TE_1_LDDR_niveau_1:Analyse

Exercice 1

- $\lim\limits_{x\to 2} \dfrac{5x(x-2)}{x^2-4} = \lim\limits_{x\to 2} \dfrac{5x\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \dfrac{10}{4} = \dfrac{5}{2}$

- $\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{2x-18} = \lim\limits_{x\to 9} \dfrac{\cancel{\sqrt{x}-3}}{2\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)} = \dfrac{1}{12}$

- $\lim\limits_{x\to \infty} \dfrac{5x^3-32x+3}{4x^3+10x-3} = \lim\limits_{x\to\infty} \dfrac{5x^3}{4x^3} = \dfrac{5}{4}$

Exercice 2 $f(x) = \dfrac{x^3+4x^2+5x+2}{x^2-4}$ $\qquad x^2-4=0 \Leftrightarrow x=\pm 2$

$D_f = \mathbb{R} \setminus \{\pm 2\}$

AV $\cdot \lim\limits_{x\to 2} f(x) = \left[\dfrac{36}{0}\right] = \infty \implies$ AV $x=2$

$\cdot \lim\limits_{x\to -2} f(x) = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to -2} \dfrac{\cancel{(x+2)}(x^2+2x+1)}{(x-2)\cancel{(x+2)}} = \dfrac{1}{-4}$

$$\begin{array}{cccc|c} 1 & 4 & 5 & 2 & -2 \\ \downarrow & -2 & -4 & -2 & \\ 1 & 2 & 1 & \boxed{0} & \end{array}$$

Trou$(-2;-\frac{1}{4})$

AH/AO

$$\begin{array}{l|l} x^3+4x^2+5x+2 & x^2-4 \\ \hline -x^3 \quad + \quad 4x & x+4 \\ \quad 4x^2+9x+2 & \\ \quad -4x^2 \quad +16 & \\ \hline \quad\quad 9x+18 & \end{array}$$

AO: $y = x+4$

Exercice 3 $f(x) = x-2x^3$

$f(x+h) = (x+h)-2(x+h)^3 = x+h-2x^3-6x^2h-6xh^2-2h^3$

$f(x+h)-f(x) = \cancel{x}+h-\cancel{2x^3}-6x^2h-6xh^2-2h^3-\cancel{x}+\cancel{2x^3}$

$\qquad = h(1-6x^2-6xh-2h^2)$

$\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{h}(1-6x^2-6xh-2h^2)}{\cancel{h}}$

$= 1-6x^2 = f'(x)$

EXERCICE 4

$f(x) = x\cos x \qquad f'(x) = \cos x - x\sin x$

$f(\pi) = -\pi \qquad f'(\pi) = -1$

$y = mx + h \rightarrow -\pi = -\pi + h \Rightarrow h = 0$

$t: \; y = -x + h$