LDDR - Niveau 1 : TE 2 Géométrie 3D

Solutions

EXERCICE 1

$$d = \begin{cases} x = 7+3\lambda \\ y = 1-\lambda \\ z = 8+2\lambda \end{cases} \qquad \pi : 2x+3y-z-6=0$$

$$d \rightarrow \pi : \quad 2(7+3\lambda)+3(1-\lambda)-(8+2\lambda)-6=0 \Leftrightarrow$$

$$\Leftrightarrow 14+6\lambda+3-3\lambda-8-2\lambda-6=0$$

$$\lambda = -3 \quad \Rightarrow \text{Sécants}$$

$$A(-2\,;\,4\,;\,2)$$

EXERCICE 2

$$a : \begin{cases} x = 1+3\lambda \\ y = 2+\lambda \\ z = 2-\lambda \end{cases} \qquad b : \begin{cases} x = 3-6\mu \\ y = 1-2\mu \\ z = 1+2\mu \end{cases}$$

1) $\vec{a} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} -6 \\ -2 \\ 2 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \parallel \vec{a} \Rightarrow a \parallel b$ ou $a = b$

$$A(1\,;2\,;2) \overset{?}{\in} b$$

$$\left.\begin{aligned} 1 &= 3-6\mu \\ 2 &= 1-2\mu \\ 2 &= 1+2\mu \end{aligned}\right\} \Rightarrow \begin{aligned} 6\mu &= 2 \Rightarrow \mu = 1/3 \\ 2\mu &= -1 \Rightarrow \mu = -1/2 \\ 2\mu &= 1 \Rightarrow \mu = 1/2 \end{aligned} \quad \not\parallel$$

$$\Rightarrow A \notin b \Rightarrow \text{confondues}$$

2) $A(1\,;2\,;2) \quad B(3\,;1\,;1) \quad \vec{a}\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \quad \vec{AB} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$

$$\pi : \begin{cases} x = 1+3t+2m \\ y = 2+t-m & *(-3) \\ z = 2-t-m & *3 \end{cases} \qquad \begin{cases} x = 1+3t+2m & (1) \\ -3y = -6-3t+3m & (2) \\ 3z = 6-3t-3m & (3) \end{cases}$$

$$(1)+(2) \quad x-3y = -5+5m \qquad\qquad x-3y = -5+5m$$

$$(1)+(3) \quad x+3z = 7-m \quad \times 5 \quad 5x+15z = 35-5m$$

$$6x-3y+15z = 30 \Leftrightarrow$$

$$\pi \Leftrightarrow 2x-y+5z=10$$

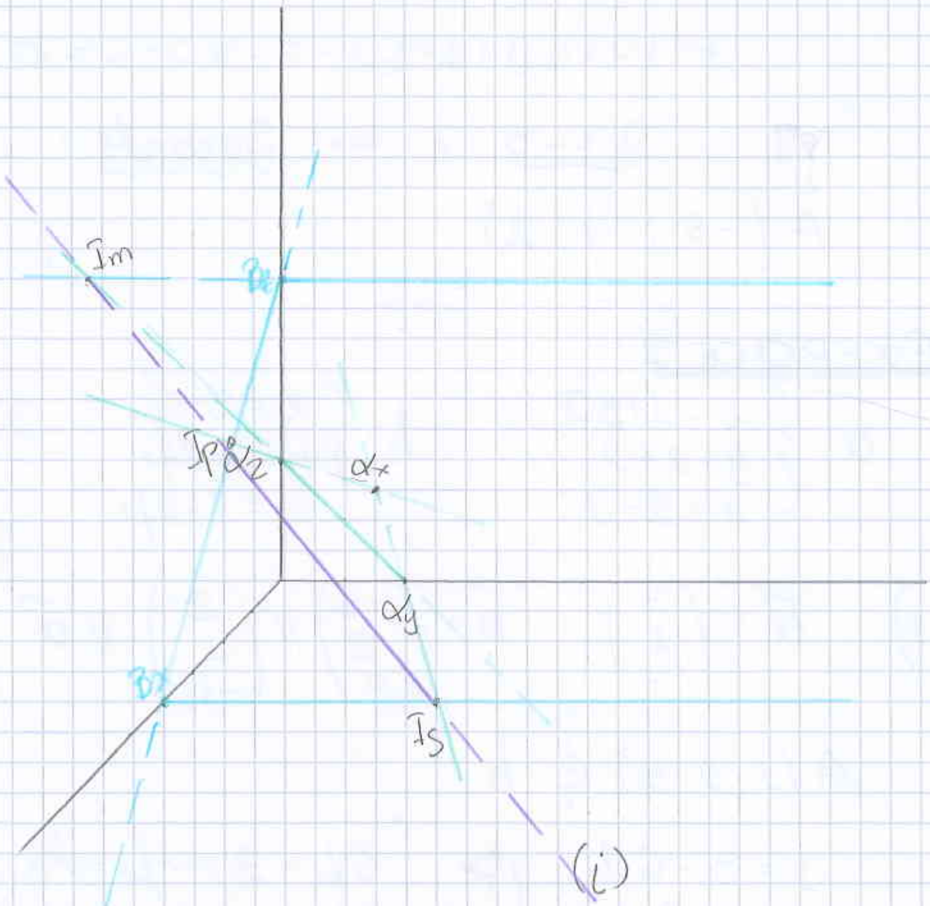
EXERCICE 3.

$\alpha : 2x-3y-3z+6=0 \qquad \beta : 5x+4z-20=0$

1) $\alpha_x(-3;0;0) \qquad \beta_x(4;0;0)$

$\alpha_y(0;2;0) \qquad \beta_y$ Impossible

$\alpha_z(0;0;2) \qquad \beta_z(0;0;5)$



$Im: (0;-3;5)$

$\begin{cases} -3y-3z+6=0 \\ 4z-20=0 \end{cases} \Rightarrow \begin{cases} y+z=2 \\ z=5 \end{cases} \Rightarrow \begin{cases} y=-3 \\ z=5 \end{cases}$

$Is \ (4;\frac{14}{3};0)$

$\begin{cases} 2x-3y=-6 \\ 5x=20 \end{cases} \Rightarrow \begin{cases} 8+6=3y \\ x=4 \end{cases} \Rightarrow \begin{cases} y=\frac{14}{3} \\ x=4 \end{cases}$

$\overrightarrow{ImIs} = \begin{pmatrix} 4 \\ \frac{14}{3}+3 \\ -5 \end{pmatrix} = \begin{pmatrix} 4 \\ \frac{13}{3} \\ -5 \end{pmatrix} \parallel \begin{pmatrix} 12 \\ 13 \\ -15 \end{pmatrix} \qquad i \begin{cases} x=0+12t \\ y=-3+13t \\ z=5-15t \end{cases}$

4) Un point $B_x(4;0;0)$ $\quad \overrightarrow{B_xB_z}$ ou $\overrightarrow{ImIs}$ ou $O_x=\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

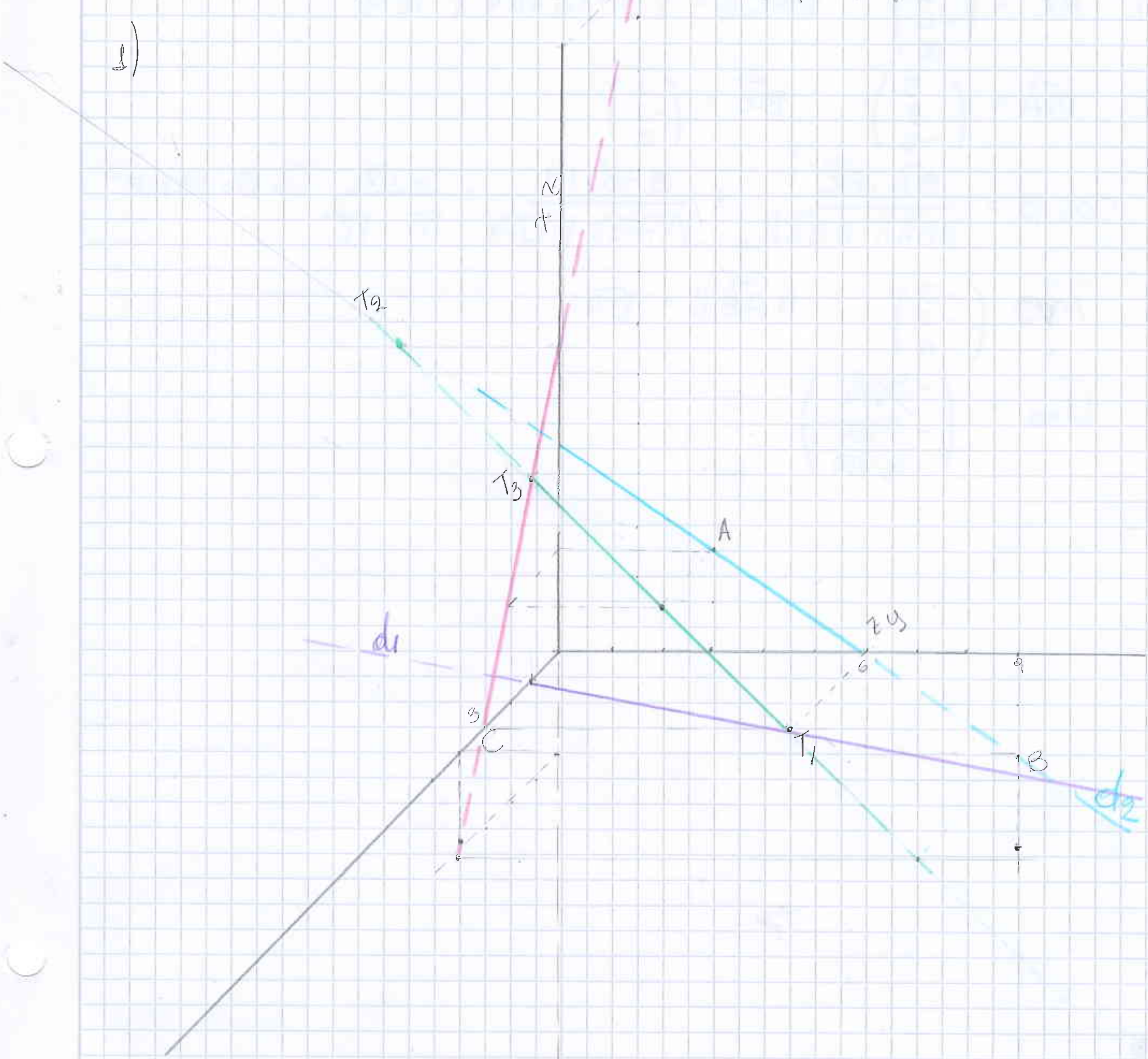
$\beta \begin{cases} x=4+12t \\ y=13t+m \\ z=-15t \end{cases}$

EXERCICE 4

d_2 A $(0;3;2)$ B$(0;9;-2)$

d_3 : C $(3;0;0)$ D $(-3;0;12)$

1)



EXERCICE 5

$A(5;-1;0) \quad B(2;-4;6) \quad C(3;-5;8)$

1) $\vec{AC} = \begin{pmatrix} -2 \\ -4 \\ 8 \end{pmatrix} \qquad \|\vec{AC}\| = \sqrt{4+16+64} = \sqrt{84}$

2) $\vec{BA} = \begin{pmatrix} 3 \\ 3 \\ -6 \end{pmatrix} \qquad \vec{BC} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

$$\cos \beta = \frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \cdot \|\vec{BC}\|} = \frac{\cancel{3} \cancel{-3} - 12}{\sqrt{9+9+36}\,\sqrt{1+1+4}} = \frac{-12}{\sqrt{54} \cdot \sqrt{6}} \Rightarrow \beta = 131{,}8^\circ$$

3) $\vec{AB} = \begin{pmatrix} -3 \\ -3 \\ 6 \end{pmatrix} \qquad \|\vec{AB}\| = \sqrt{54}$

$$\vec{u}_{AB} = \begin{pmatrix} -3/\sqrt{54} \\ -3/\sqrt{54} \\ 6/\sqrt{54} \end{pmatrix}$$