LDDR - Niveau 1 : TE 4 Analyse.

Exercice 1

$f(x) = \frac{x^3-1}{2x^2+x-3}$

$2x^2+x-3=0 \quad \Delta = 1+24=25$

$x_{1,2} = \frac{-1 \pm 5}{4} = \begin{cases} x_1 = \frac{4}{4} \\ x_2 = -\frac{6}{4} = -\frac{3}{2} \end{cases}$

$D_f = \mathbb{R} \setminus \{ 5; -\frac{3}{2} \}$.

- $\cap\, O_x$: $f(x) = 0 \Leftrightarrow x^3 = 1 \Leftrightarrow x = 1 \quad I_x(1;0)$
- $\cap\, O_y$: $x = 0 \quad y = -\frac{1}{-3} = \frac{1}{3} \quad I_y(0; \frac{1}{3})$

AV : $\lim\limits_{x \to 1} f(x) = \left[\frac{0}{0}\right] = \lim\limits_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{2\cancel{(x-1)}(x+\frac{3}{2})} = \frac{3}{5}$

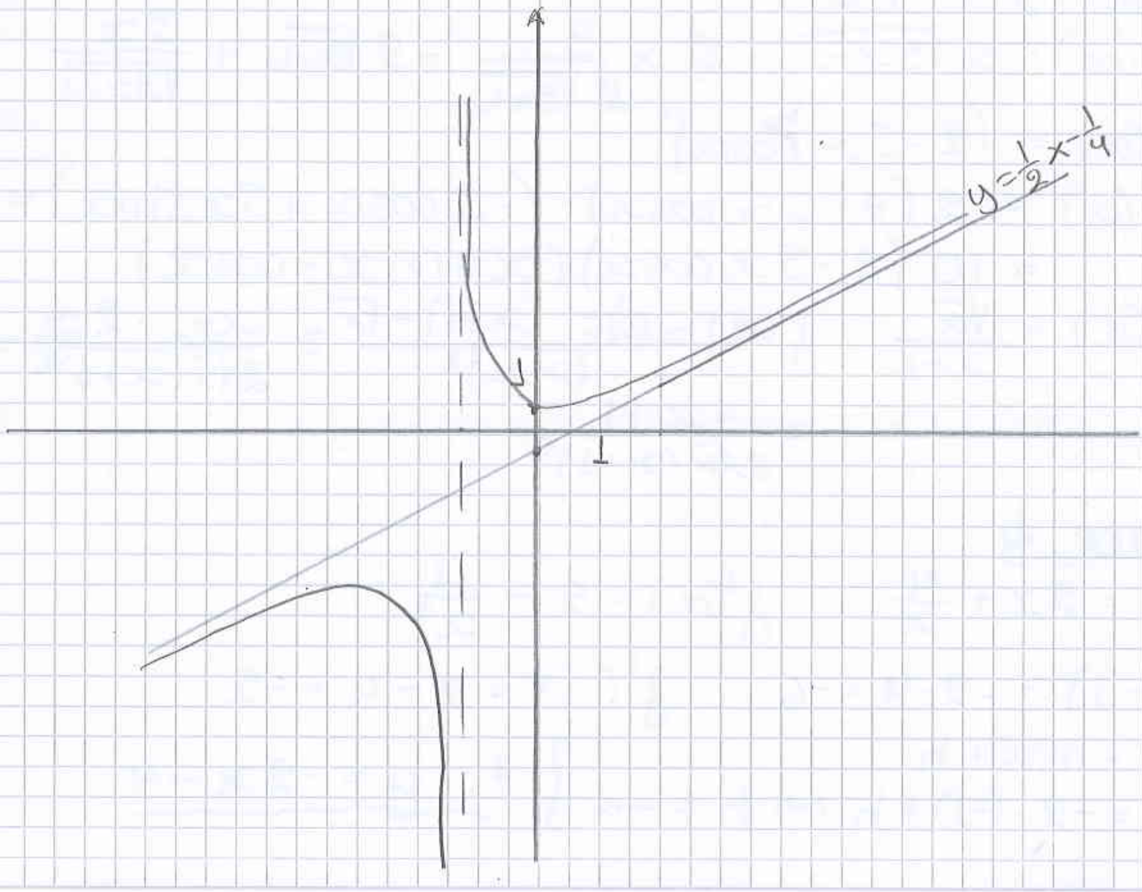
Trou $(1; \frac{3}{5})$

$\lim\limits_{x \to -\frac{3}{2}} f(x) = \left[\frac{-\frac{27}{8}-1}{0}\right] = \infty \qquad$ AV $x = -\frac{3}{2}$

AO

$$\begin{array}{rl|l} \cancel{x^3} & -1 & 2x^2+x-3 \\ \hline -x^3 - \frac{1}{2}x^2 + \frac{3}{2}x & & \frac{1}{2}x - \frac{1}{4} \\ \hline -\frac{1}{2}\cancel{x^2} + \frac{3}{2}x - 1 & & \\ \frac{1}{2}\cancel{x^2} + \frac{1}{4}x - \frac{3}{4} & & \\ \hline \frac{7x}{4} - \frac{7}{4} & & \end{array}$$

AO : $y = \frac{1}{2}x - \frac{1}{4}$



EXERCICE 2

1) $f(x) = 3x^2$

$f(x+h) = 3(x+h)^2 = 3x^2 + 6xh + 3h^2$

$f(x+h) - f(x) = \cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2} = h(6x+3h)$

$\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \frac{\cancel{h}(6x+3h)}{\cancel{h}} = \underline{6x = f'(x)}$

2) $f(x) = \frac{1}{x+1}$ $\quad f(x+h) = \frac{1}{x+h+1}$

$f(x+h) - f(x) = \frac{1}{x+h+1} - \frac{1}{x+1} = \frac{\cancel{x}+\cancel{1}-\cancel{x}-h-\cancel{1}}{(x+h+1)(x+1)} = \frac{-h}{(x+h+1)(x+1)}$

$\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \frac{-\cancel{h}}{(x+h+1)(x+1)\cdot \cancel{h}} = \underline{\frac{-1}{(x+1)^2} = f'(x)}$

EXERCICE 3

1) $f(x) = 3x^4 - 7x + 2$ $\quad f'(x) = 12x^3 - 7$

2) $f(x) = \frac{\cos x}{\sin x}$ $\quad f'(x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$

3) $f(x) = \frac{x}{\sqrt[4]{x^3}} = x \cdot x^{3/4} = x^{7/4}$

$f'(x) = \frac{7}{4} x^{3/4} = \frac{7}{4} \sqrt[4]{x^3}$

4) $f(x) = 2x \cdot \sqrt{3x+1}$

$f'(x) = 2\sqrt{3x+1} + \cancel{2}x \frac{3}{\cancel{2}\sqrt{3x+1}} = 2\sqrt{3x+1} + \frac{3x}{\sqrt{3x+1}}$

5) $f(x) = (2 - 5x\cos x)^2$

$f'(x) = 2(2 - 5x\cos x) \cdot (-5\cos x + 5x\sin x) =$

$= 10(2 - 5x\cos x)(x\sin x - \cos x)$

6) $f(x) = \frac{\sqrt{x}}{x+1}$ $\quad f'(x) = \frac{\frac{1}{2\sqrt{x}}(x+1) - \sqrt{x}}{(x+1)^2} = \frac{x+1-2x}{2\sqrt{x}(x+1)^2} =$

$= \frac{-x+1}{2\sqrt{x}(x+1)^2}$

EXERCICE 4

1) $f(x) = 2x + \frac{4}{x}$ $\quad f'(x) = 2 - \frac{4}{x^2}$

• $f(-1) = -2 - 4 = -6$ $\quad f'(-1) = 2 - 4 = -2$

$\left.\begin{array}{l} y = mx + h \\ -6 = -2\cdot(-1) + h \Leftrightarrow h = -4 \end{array}\right\}$ t: $\underline{y = -2x - 4}$

2) $f'(x) = 0 \Leftrightarrow 2 - \frac{4}{x^2} = 0 \Leftrightarrow 2x^2 - 4 = 0 \Leftrightarrow x^2 - 2 = 0$

$\Leftrightarrow x = \pm 2$ $A(2;6)$ $B(-2;-6)$