x\cos x + \sin x = 0 \Leftrightarrow \sin x\,(2\cos x + 1) = 0$

$\sin x = 0 \Rightarrow x = \pi$

$\cos x = -\frac{1}{2} \Rightarrow x = \dfrac{4\pi}{3}$

- $\underline{x = \pi}$ $\quad f'(x) = 2\cos(2x)$ $\quad f'(\pi) = 2$ $\quad u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$g'(x) = -\cos x$ $\quad g'(\pi) = 1$ $\quad v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

On utilise la formule: $\cos\alpha = \dfrac{|u\cdot v|}{\|u\|\cdot\|v\|}$

$u\cdot v = 1\cdot 2 + 1\cdot 1 = 3$

$\|u\| = \sqrt{1+2^2} = \sqrt{5}$ $\quad \|v\| = \sqrt{2}$

$\cos\alpha = \dfrac{3}{\sqrt{5}\cdot\sqrt{2}} \Rightarrow \alpha = 18{,}43^\circ$

- $x = \dfrac{4\pi}{3}$ $\quad f'\left(\dfrac{4\pi}{3}\right) = 2\cos\left(\dfrac{8\pi}{3}\right) = 2\cos\left(2\pi + \dfrac{2\pi}{3}\right) = 2\cdot\left(-\dfrac{1}{2}\right) = -1$

$g'\left(\dfrac{4\pi}{3}\right) = -\cos\left(\dfrac{4\pi}{3}\right) = -\left(-\dfrac{1}{2}\right) = \dfrac{1}{2}$ $\quad u = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\quad v = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$

$u\cdot v = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

$\|u\| = \sqrt{2}$ $\quad \|v\| = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \dfrac{\sqrt{5}}{2}$

$\cos\alpha = \dfrac{\cancel{1/2}}{\sqrt{2}\cdot\sqrt{5}/\cancel{2}} = \dfrac{1}{\sqrt{10}} \Rightarrow \alpha = 71{,}56^\circ$

EXERCICE 3

1) $f(x) = (2x+1)^2 (3-x)^3 \qquad D_f = \mathbb{R}$

$f'(x) = 2(2x+1)\cdot 2\,(3-x)^3 + (2x+1)^2 \cdot 3(3-x)^2(-1) =$

$= (2x+1)(3-x)^2\left(4(3-x) - 3(2x+1)\right) =$

$= (2x+1)(3-x)^2(12 - 4x - 6x - 3) =$

$= (2x+1)(3-x)^2(-10x+8) = 2(2x+1)(3-x)^2(-5x+4)$

$f'(x) = 0 \Leftrightarrow x = -\frac{1}{2} \quad x = 3 \quad x = \frac{4}{5}$

x	$-\infty$		$-1/2$		$4/5$		3		$+\infty$
$(2x+1)$		$-$	0	$+$		$+$		$+$	
$(3-x)^2$		$+$		$+$		$+$	0	$+$	
$(-5x+4)$		$+$		$+$	0	$-$		$-$	
f'		$-$	0	$+$	0	$-$	0	$-$	
f		$\searrow$	Min	$\nearrow$	Max	$\searrow$	PI	$\searrow$	

Min $(-\frac{1}{2};0)$ Max $(\frac{4}{5};34.5)$ PI $(3;0)$

2) $f(x) = \sin(x) - 2\cos x \qquad [0;2\pi]$

$f'(x) = \cos x + 2\sin x = 0 \Leftrightarrow \frac{\sin x}{\cos x} = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \tan x = -\frac{1}{2} \Rightarrow x_1 = 2.68$ rad

$x_2 = 5.82$ rad

	0		2.68		5.82		2π
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$	Max	$\searrow$	Min	$\nearrow$	

Max $(2.68;-0.005)$ Min $(5.82;-2.23)$

EXERCICE 4

$f(x) = \frac{2x^2-5x+2}{x^2-x-2} \qquad x^2-x-2=0 \Leftrightarrow x_1 = 2, \; x_2 = -1$

- $D_f = \mathbb{R} \setminus \{-1;2\}$
- $\cap O_x$: $y=0 \Leftrightarrow 2x^2-5x+2=0 \quad \Delta = 9 \quad x_{1,2} = \frac{5\pm3}{4} = x_1 = 2$ Imp. ; $x_2 = \frac{1}{2}$

  $I_{1x}(2;0)$ $I_{2x}(\frac{1}{2};0)$

  $\cap O_y$: $x=0 \quad y=-1 \quad I_y(0;-1)$

TS

x	$-\infty$		-1		$\frac{1}{2}$		2		$+\infty$
$2x^2-5x+2$		$+$	|	$+$	0	$-$	0	$+$	
x^2-x-2		$+$	0	$-$	|	$-$	0	$+$	
f		$+$	//	$-$	0	$+$	//	$+$	

AV $\lim\limits_{x\to -1} f(x) = \left[\frac{9}{0}\right] = \begin{cases} +\infty & x<-1 \\ -\infty & x>-1 \end{cases}$ AV $x=-1$

$\lim\limits_{x\to 2} f(x) = \left[\frac{0}{0}\right] = \lim\limits_{x\to 2} \frac{2\cancel{(x-2)}(x-\frac{1}{2})}{\cancel{(x-2)}(x+1)} = \frac{2}{3}$

Troue $(2;\frac{2}{3})$

AH: $y=2$

Dérivée : $f(x) = \frac{2\cancel{(x-2)}(x-\frac{1}{2})}{\cancel{(x-2)}(x+1)} = \frac{2x-1}{x+1}$

$f'(x) = \frac{2(x+1)-(2x-1)}{(x+1)^2} = \frac{\cancel{2x}+2-\cancel{2x}+1}{(x+1)^2} = \frac{3}{(x+1)^2}$

$f'(x)=0 \Leftrightarrow 3=0$ impossible

x	$-\infty$		-1		2		$+\infty$
f'		$+$	//	$+$	//	$+$	
f		$\nearrow$		$\nearrow$		$\nearrow$