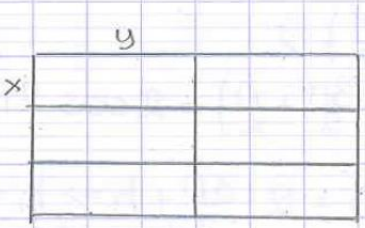
LDDR_Niveau_1:TE3 Analyse solutions

EXERCICE 1



$288 = 8y + 9x \Leftrightarrow y = \frac{288-9x}{8}$

$S = xy$

$S(x) = x \cdot \frac{288-9x}{8} = \frac{1}{8}x(288-9x)$

$= \frac{1}{8}(288x - 9x^2)$

$S'(x) = \frac{1}{8}(-18x+288) = \frac{1}{8} \cdot 18(-x+16) = \frac{9}{4}(-x+16)$

$S'(x) = 0 \Leftrightarrow x = 16$

x	0		16	
S'		$+$	0	$-$
S		↗	Max	↘

Max pour $x = 16$ $y = 18$

EXERCICE 2

$y = f(x) = x^3 + ax^2 + x$

$y' = 3x^2 + 2ax + 1$

On veut aucun PTH $\Leftrightarrow$ l'équation $f'(x) = 0$ n'a pas de solution $\Leftrightarrow$

$\Delta < 0 \Leftrightarrow 4a^2 - 12 < 0 \Leftrightarrow 4(a^2-3) < 0$

	$-\sqrt{3}$		$\sqrt{3}$	
	$+$	0	$-$	0 $+$

$a \in]-\sqrt{3}; \sqrt{3}[$

EXERCICE 3

$f(x) = \sin(2x + \frac{\pi}{2})$

a) $f(x) = 0 \Leftrightarrow \sin(2x + \frac{\pi}{2}) = 0 \Leftrightarrow$

$2x + \frac{\pi}{2} = 0 + k\pi$

pour $k = 0$ $2x + \frac{\pi}{2} = 0 \Leftrightarrow x = -\frac{\pi}{4}$

$k = 1$ $2x + \frac{\pi}{2} = \pi \Leftrightarrow 2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$

$k = 2$ $2x + \frac{\pi}{2} = 2\pi \Leftrightarrow 2x = \frac{3\pi}{2} \Leftrightarrow x = \frac{3\pi}{4}$

Comme x_A est la deuxième solution positive $x_A = \frac{3\pi}{4}$

b) $x_A = \frac{3\pi}{4} \quad y_A = 0$

$f'(x) = 2\cos\left(2x + \frac{\pi}{2}\right) \cdot 2$

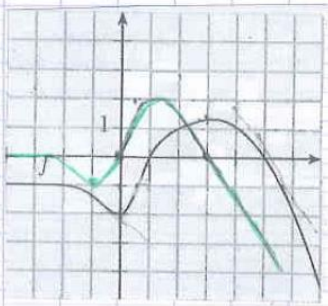
$f'\left(\frac{3\pi}{4}\right) = 2 \cdot \cos\left(\frac{3\pi}{2} + \frac{\pi}{2}\right) = 2\cos 2\pi = 2$

$y = mx + h \rightarrow 0 = 2 \cdot \frac{3\pi}{4} + h \Rightarrow h = -\frac{3\pi}{2}$

$t: y = 2x - \frac{3\pi}{2}$

c) $\alpha = \tan^{-1}(2) \Rightarrow \alpha = 63,43^\circ$

EXERCICE 4



EXERCICE 5

$f(x) = \frac{3x-2}{8-bx}$

- $f'(2) = \frac{7}{2}$

$f'(x) = \frac{3(8-bx) - (3x-2)(-b)}{(8-bx)^2} =$

$= \frac{24 - 3bx + 3bx - 2b}{(8-bx)^2} = \frac{24-2b}{(8-bx)^2}$

$f'(2) = \frac{7}{2} \Leftrightarrow \frac{24-2b}{(8-2b)^2} = \frac{7}{2} \Leftrightarrow$

$\Leftrightarrow 48 - 4b = 7(8-2b)^2 \Leftrightarrow$

$\Leftrightarrow 48 - 4b = 448 - 224b + 28b^2 \Leftrightarrow$

$\Leftrightarrow 28b^2 - 220b + 400 = 0 \Leftrightarrow$

$7b^2 - 55b + 100 = 0$

$\Delta = 225 \qquad b_{1,2} = \frac{55 \pm 15}{14} = \begin{cases} b_1 = \frac{40}{14} = \frac{20}{7} \\ b_2 = \frac{70}{14} = 5 \end{cases}$