EXERCICE 5.1

$$\begin{pmatrix}3\\-8\\m\end{pmatrix} \parallel \begin{pmatrix}n\\9,6\\p\end{pmatrix} \parallel \begin{pmatrix}-9\\q\\-15\end{pmatrix}$$
$$\quad \vec{a} \qquad\quad \vec{b} \qquad\quad \vec{c}$$

$\vec{a} \parallel \vec{b} \Rightarrow \exists\ t \in \mathbb{R} : \vec{a} = t \cdot \vec{b} \Rightarrow \begin{pmatrix}3\\-8\\m\end{pmatrix} = t\begin{pmatrix}n\\9,6\\p\end{pmatrix} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 3 = t \cdot n & \Rightarrow 3 = -0.833 \cdot n \Leftrightarrow n = -3.6 \\ -8 = t \times 9,6 & \Rightarrow t = -0.833 \\ m = t \cdot p & \Rightarrow m = -0.833 \cdot p \Rightarrow m = -0.833(-6) = +5 \end{cases}$

$\vec{b} \parallel \vec{c} \Rightarrow \begin{cases} n = k \cdot (-9) \Rightarrow -3,6 = -9k \Rightarrow k = +0,4 \\ 9,6 = k \cdot q \Rightarrow 9,6 = +0,4 \cdot q \Rightarrow q = +24 \\ p = k \cdot (-15) \Rightarrow p = +0,4(-15) \Leftrightarrow p = -6 \end{cases}$

EXERCICES 5.2

a) $\vec{a} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ $\vec{b} = \begin{pmatrix}2\\3\\4\end{pmatrix}$ $\vec{c} = \begin{pmatrix}2\\1\\3\end{pmatrix}$

On suppose que $\vec{c} = \lambda\vec{a} + \mu\vec{b} \Rightarrow$

$$\begin{pmatrix}2\\1\\3\end{pmatrix} = \lambda\begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}2\\3\\4\end{pmatrix} \Leftrightarrow \begin{cases} 2 = \lambda + 2\mu & (1) \\ 1 = \lambda + 3\mu & (2) \\ 3 = \lambda + 4\mu & (3) \end{cases}$$

(1) − (2) donne $1 = -\mu \Leftrightarrow \mu = -1$.

D'où (2) $1 = \lambda - 3 \Leftrightarrow \lambda = 4$.

On remplace en (3) $3 = 4 + (-4)$ impossible

Donc $\vec{c}$ n'est pas une combinaison linéaire de $\vec{a}$ et $\vec{c}$. Càd que n'est pas lin dép avec $\vec{a}$ et $\vec{b}$.

On doit aussi justifier que $\vec{a} \not\parallel \vec{b}$.

Soit $\vec{a} \parallel \vec{b} \Rightarrow \vec{a} = t \cdot \vec{b} \Leftrightarrow \begin{pmatrix}1\\1\\1\end{pmatrix} = t\begin{pmatrix}2\\3\\4\end{pmatrix} \Rightarrow \begin{cases} t = 1/2 \\ t = 1/3 \\ t = 1/4 \end{cases}$ impossible

Donc $\vec{a}, \vec{b}, \vec{c}$ sont lin. ind et ils forment une base de $\mathbb{R}^3$

b) $\vec{a} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} 7 \\ -2 \\ -11 \end{pmatrix}$

Soit $\vec{c} = \lambda\vec{a} + \mu\vec{b} \Rightarrow \left\{ \begin{pmatrix} 7 \\ -2 \\ -11 \end{pmatrix} = \lambda \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \right. \Leftrightarrow$

$\Leftrightarrow \begin{cases} 7 = -\lambda + 2\mu & (1) \\ -2 = 4\lambda + 5\mu & (2) \\ -11 = 3\lambda - \mu & (3) \end{cases}$

(1) + 2(3)

$$\begin{array}{rl} 7 = & -\lambda + 2\mu \\ + \; -22 = & 6\lambda - 2\mu \\ \hline -15 = & 5\lambda \Leftrightarrow \lambda = -3 \end{array}$$

(1): $7 = 3 + 2\mu \Leftrightarrow 4 = 2\mu \Leftrightarrow \mu = 2$

(2): $-2 = 4(-3) + 5\cdot 2$

$-2 = -12 + 10$ ✓

Donc $\vec{c}, \vec{a}, \vec{b}$ sont linéairement dépendants $\Rightarrow$ ils ne forment pas une base de $\mathbb{R}^3$

b. $\vec{a} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 4 \\ -5 \\ z \end{pmatrix}$ $\vec{c} = \begin{pmatrix} 2 \\ -4 \\ 15 \end{pmatrix}$

$\vec{c} = \lambda\vec{a} + \mu\vec{b} \Rightarrow \begin{pmatrix} 2 \\ -4 \\ 15 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -5 \\ z \end{pmatrix} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 2 = 2\lambda + 4\mu \\ -4 = -3\lambda - 5\mu \\ 15 = 5\lambda + z\mu \end{cases} \Leftrightarrow \begin{cases} 1 = \lambda + 2\mu & *(-3) \; (1) \\ 4 = 3\lambda + 5\mu & (2) \\ 15 = 5\lambda + z\mu & (3) \end{cases}$

(1), (2)

$$\begin{array}{rl} -3 = & -3\lambda - 6\mu \\ + \quad 4 = & 3\lambda + 5\mu \\ \hline 1 = & -\mu \Leftrightarrow \mu = -1 \end{array}$$

(1) $1 = \lambda - 2 \Leftrightarrow \lambda = 3$

On remplace en (3)

$15 = 5\cdot 3 + z\cdot(-1) \Leftrightarrow 15 = 15 - z \Leftrightarrow \underline{z = 0}$

c) $\vec{a} = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

$\vec{c} = \frac{1}{2}\vec{a} - \frac{1}{3}\vec{b} \Rightarrow c = \frac{1}{2}\begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1/2 \\ 5/2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -1/3 \\ 2/3 \end{pmatrix} =$

$\Rightarrow \vec{c} = \begin{pmatrix} 1/2 - 1 \\ \frac{5}{2} + \frac{1}{3} \\ 2 - \frac{2}{3} \end{pmatrix} \Rightarrow \vec{c} = \begin{pmatrix} -1/2 \\ 17/6 \\ 4/3 \end{pmatrix}$

$\vec{d} = -5\vec{a} + 7\vec{b} \Rightarrow \vec{d} = \begin{pmatrix} -5 \\ -25 \\ -20 \end{pmatrix} + \begin{pmatrix} 21 \\ -7 \\ 14 \end{pmatrix} \Rightarrow \vec{d} = \begin{pmatrix} 16 \\ -32 \\ -6 \end{pmatrix}$

$2\vec{a} - 3\vec{b} - 5\vec{e} = 0 \Leftrightarrow 2\vec{a} - 3\vec{b} = 5\vec{e} \Leftrightarrow$

$\Leftrightarrow \frac{1}{5}(2\vec{a} - 3\vec{b}) = e \Leftrightarrow \vec{e} = \frac{1}{5}\left[\begin{pmatrix} 2 \\ 10 \\ 8 \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix}\right] = \frac{1}{5}\begin{pmatrix} -7 \\ 13 \\ 2 \end{pmatrix} \Rightarrow$

$\Rightarrow \vec{e} = \begin{pmatrix} -7/5 \\ 13/5 \\ 2/5 \end{pmatrix}$

EXERCICE 5.3

A	$(1;-2;5)$	$(-5;\frac{1}{2};6)$	$(4;-5;-13)$ ③	$(0;-1;6)$	$(4;\ldots;\frac{1}{4})$
B	$(5..;1..;4)$ ①	$(4;1;1/3)$	$(6;2;-4)$	$(a,3;b,)$ ④	$(\ldots;\frac{1}{2};\frac{1}{2})$ ⑥
$\vec{AB}$	$\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \\ \ldots \end{pmatrix}$ ②	$\begin{pmatrix} 2 \\ 7 \\ 9 \end{pmatrix}$	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} // \begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix}$ ⑤	$\begin{pmatrix} 0 \\ 5 \\ \ldots \end{pmatrix}$

① $\vec{AB} = \vec{OB} - \vec{OA} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \Leftrightarrow \begin{cases} 4 = x - 1 \\ 3 = y + 2 \\ -1 = z - 5 \end{cases} \Leftrightarrow \begin{cases} x = 5 \\ y = 1 \\ z = 4 \end{cases}$

② $\vec{AB} = \begin{pmatrix} 4 \\ 1 \\ 1/3 \end{pmatrix} - \begin{pmatrix} -5 \\ 1/2 \\ 6 \end{pmatrix} \Rightarrow \vec{AB} = \begin{pmatrix} 9 \\ 1/2 \\ -17/3 \end{pmatrix}$

③ $\vec{AB} = \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix} - \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2 \\ 7 \\ 9 \end{pmatrix} = \begin{pmatrix} 6-x \\ 2-y \\ -4-z \end{pmatrix} \Leftrightarrow \begin{cases} 2 = 6 - x \\ 7 = 2 - y \\ 9 = -4 - z \end{cases} \Leftrightarrow \begin{cases} x = 4 \\ y = -5 \\ z = -13 \end{cases}$

④,⑤ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = t\begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix} \Rightarrow \begin{matrix} x = 5t \\ y = -8t \\ z = 2t \end{matrix}$ et $\begin{pmatrix} 5t \\ -8t \\ 2t \end{pmatrix} = \begin{pmatrix} a \\ 3 \\ b \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix} \Rightarrow \begin{cases} 5t = a \\ -8t = 3+1 \\ 2t = b - 6 \end{cases}$

$-8t = 4 \Leftrightarrow t = -1/2 \Rightarrow -5\frac{1}{2} = a \Rightarrow a = -5/2$ $\quad \frac{-2}{2} = b - 6 \Leftrightarrow b = 5$

⑥ $(4\,;\,a\,;\,\tfrac{1}{4})$ $(b\,;\,\tfrac{1}{2}\,;\,\tfrac{1}{2})$ $(0\,;\,5\,;\,c)$

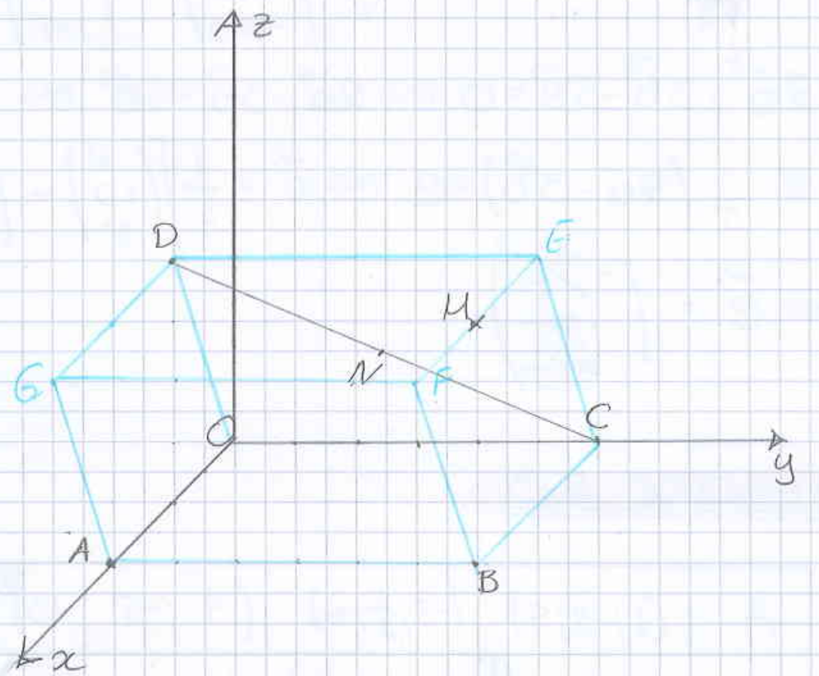
$\quad\; \| \qquad\qquad\quad \| \qquad\qquad\quad \|$

$\quad\; A \qquad\qquad\quad B \qquad\qquad\quad \vec{AB}$

$$\begin{pmatrix}0\\5\\c\end{pmatrix} = \begin{pmatrix}b\\ \frac{1}{2}\\ \frac{1}{2}\end{pmatrix} - \begin{pmatrix}4\\a\\ \frac{1}{4}\end{pmatrix} \Leftrightarrow \begin{cases} 0 = b-4 \\ 5 = \frac{1}{2} - a \\ c = \frac{1}{2} - \frac{1}{4}\end{cases} \Leftrightarrow \begin{cases} b = 4 \\ a = -\frac{9}{2} \\ c = \frac{1}{4}\end{cases}$$

EXERCICE 5.4

$O(0;0;0)$ $A(4;0;0)$ $B(4;6;0)$ $C(0;6;0)$ $D(2;0;4)$



$$\vec{OC} = \vec{DE} \Rightarrow \begin{pmatrix}0\\6\\0\end{pmatrix} = \begin{pmatrix}x_E-2\\y_E-0\\z_E-4\end{pmatrix} \Leftrightarrow \begin{cases} x_E = 2\\ y_E = 6\\ z_E = 4\end{cases} \qquad \underline{E(2;6;4)}$$

$$\vec{EF} = \vec{CB} \Rightarrow \begin{pmatrix}x_F-2\\y_F-6\\z_F-4\end{pmatrix} = \begin{pmatrix}4\\0\\0\end{pmatrix} \Leftrightarrow \begin{cases} x_F = 6\\ y_F = 6\\ z_F = 4\end{cases} \qquad \underline{F(6;6;4)}$$

$$\vec{DG} = \vec{OA} \Rightarrow \begin{pmatrix}x_G-2\\y_G-0\\z_G=4\end{pmatrix} = \begin{pmatrix}4\\0\\0\end{pmatrix} \Leftrightarrow \begin{cases} x_G = 6\\ y_G = 0\\ z_G = 4\end{cases} \qquad \underline{G(6;0;4)}$$

M: $x_M = \dfrac{x_E + x_F}{2} \Rightarrow x_M = \dfrac{2+6}{2} \Rightarrow x_M = 4$

$y_M = \dfrac{y_E + y_F}{2} \Rightarrow y_M = \dfrac{6+6}{2} \Rightarrow y_M = 6 \qquad \underline{M(4;6;4)}$

$z_M = \dfrac{z_E + z_F}{2} \Rightarrow z_M = \dfrac{4+4}{2} \Rightarrow z_M = 4$

$\underline{\underline{N}}$: $x_N = \frac{x_D + x_C}{2} \Rightarrow x_N = \frac{0+2}{2} \Rightarrow x_N = 1$

$y_N = \frac{y_D + y_C}{2} \Rightarrow y_N = \frac{6+0}{2} \Rightarrow y_N = 3$ $\qquad N = (1;3;2)$

$z_N = \frac{z_D + z_C}{2} \Rightarrow z_N = \frac{4+0}{2} \Rightarrow z_N = 2$

DEFG rectangle

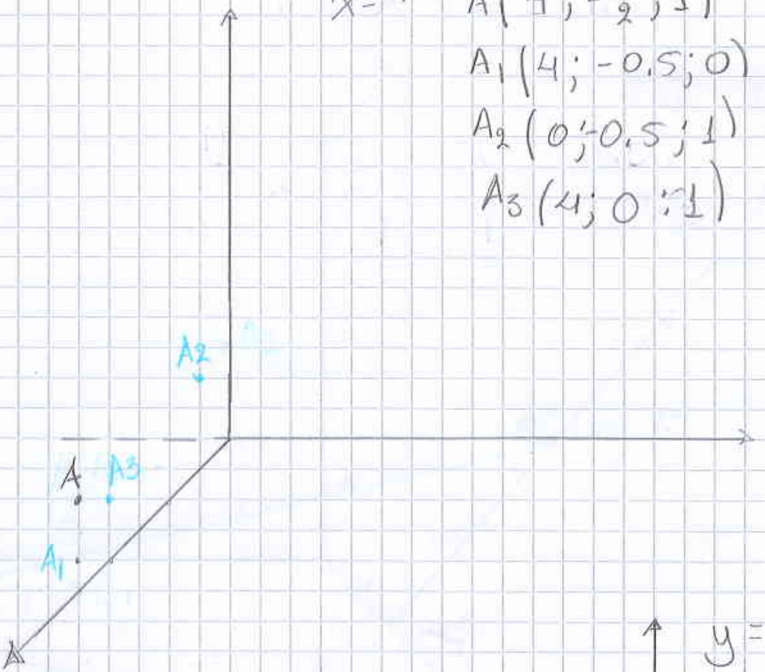
EXERCICE 5.5

$x = 4$ $\quad A(4; -\frac{1}{2}; 1)$

$A_1(4; -0.5; 0)$

$A_2(0; -0.5; 1)$

$A_3(4; 0; 1)$

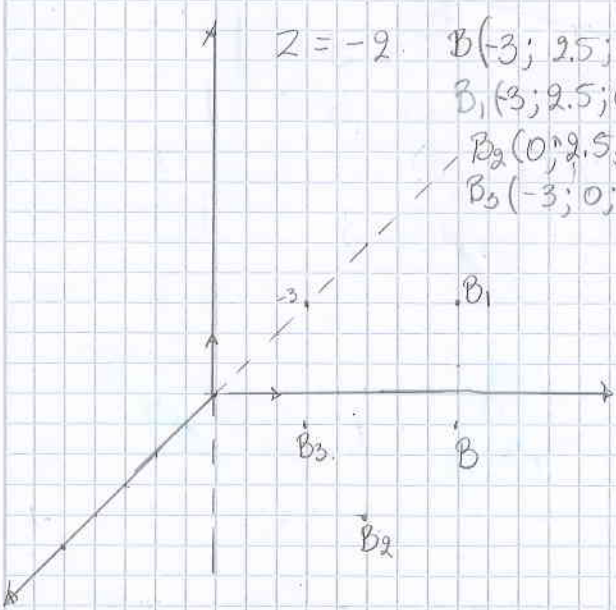


$z = -2$ $\quad B(-3; 2.5; -2)$

$B_1(-3; 2.5; 0)$

$B_2(0; 2.5; -2)$

$B_3(-3; 0; -2)$



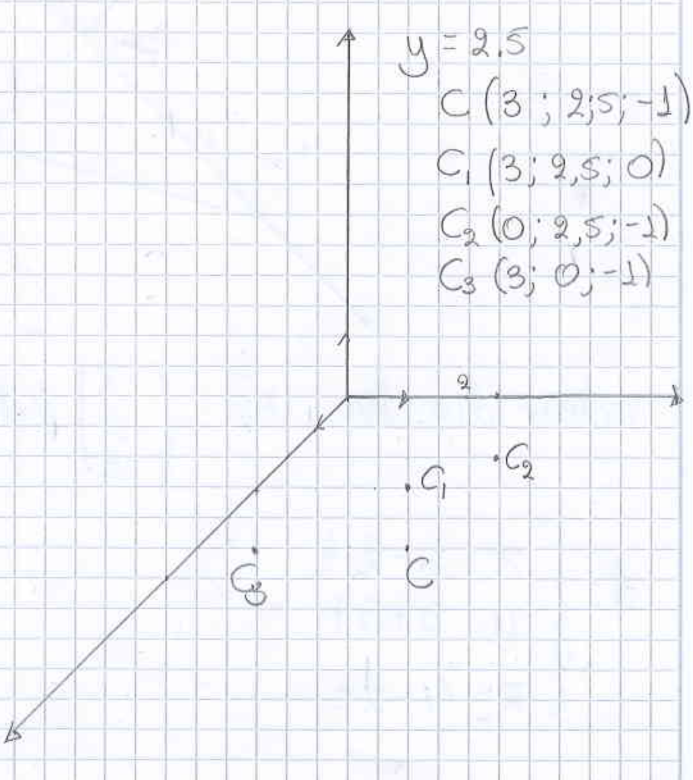
$y = 2.5$

$C(3; 2.5; -1)$

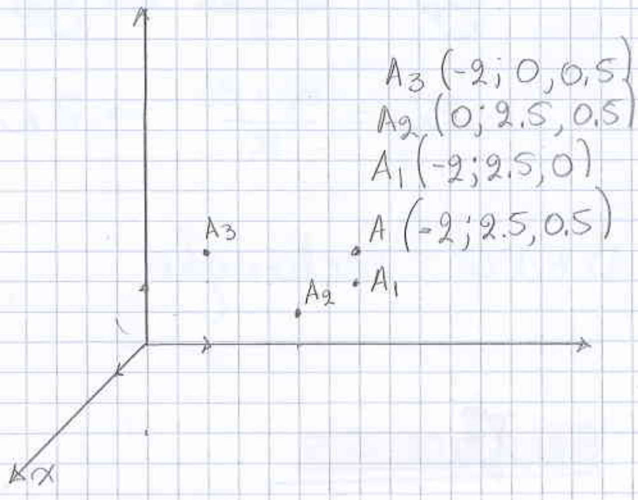
$C_1(3; 2.5; 0)$

$C_2(0; 2.5; -1)$

$C_3(3; 0; -1)$

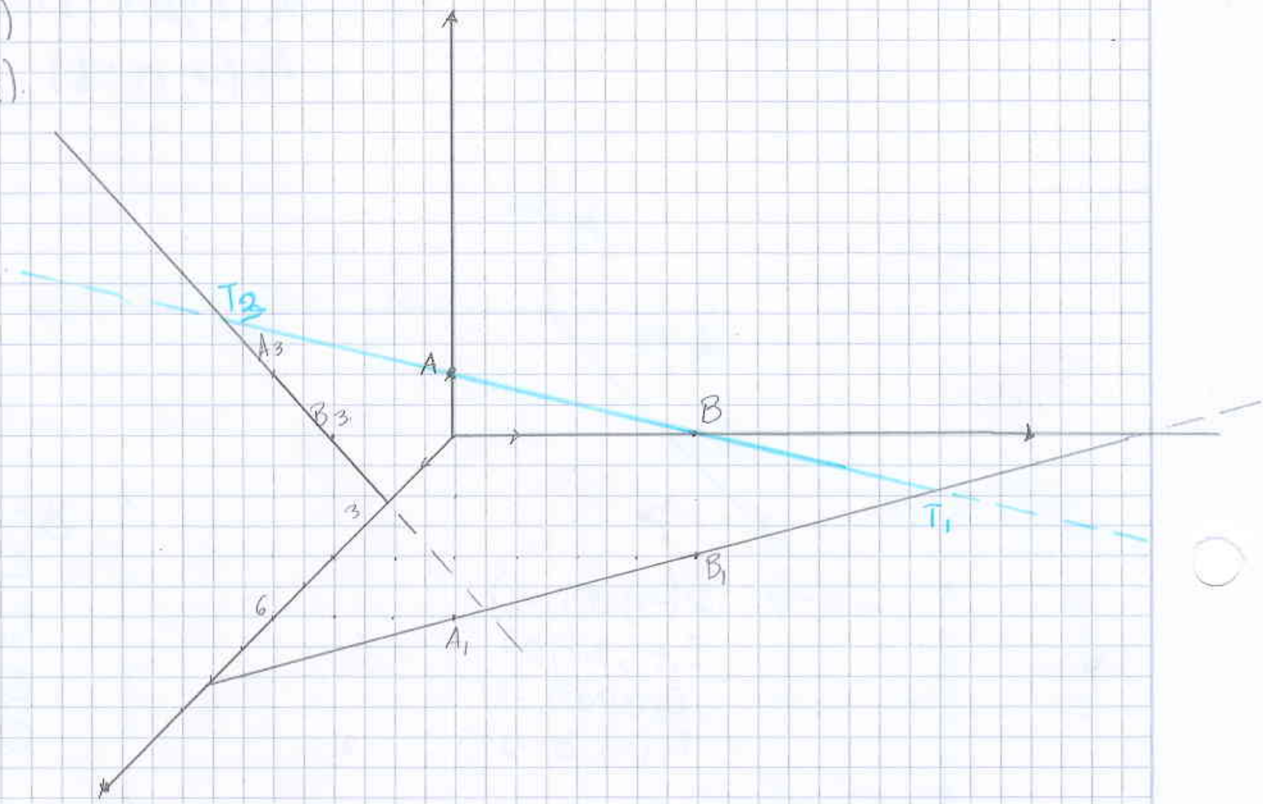


EXERCICE 5.6



EXERCICE 5.7

$A(6;3;4)$
$B(4;6;2)$

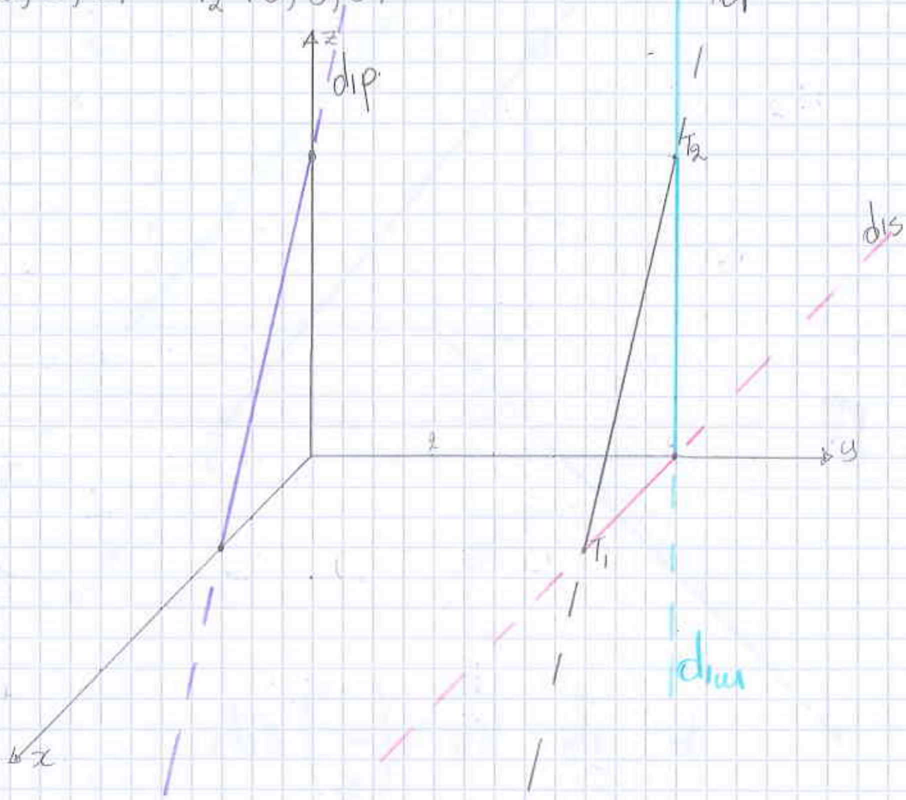


vecteur directeur $\vec{AB} = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix} // d$

$$d: \begin{cases} x = 6 - 2t \\ y = 3 + 3t \\ z = 4 - 2t \end{cases}$$

EXERCICE 5.8

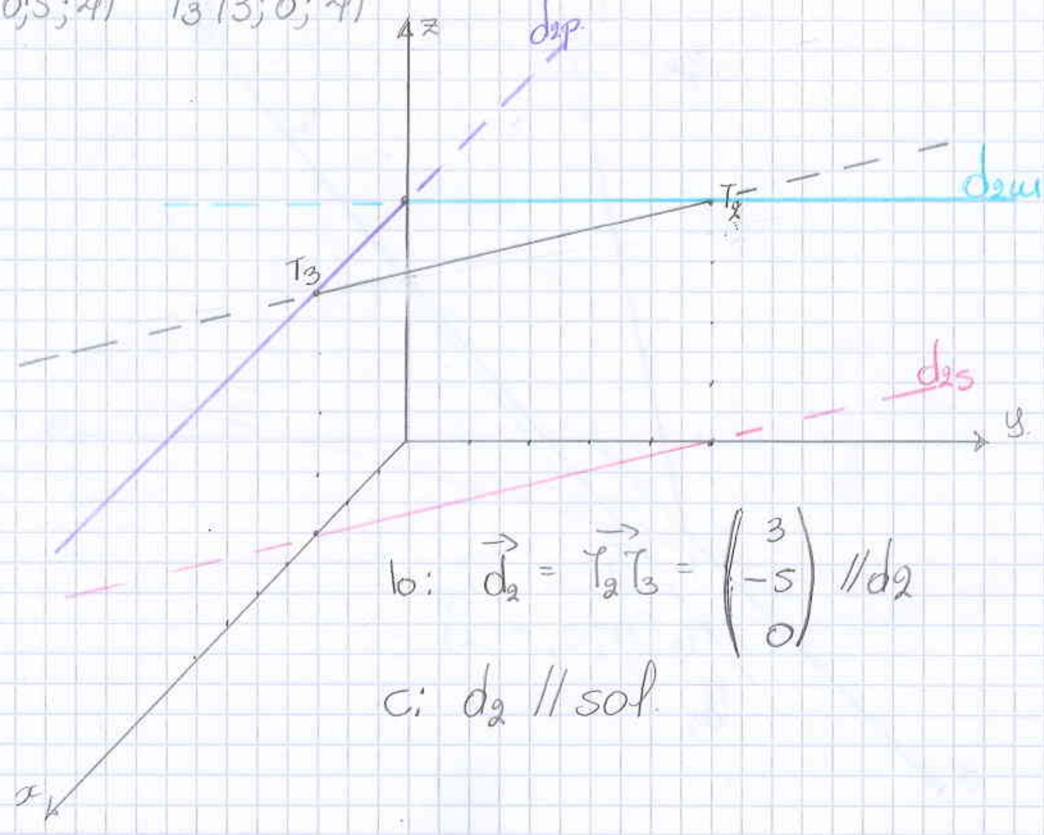
a) $d_1 : T_1 (3;6;0) \quad T_2 (0;6;5)$



b): vecteur $// \vec{d_1} = \overrightarrow{T_1 T_2} = \begin{pmatrix} -3 \\ 0 \\ 5 \end{pmatrix}$

c) $d_1 // Oz$ (paroi)

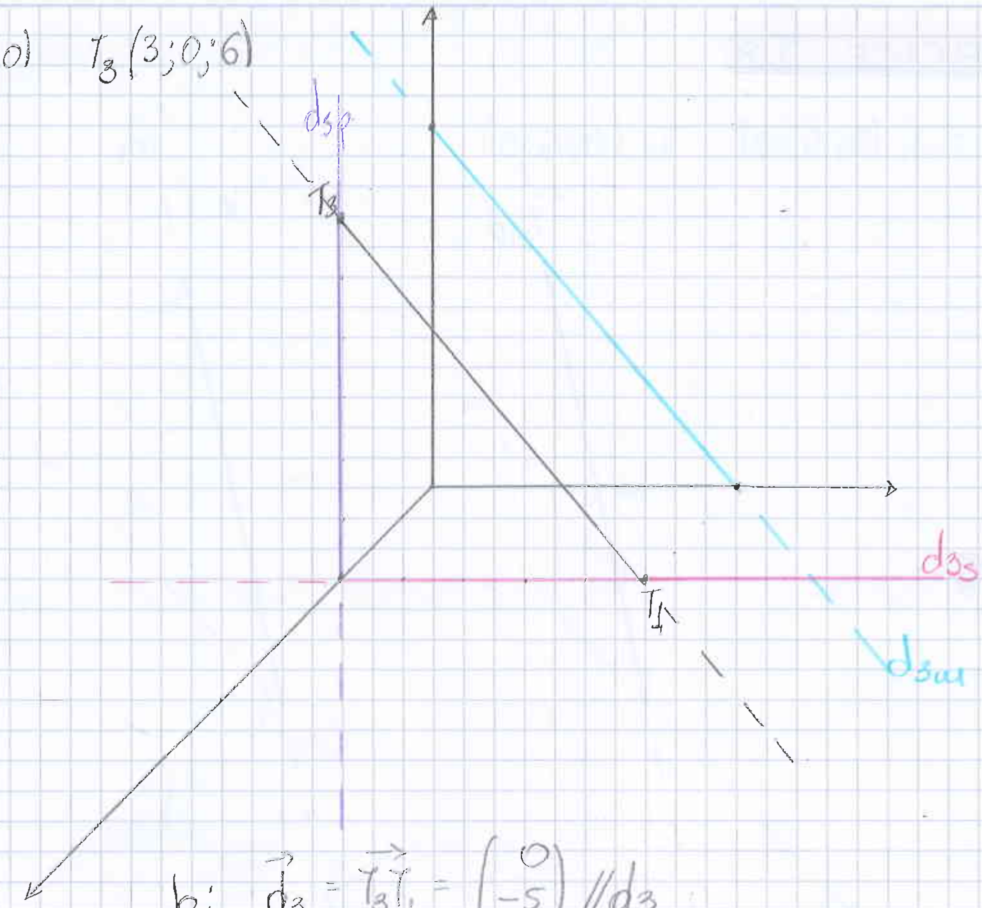
$T_2 : (0;5;4) \quad T_3 (3;0;4)$



b: $\vec{d_2} = \overrightarrow{T_2 T_3} = \begin{pmatrix} 3 \\ -5 \\ 0 \end{pmatrix} // d_2$

c: d_2 // sol

▶ $T_1(3;5;0)$ $T_3(3;0;6)$



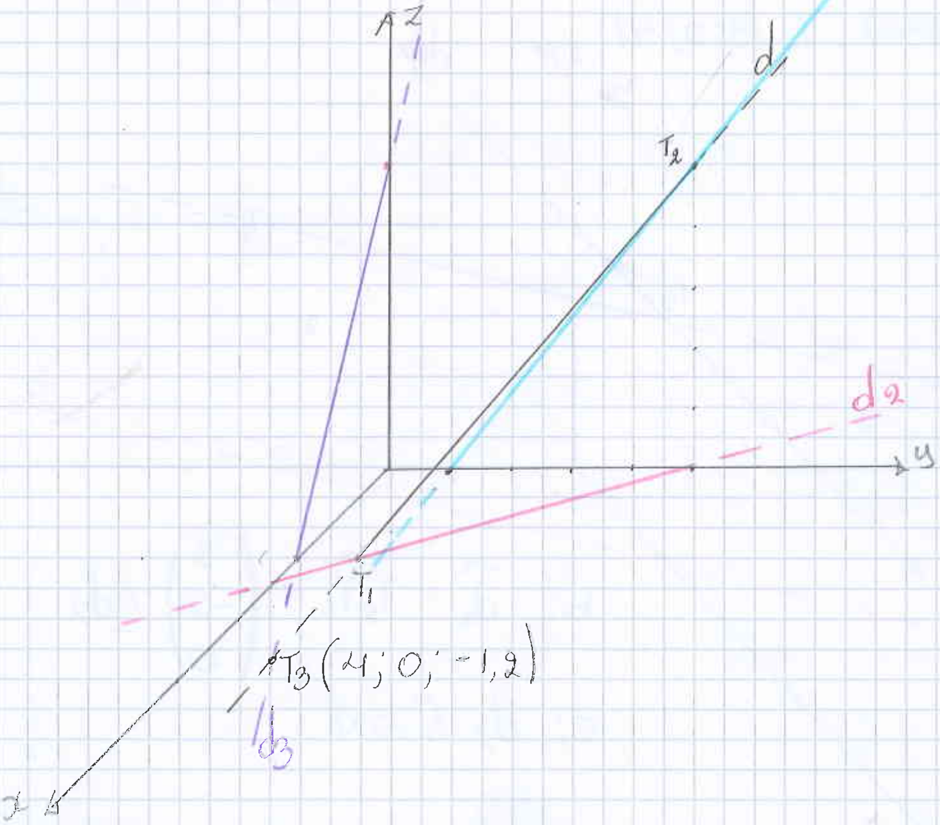
b: $\vec{d_3} = \overrightarrow{T_3T_1} = \begin{pmatrix} 0 \\ -5 \\ 6 \end{pmatrix} // d_3$

c: d_3 // mur.

EXERCICE 5.9

$T_1(3;1;0)$ $T_2(0;5;4)$

a.



c. $\overrightarrow{T_2T_1} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix} // d$

$$d = \begin{cases} x = 3 + 3t \\ y = 1 - 4t \\ z = 0 - 4t \end{cases}$$

T_3 ? $\quad y = 0 \Leftrightarrow 1 - 4t = 0 \Leftrightarrow t = 1/4$

$x = 3 + 3 \cdot \frac{1}{4} \Rightarrow x = \frac{15}{4} = 3,75$

$y = 0$

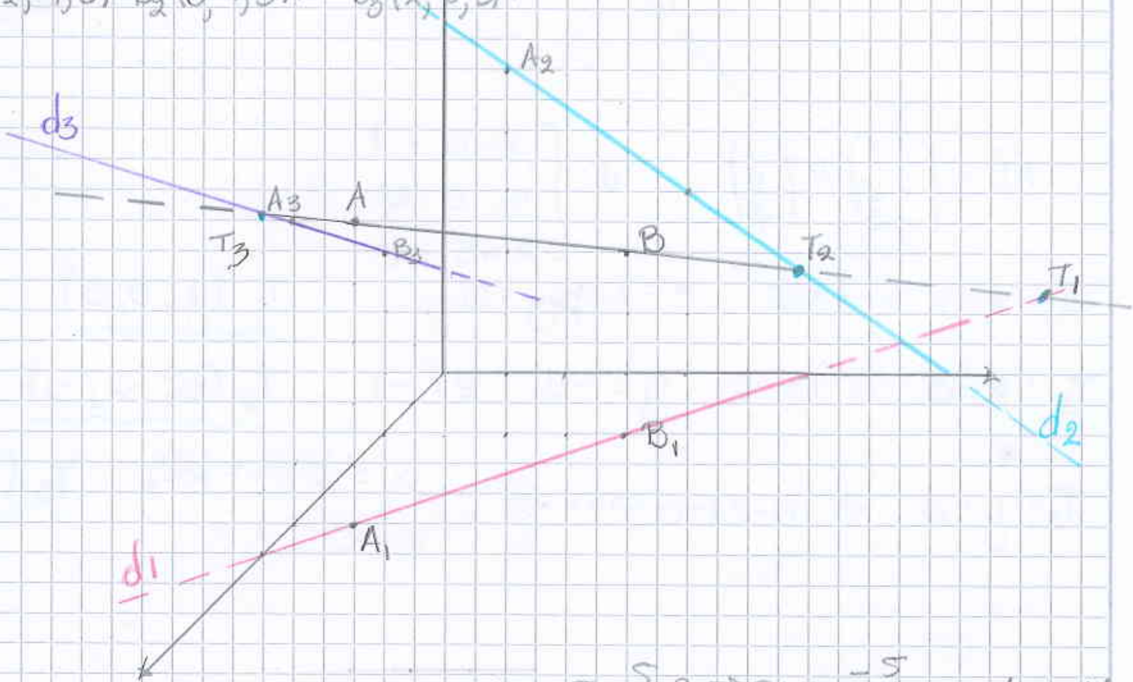
$z = -4 \cdot \frac{1}{4} = -1 \qquad$ Donc $T_3 = (3,75 ; 0 ; -1)$

EXERCICE 5.10

Cas a :

$A(5;1;5)$ $\quad A_1(5;1;0)$ $\quad A_2(0;1;5)$ $\quad A_3(5;0;5)$

$B(2;4;3)$ $\quad B_1(2;4;0)$ $\quad B_2(0;4;3)$ $\quad B_3(2;0;3)$



$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$

$$d : \begin{cases} x = 5 - 3t \\ y = 1 + 3t \\ z = 5 - 2t \end{cases}$$

T_1 : $z = 0 \Leftrightarrow t = \frac{5}{2}$ $\quad x = 5 - \frac{5}{2} \cdot 3 \Rightarrow x = -\frac{5}{2}$ $\quad y = 1 + 3 \cdot \frac{5}{2} \Rightarrow y = \frac{17}{2}$ $\quad T_1\left(-\frac{5}{2} ; \frac{17}{2} ; 0\right)$

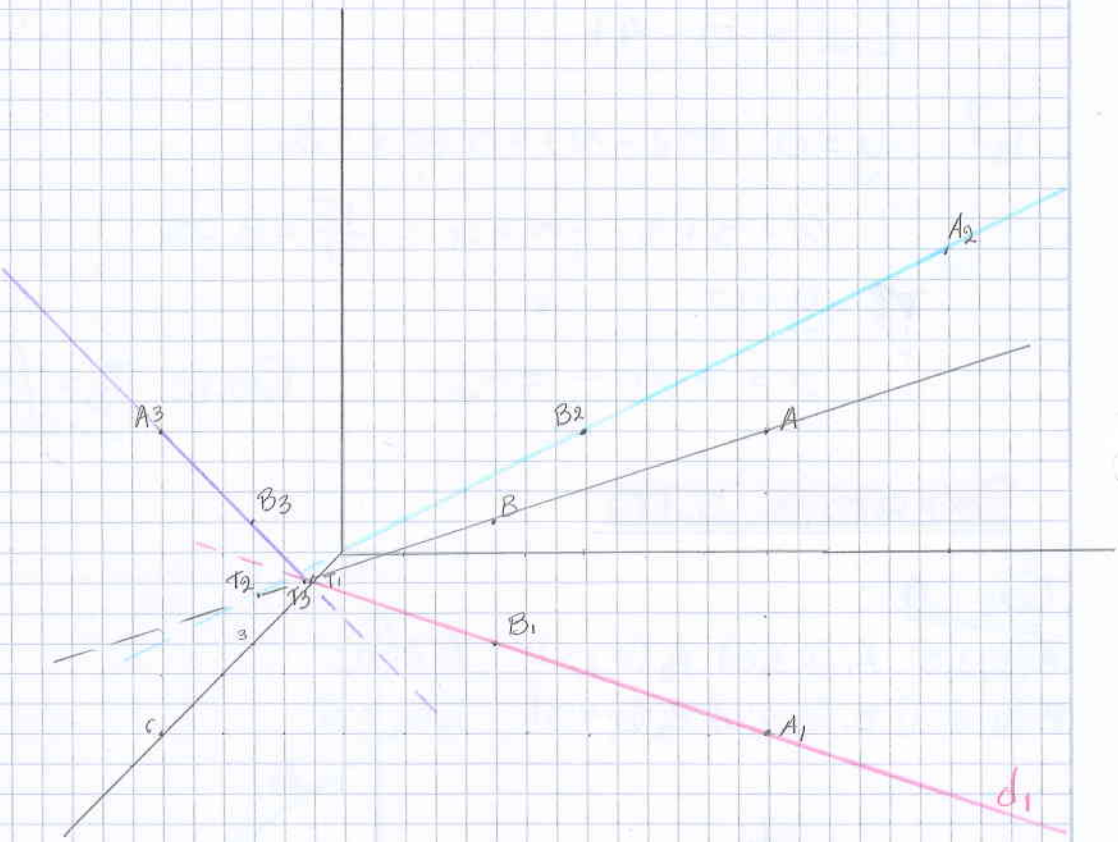
T_3 : $y = 0 \Leftrightarrow t = -1/3$ $\quad x = 5 + 3\frac{1}{3} \Rightarrow x = 6$ $\quad z = 5 + 2\frac{1}{3} \Rightarrow z = \frac{17}{3}$ $\quad T_3\left(6 ; 0 ; \frac{17}{3}\right)$

T_2 : $x = 0 \Leftrightarrow t = 5/3$ $\quad y = 1 + 3 \cdot \frac{5}{3} \Rightarrow y = 6$ $\quad z = 5 - 2 \cdot \frac{5}{3} \Rightarrow z = \frac{5}{3}$ $\quad T_2\left(0 ; 6 ; \frac{5}{3}\right)$

▶ Cas 6

$A(6;10;5)$ $A_1(6;10;0)$ $A_2(0;10;5)$ $A_3(6;0;5)$

$B(3;4;2)$ $B_1(3;4;0)$ $B_2(0;4;2)$ $B_3(3;0;2)$



$\vec{AB} = \begin{pmatrix}-3\\-6\\-3\end{pmatrix} \parallel \begin{pmatrix}1\\2\\1\end{pmatrix}$ $\quad d: \begin{cases} x = 6 + t \\ y = 10 + 2t \\ z = 5 + t \end{cases}$

$T_1 : z = 0 \Rightarrow t = -5 \quad x = 1 \quad y = 0 \qquad T_1\ (1;0;0)$

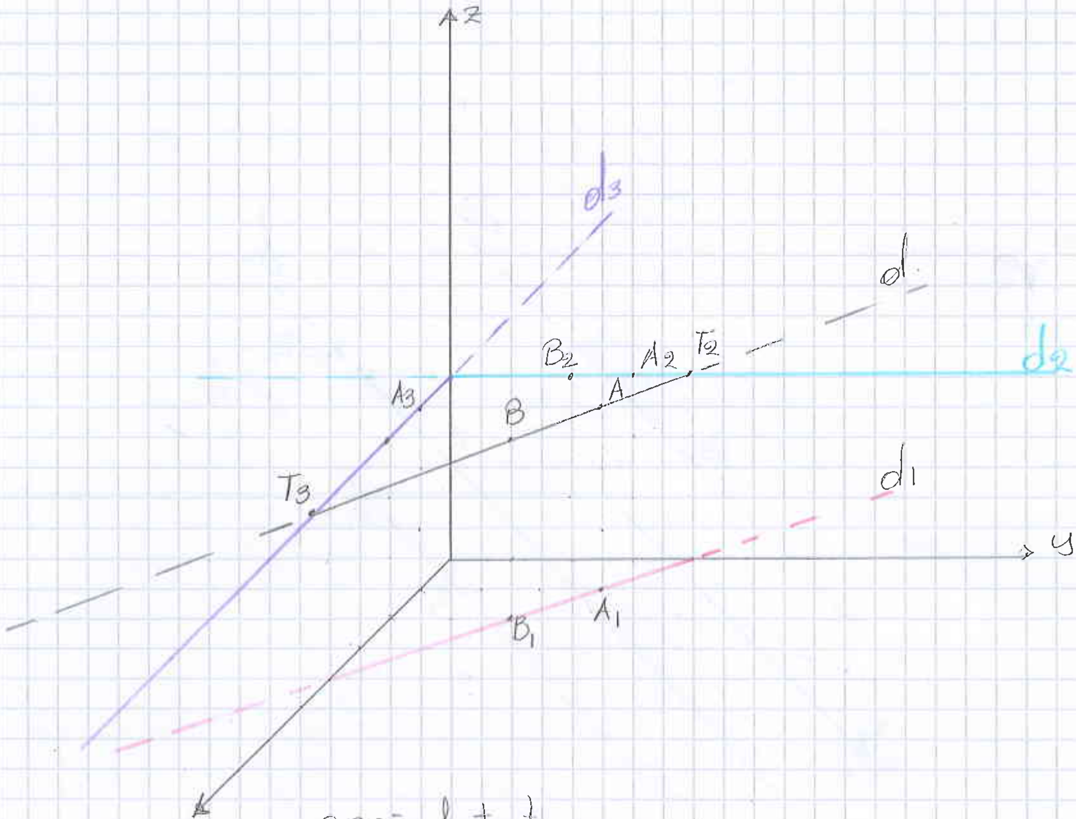
$T_2 : x = 0 \Rightarrow t = -6 \quad y = -2 \quad z = -1 \qquad T_2\ (0;-2;-1)$

$T_3 : y = 0 \Rightarrow 10+2t = 0 \Leftrightarrow t = -5 \quad x = 6-5 \Rightarrow x = 1 \quad z = 0 \qquad T_3\ (1;0;0)$

▶ Cas C.

A (1;3;3) A_1 (1;3;0) A_2 (0;3;3) A_3 (1;0;3)

B (2;2;3) B_1 (2;2;0) B_2 (0;2;3) B_3 (2;0;3)



$\vec{AB} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ $d: \begin{cases} x = 1 + t \\ y = 3 - t \\ z = 3 \end{cases}$

T_1: $z = 0$ Impossible $\Rightarrow \nexists\ T_1$

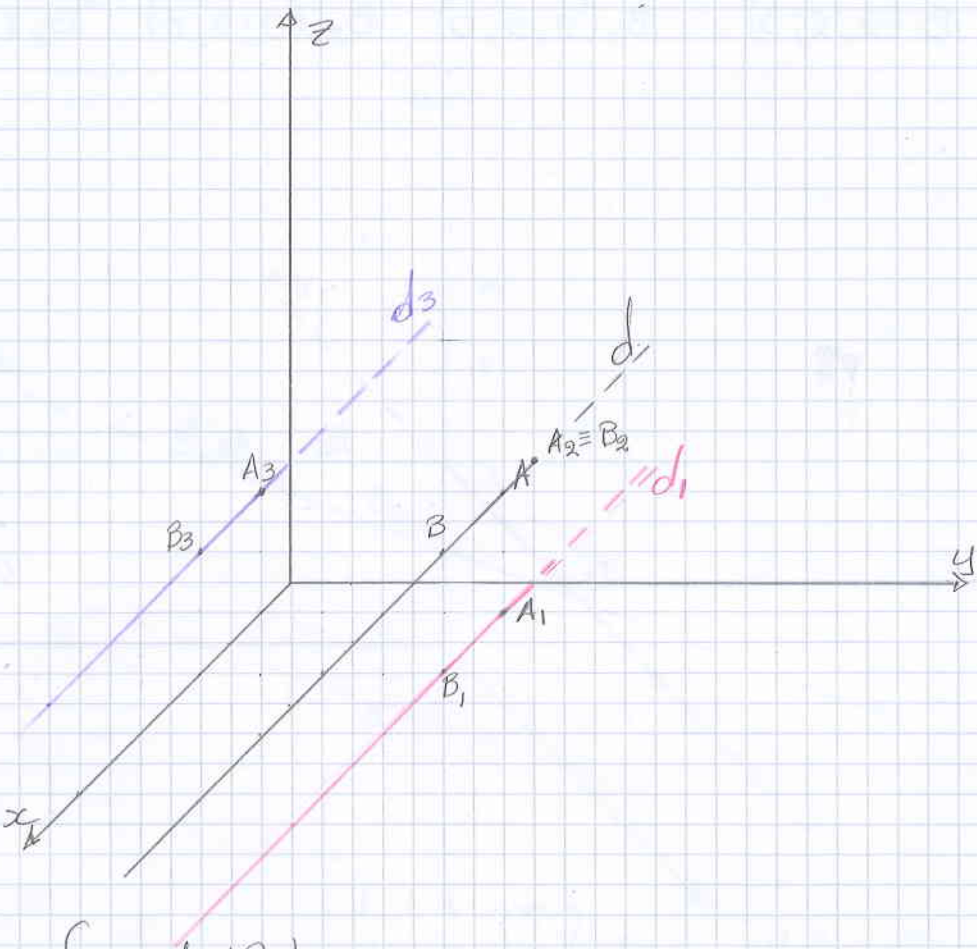
T_2: $x = 0 \Rightarrow t = -1$ $y = 4$, $z = 3$ $T_2 (0;4;3)$

T_3: $y = 0 \Rightarrow t = 3$ $x = 4$, $z = 3$ $T_3 (4;0;3)$

Cas d:

$A(1;4;2)$ $A_1(1;4;0)$ $A_2(0;4;2)$ $A_3(1;0;2)$

$B(3;4;2)$ $B_1(3;4;0)$ $B_2(0;4;2)$ $B_3(3;0;2)$



$\vec{AB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ $d: \begin{cases} x = 1 + 2t \\ y = 4 \\ z = 2 \end{cases}$

T_1: $z = 0$ Impossible

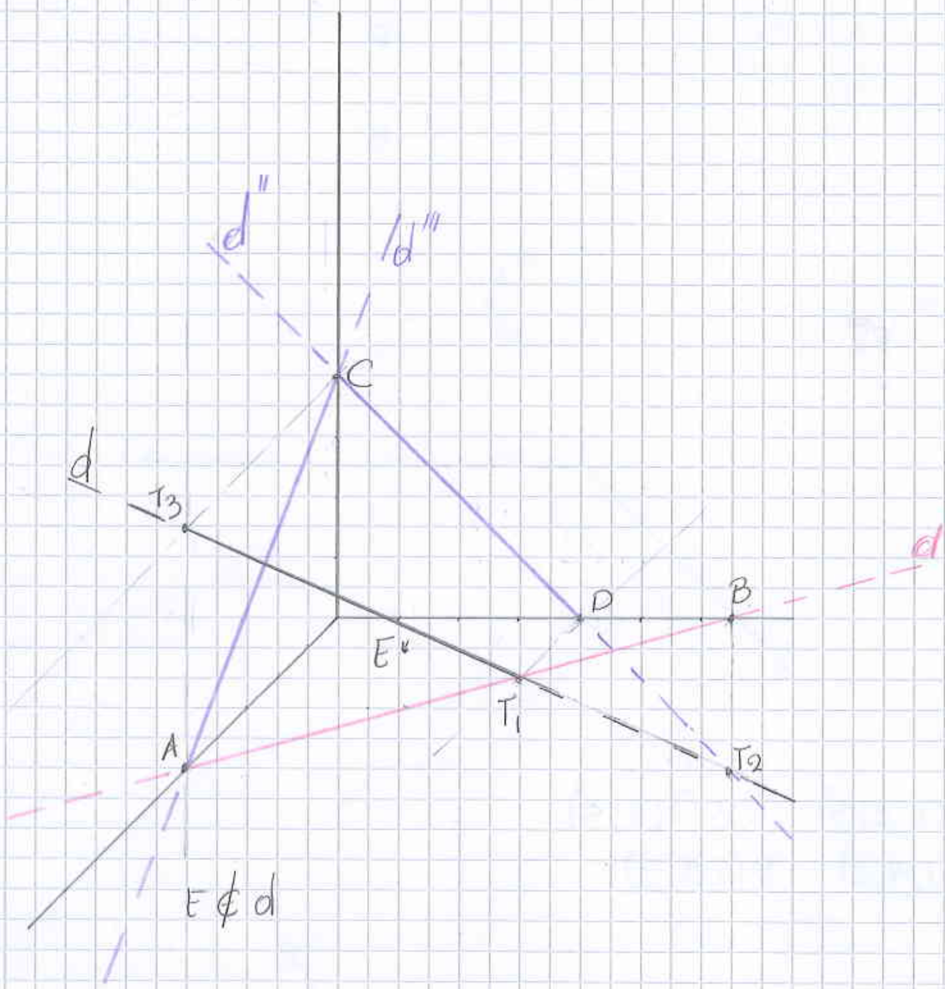
T_2: $x = 0 \Leftrightarrow t = -\frac{1}{2}$ $T_2(0;4;2) = A_2 \equiv B_2$

T_3: $y = 0$ Impossible

EXERCICE 5.11

d' $A(5;0;0)$ $B(0;6,5;0)$

d'' $C(0;0;4)$ $D(0;4;0)$



EXERCICE 5.12

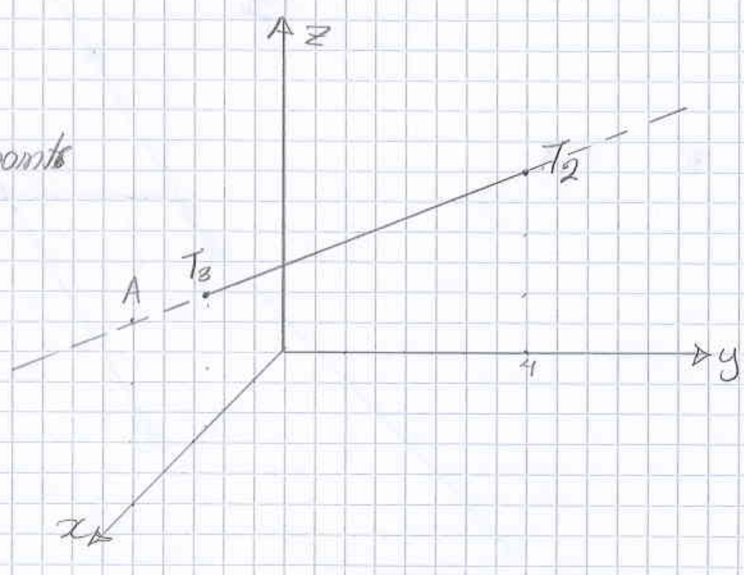
$A(3;-1;2)$

d // sol => $z=2$ pour tout points

$T_2(0;4;2)$

$\overrightarrow{AA_2} = \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix}$

$d = \begin{cases} x = 3 - 3t \\ y = -1 + 5t \\ z = 2 \end{cases}$



Facultatif:

T_1: Impossible

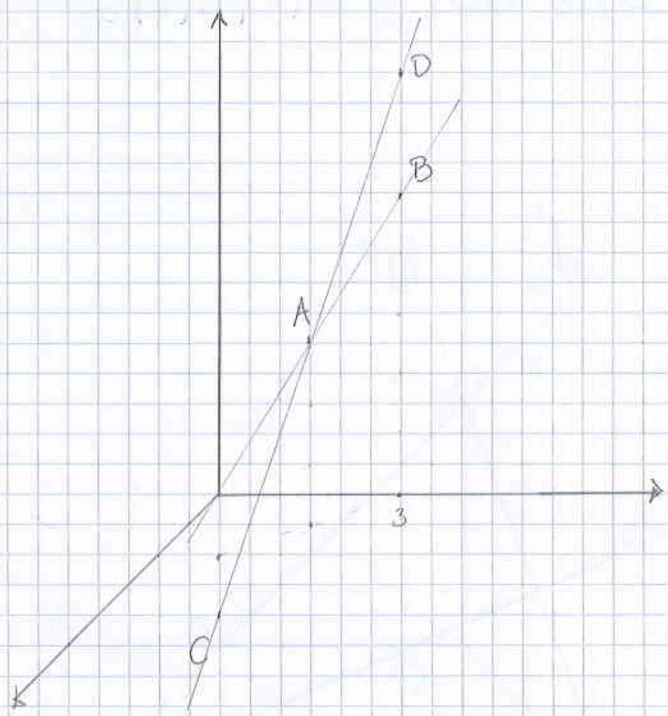
T_2: $(0;4;2)$

T_3: $y=0$ → $t=\frac{1}{5}$ $x = 3 - \frac{3}{5} = \frac{12}{5} = x$ $z=2$

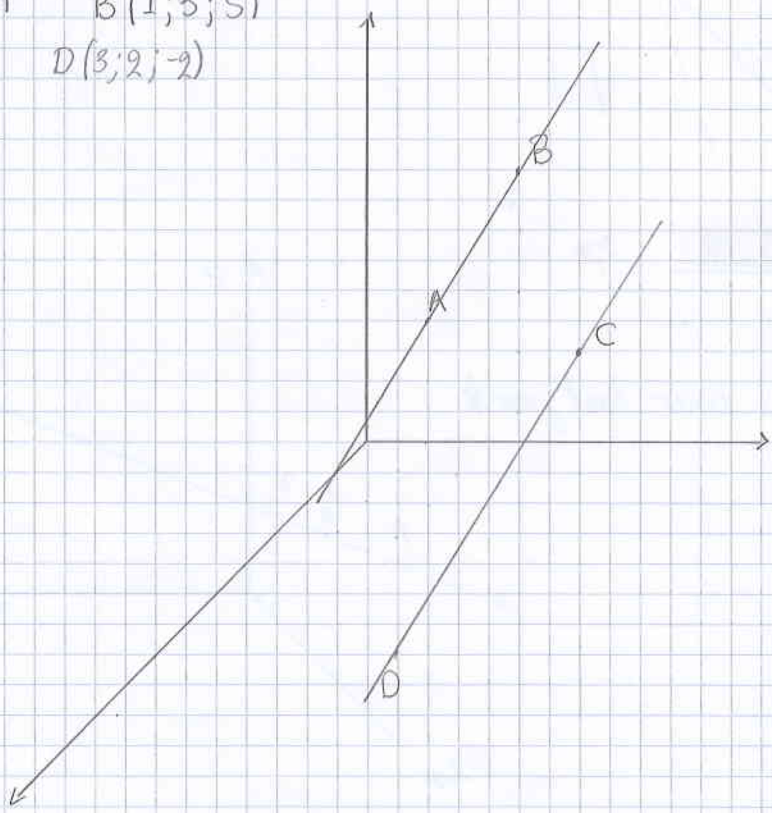
$T_3\left(\frac{12}{5};0;2\right)$

EXERCICE 5.14

a. d_1 : $A(1;2;3)$ $B(2;4;6)$
d_2 : $C(2;1;-1)$ $D(0;3;7)$



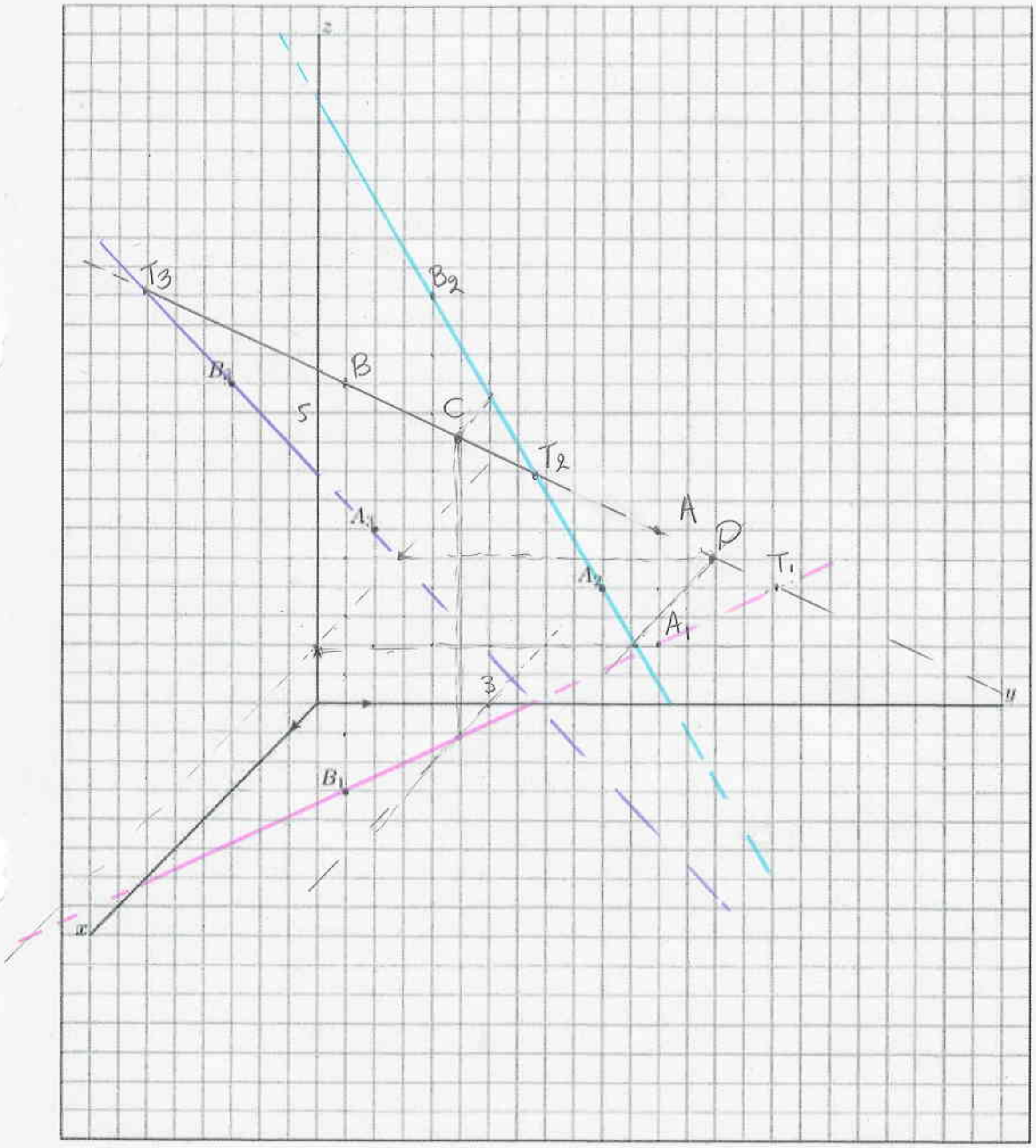
b. d_1 $A(2;2;3)$ $B(1;3;5)$
d_2 $C(1;4;2)$ $D(3;2;-2)$



$A_2(0;5;2)$ $A_3(-2;0;2) \Rightarrow A_1(-2;5;0)$ et $A(-2;5;2)$
$B_1(3;2;0)$ $B_3(3;0;7) \Rightarrow B_2(0;2;7) \Rightarrow B(3;2;7)$

Exercice 5.13

Construire A_1, B_2, A et B. Dessiner la droite d passant par A et B et ses projections.
Construire les deux points de d suivants : C d'ordonnée 3 et D de cote 1.



Exercice 5.14

On donne deux droites d_1 et d_2 par deux points.
Déterminer géométriquement la position relative de ces deux droites :

a. $d_1 : A(1;2;3)$ et $B(2;4;6)$ $\quad d_2 : C(2;1;-1)$ et $D(0;3;7)$

b. $d_1 : A(2;2;3)$ et $B(1;3;5)$ $\quad d_2 : C(1;4;2)$ et $D(3;2;-2)$

EXERCICE 5.15

a) $d_1 : A(6;3;0)$ $\vec{a} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ $d_1 \begin{cases} x = 6 - t \\ y = 3 + t \\ z = t \end{cases}$

$d_2 : B(0;0;4)$ $\vec{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ $d_2 : \begin{cases} x = k \\ y = k \\ z = 4 - k \end{cases}$

$\begin{cases} 6 - t = k & ① \\ 3 + t = k & ② \\ t = 4 - k & ③ \end{cases}$ $① + ② \Rightarrow 9 = 2k \Rightarrow k = 4{,}5$

$① \Rightarrow 6 - t = 4{,}5 \Leftrightarrow t = 1{,}5$

On remplace en ③

$1{,}5 = 4 - 4{,}5$ ↯

Donc il n'y a pas de points communs

$\Rightarrow$ $d_1 // d_2$ / gauches

$\vec{a} \not\parallel \vec{b} \Rightarrow d_1, d_2$ gauches

b) $d_1 : A(-3;-1;2)$ $\vec{a} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ $d_1 = \begin{cases} x = -3 + 2t \\ y = -1 + t \\ z = 2 - t \end{cases}$

$d_2 : B(4;-1;0)$ $\vec{b} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} // \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ $d_2 = \begin{cases} x = 4 + 2k \\ y = -1 + k \\ z = -k \end{cases}$

$\begin{cases} -3 + 2t = 4 + 2k \\ -1 + t = -1 + k \\ 2 - t = k \end{cases} \Leftrightarrow \begin{cases} 2t = 2k + 7 \\ t = k \\ 2 - t = k \end{cases} \Rightarrow ①\ 2t = 2t + 7 \Rightarrow$

$0 = 7$ ↯

$\Rightarrow$ gauches / $d_1 // d_2$

Mais $\vec{a} // \vec{b}$ en particulier $\vec{b} = 2\vec{a} \Rightarrow d_1 // d_2$

c) $d_1 : A(1,4;1)$ $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $d_1 \begin{cases} x = 1 + 2t \\ y = 4 \\ z = 1 + t \end{cases}$

$d_2 : B(5;-1;0)$ $\vec{b} = \begin{pmatrix} -2 \\ 5 \\ 2 \end{pmatrix}$ $d_2 = \begin{cases} x = 5 - 2k \\ y = -1 + 5k \\ z = 2k \end{cases}$

$$\begin{cases} 1+2t = 5-2k \\ 4 = -1+5k \\ 1+t = 2k \end{cases} \Leftrightarrow \begin{cases} 2t = 4-2k \\ 5 = 5k \\ t = 2k-1 \end{cases} \Leftrightarrow \begin{cases} t = 2-k \\ k = 1 \\ t = 2k-1 \end{cases} \Rightarrow$$

$$\Leftrightarrow \begin{cases} t = 1 \\ k = 1 \\ t = 1 \end{cases}$$ ✓ Donc le point $(3;4;2)$ est le point commun

$\Rightarrow$ sécantes

d) $d_1 : A(2;-1;-4)$ $\vec{a} = \begin{pmatrix} -4 \\ -2 \\ 2 \end{pmatrix}$ $d_1 \begin{cases} x = 2-4t \\ y = -1-2t \\ z = -4+2t \end{cases}$

$d_2 : B(4;0;-5)$ $\vec{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ $d_2 = \begin{cases} x = 4+2k \\ y = k \\ z = -5-k \end{cases}$

$$\left.\begin{matrix} 2-4t = 4+2k \\ -1-2t = k \\ -4+2t = -5-k \end{matrix}\right\} \Leftrightarrow \begin{matrix} -4t = 2+2k \\ k = -1-2t \\ k = -1-2t \end{matrix} \Leftrightarrow \begin{cases} -2k = -1-2t \\ k = -1-2t \\ k = -1-2t \end{cases}$$

Donc on a une infinité de solutions

$\Rightarrow$ d_1, d_2 confondues

EXERCICE 5.16

$d_1 : A(1;1;1)\quad \vec{a} = \begin{pmatrix}2\\-1\\1\end{pmatrix}$ $\qquad d_2 : B(1;4;3)\quad \vec{b} = \begin{pmatrix}3\\3\\p\end{pmatrix}$

$d_1 : \begin{cases} x = 1 + 2t \\ y = 1 - t \\ z = 1 + t \end{cases}$ $\qquad d_2 : \begin{cases} x = 1 + 3k \\ y = 4 + 3k \\ z = 3 + pk \end{cases}$

$\begin{cases} 1 + 2t = 1 + 3k & ① \\ 1 - t = 4 + 3k & ② \\ 1 + t = 3 + pk & ③ \end{cases}$ $\qquad ① - ② \quad 3t = -3 \Rightarrow t = -1$

$② : 1 + 1 = 4 + 3k \Rightarrow 3k = -2 \Rightarrow k = -\frac{2}{3}$

On remplace en ③: $1 - 1 = 3 + p\left(-\frac{2}{3}\right) \Leftrightarrow$

$\Leftrightarrow \frac{2}{3}p = 3 \Rightarrow p = \frac{9}{2} \Rightarrow \boxed{p = 4.5}$

EXERCICE 5.17

$\Pi : A(1;2;2)\quad \vec{r} = \begin{pmatrix}1\\-1\\2\end{pmatrix}\quad \vec{s} = \begin{pmatrix}2\\2\\-1\end{pmatrix}$

$\Pi : \begin{cases} x = 1 + t + 2k & ① \\ y = 2 - t + 2k & ② \\ z = 2 + 2t - k \end{cases}$

On trouve l'équation cartésienne

$① + ② \quad x + y = 3 + 4k$

$2 \times ② \quad 2y = 4 - 2t + 4k$

$③ \quad + \; z = 2 + 2t - k$

$2y + z = 6 + 3k \quad * (-4)$

$x + y = 3 + 4k \quad * (3)$

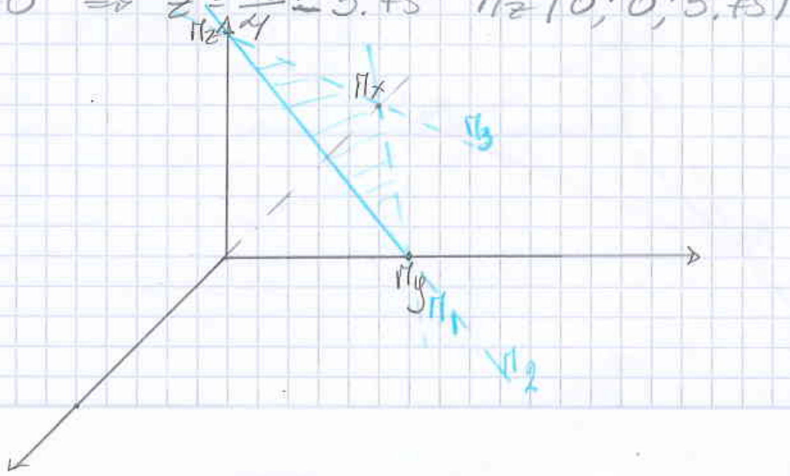
$\begin{cases} -8y - 4z = -24 - 12k \\ 3x + 3y = 9 + 12k \end{cases}$

$3x - 5y - 4z = -15 : \Pi$

$\Pi_x : y = z = 0 \Rightarrow x = -5 \quad \Pi_x(-5;0;0)$

$\Pi_y : x = z = 0 \Rightarrow y = 3 \quad \Pi_y(0;3;0)$

$\Pi_z : x = y = 0 \Rightarrow z = \frac{15}{4} = 3.75 \quad \Pi_z(0;0;3.75)$



EXERCICE 5.18

Π : $A(4;-8;4)$ $B(0;1;1)$ $C(-2;4.5;0)$

a) $\vec{AB} = \begin{pmatrix} -4 \\ +9 \\ -3 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} -6 \\ 12.5 \\ -4 \end{pmatrix}$

$$\Pi : \begin{cases} x = 0 - 4t - 6k & (1) \\ y = 1 + 9t + 12.5k & (2) \\ z = 1 - 3t - 4k & (3) \end{cases}$$

$9 \times (1)$ $9x = -36t - 54k$
$+ \ 4 \times (2)$ $4y = 4 + 36t + 50k$
$9x + 4y = 4 - 4k$

$-3 \times (1)$: $-3x = 12t + 18k$
$+ \ 4 \times (3)$: $4z = 4 - 12t - 16k$
$-3x + 4z = 4 + 2k$

$$\begin{cases} 9x + 4y = 4 - 4k \\ -3x + 4z = 4 + 2k \quad \times 2 \end{cases}$$

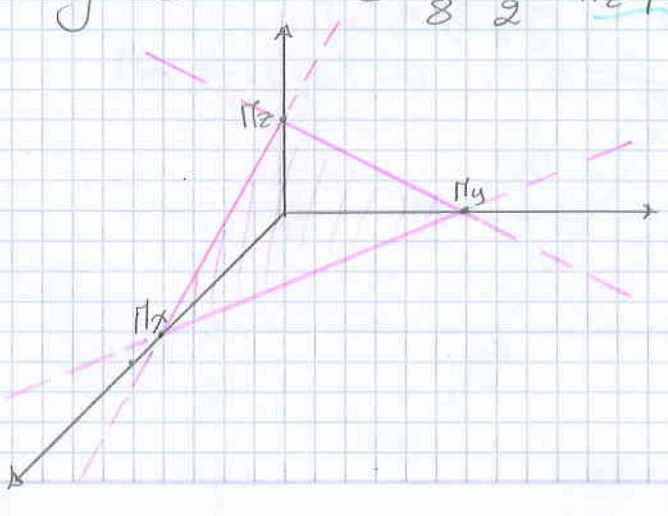
$9x + 4y = 4 - 4k$
$+ \ 6x + 8z = 8 + 4k$
$3x + 4y + 8z = 12$ (Π)

b) $D(5;-2;0.5) \overset{?}{\in} \Pi$
On remplace :
$3 \cdot 5 + 4(-2) + 8 \cdot 0.5 = 15 - 8 + 4 = 11 \neq 12 \Rightarrow D \notin \Pi$

c) k ? $E(8;k;-1) \in \Pi$

$3 \cdot 8 + 4 \cdot k + 8(-1) = 12 \Rightarrow 24 + 4k - 8 = 12 \Leftrightarrow$
$\Leftrightarrow 4k = -4 \Leftrightarrow k = -1$

d) Π_x : $y = z = 0 \Rightarrow x = 4$ $\Pi_x(4;0;0)$
Π_y : $x = z = 0 \Rightarrow y = 3$ $\Pi_y(0;3;0)$
Π_z : $x = y = 0 \Rightarrow z = \frac{12}{8} = \frac{3}{2}$ $\Pi_z(0;0;1.5)$



EXERCICE 5.19

a) $A(6;0;0)$ $B(0;4;0)$ $C(0;0;3)$

$\vec{AB} = \begin{pmatrix}-6\\4\\0\end{pmatrix} // \begin{pmatrix}-3\\2\\0\end{pmatrix}$ $\vec{AC} = \begin{pmatrix}-6\\0\\3\end{pmatrix} // \begin{pmatrix}-2\\0\\1\end{pmatrix}$ $\Pi = \begin{cases} x = 6 - 3t & (1) \\ y = 2t & (2) \\ z = k & (3) \end{cases}$

$2 \times (1)$ $2x = 12 - 6t$

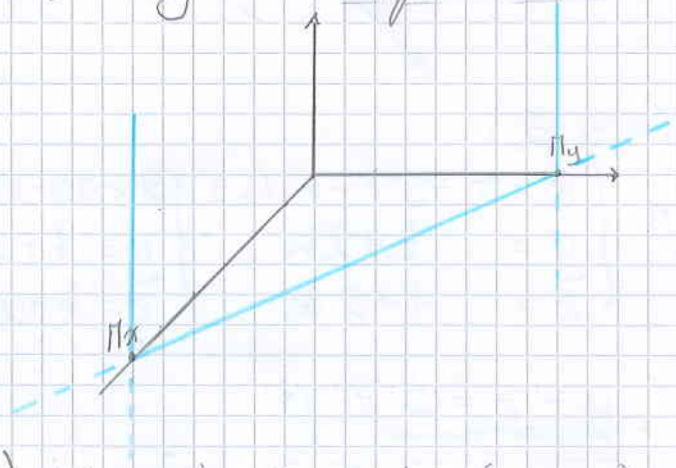
$3 \times (2)$ $+\ 3y = 6t$

$2x + 3y = 12$ (Π)

$\Pi_x : y = z = 0$ $\Pi_x(6;0;0)$

$\Pi_y : x = z = 0$ $\Pi_y(0;4;0)$

$\Pi_z : x = y = 0$ Impossible $\Pi // Oz$



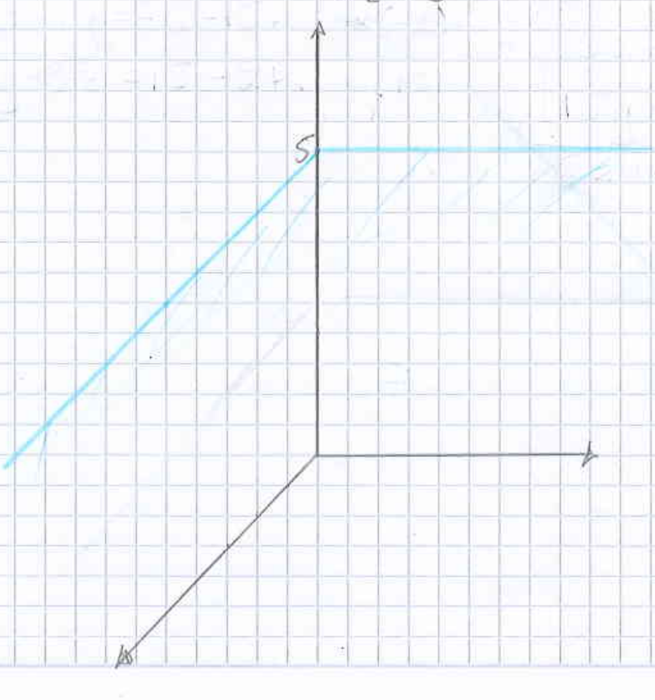
b) $A(2;3;5)$ $B(1;0;5)$ $C(6;-2;5)$

$\vec{AB} = \begin{pmatrix}-1\\-3\\0\end{pmatrix}$ $\vec{AC} = \begin{pmatrix}4\\-5\\0\end{pmatrix}$ $\Pi = \begin{cases} x = 1 - t + 4k \\ y = -3t - 5k \\ z = 5 \end{cases}$ $\Pi : z - 5 = 0$

Π_y, Π_x Impossible

$\Pi_z : (0;0;5)$

$\Pi //$ sol

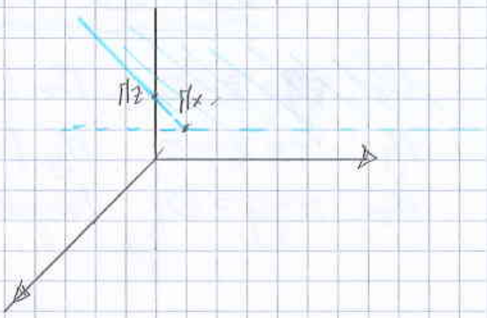


c) $A(2;1;3)$ $B(5;-1;6)$ $C(1;4;2)$

$\vec{AB} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$ $\Pi: \begin{cases} x = 2+3t-k & ① \times(3) \\ y = 1-2t+3k & ② \\ z = 3+3t-k & ③ \times(3) \end{cases}$

$\Leftrightarrow \begin{cases} 3x = 6+9t-3k & ① \\ y = 1-2t+3k & ② \\ 3z = 9+9t-3k & ③ \end{cases}$ $\begin{aligned} ①+② \quad & 3x+y = 7+7t \\ ②+③ \quad & y+3z = 10+7t \\ \hline & 3x-3z = -3 \Leftrightarrow x-z=-1 \ (\Pi) \end{aligned}$

Π_x: $y=z=0$ $x=-1$ $\Pi_x(-1;0;0)$
Π_y: $x=z=0$ Impossible
Π_z: $x=y=0$ $z=1$ $\Pi_z(0;0;1)$

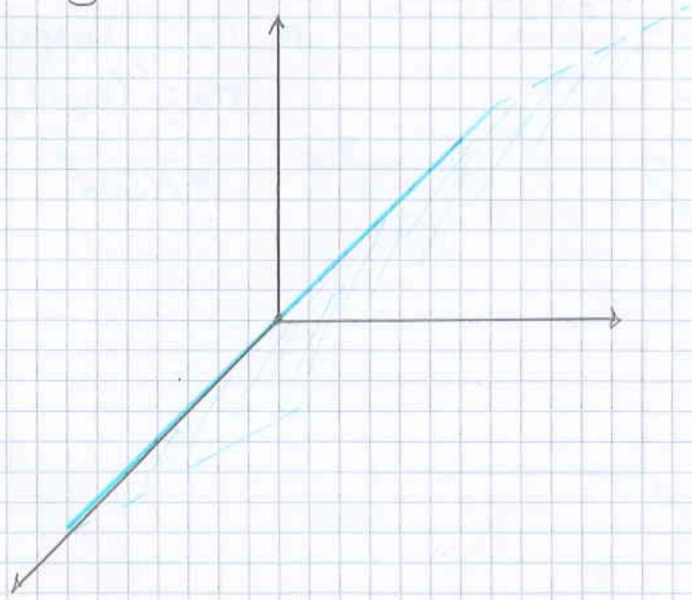


d) $A(5;0;0)$ $B(0;1;1)$ $C(4;2;2)$

$\vec{AB} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} -1 \\ 2 \\ +2 \end{pmatrix}$ $\Pi: \begin{cases} x = 5-5t-k \ (\times 2) \\ y = t+2k \\ z = t+2k \end{cases}$ $\begin{cases} 2x = 10-10t-2k & ① \\ y = t+2k & ② \\ z = t+2k & ③ \end{cases}$

$①+②$ $2x+y = 10-9t$
$②-③$ $y-z = 0$ $\Rightarrow$ $y-z=0$ Π

Π_x: $y=z=0$ $\Rightarrow x = \lambda \in \mathbb{R}$ $\Rightarrow$ $O_x \in \Pi$
Π_y: $x=z=0$ $\Rightarrow y=0$ $\Pi_y(0;0;0)$
Π_z: $x=0=y$ $\Rightarrow z=0$ $\Pi_z(0;0;0)$

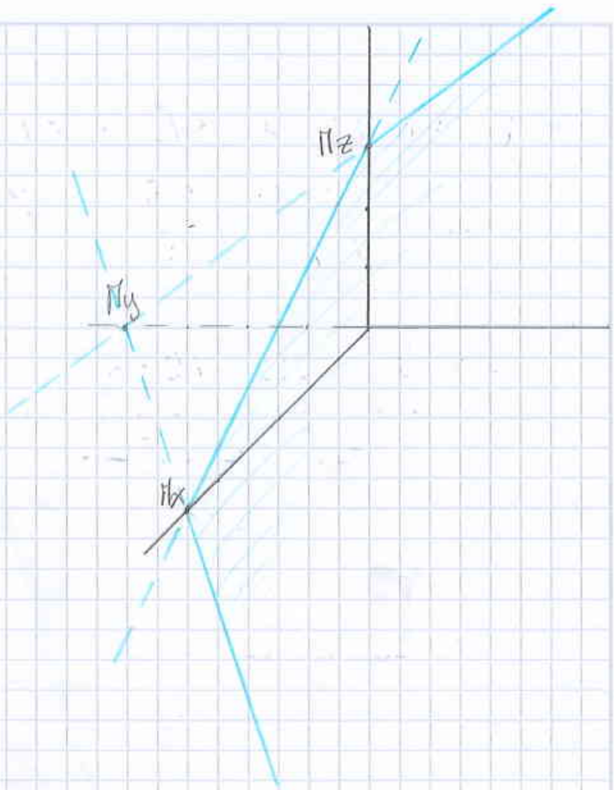


c) $2x-3y+4z-12=0$

Π_x: $y=z=0$ $\Pi_x(6;0;0)$

Π_y: $x=z=0$ $\Pi_y(0;-4;0)$

Π_z: $x=y=0$ $\Pi_z(0;0;3)$



EXERCICE 5.20

d: $A(2;3;3)$ $\vec{d}=\begin{pmatrix}2\\-2\\-1\end{pmatrix}$ d: $\begin{cases} x=2+2t \\ y=3-2t \\ z=3-t \end{cases}$ p: $\begin{cases} x=4+2t \\ y=4-2t \\ z=-t \end{cases}$

$p \,//\, d$ $B(4;4;0)$

a)

d $\begin{cases} T_1: z=0 \Leftrightarrow t=3 \quad x=8 \quad y=-3 \quad T_1(8;-3;0) \\ T_2: x=0 \Rightarrow t=-1 \quad y=5 \quad z=4 \quad T_2(0;5;4) \\ T_3: y=0 \Rightarrow t=3/2 \quad x=5 \quad z=3-\frac{3}{2}=\frac{3}{2} \quad T_3(5;0;1.5) \end{cases}$

p $\begin{cases} T_1: z=0 \Rightarrow t=0 \quad x=4 \quad y=4 \quad T_1(4;4;0) \\ T_2: x=0 \Rightarrow t=2 \quad y=8 \quad z=2 \quad T_2(0;8;2) \\ T_3: y=0 \Rightarrow t=2 \quad x=8 \quad z=-2 \quad T_3(8;0;-2) \end{cases}$

b) Vecteurs directeurs: $\vec{d}$ et $\begin{pmatrix}0\\0\\1\end{pmatrix}$ car $\pi \perp$ sol

π: $\begin{cases} x=2+2t \\ y=3-2t \\ z=3-t+k \end{cases}$ + $x+y=5$ (π)

c) 6: $\vec{d}=\begin{pmatrix}2\\-2\\-1\end{pmatrix}$ $\vec{AB}=\begin{pmatrix}2\\1\\-3\end{pmatrix}$ 6: $\begin{cases} x=2+2t+2k & (1) \\ y=3-2t+k & (2) \\ z=3-t-3k & (3) \end{cases}$

(1)+(2) $x+y=5+3k$

(1)+2×(3) $x=2+2t+2k$

$\underline{2z=6-2t-6k}$

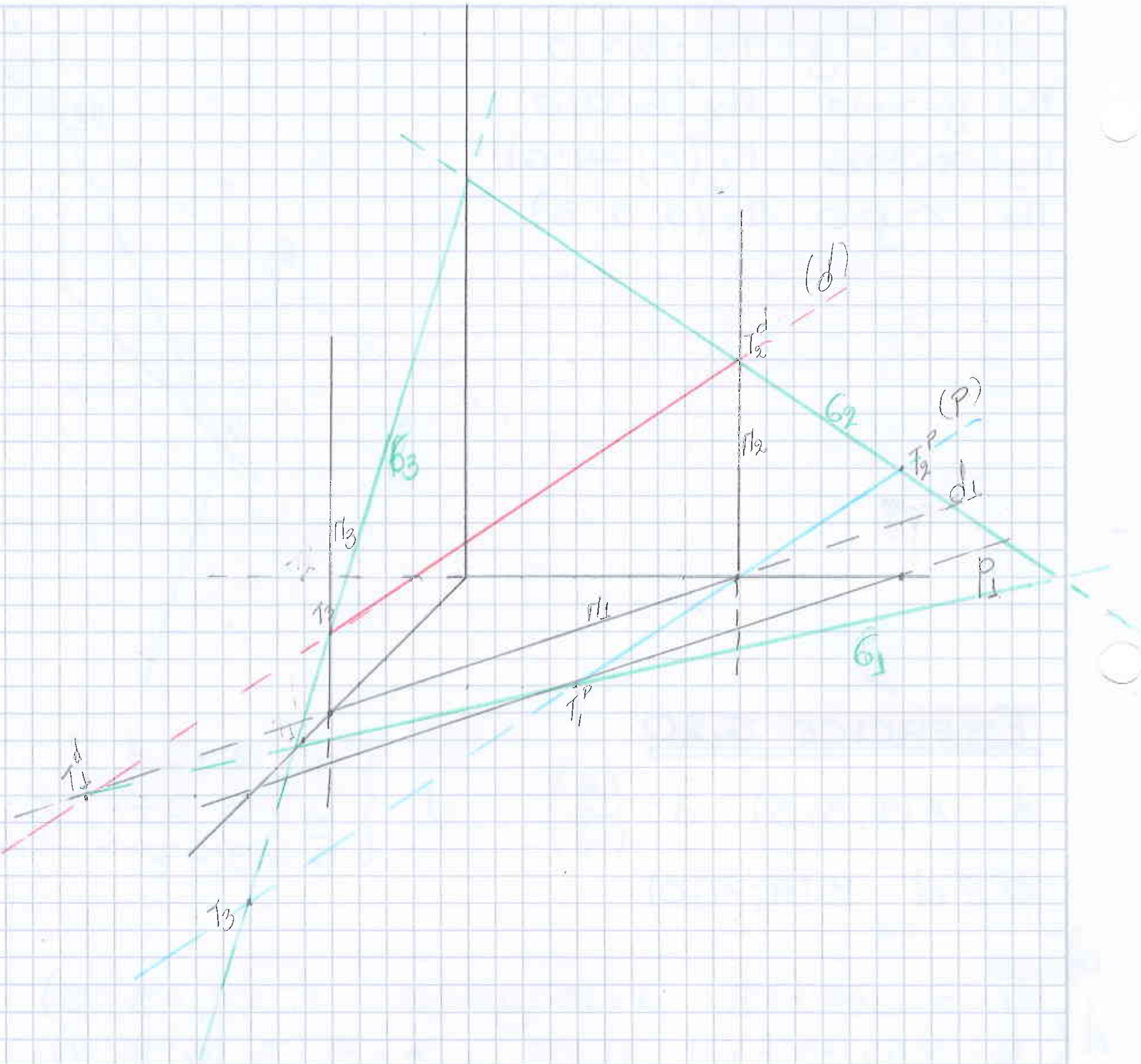
$x+2z=8-4k$.

$\begin{cases} x+y=5+3k & *4 \\ x+2z=8-4k & *3 \end{cases}$

$4x+4y=20+12k$

$+\ \underline{3x+6z=24-12k}$

$7x+4y+6z=44$ (6)


(d)
T_2^d
$б_2$
(P)
T_2^P
Π_2
d_1
P_1
$б_1$
$б_3$
Π_3
Π_1
T_1^P
T_1^d
T_3

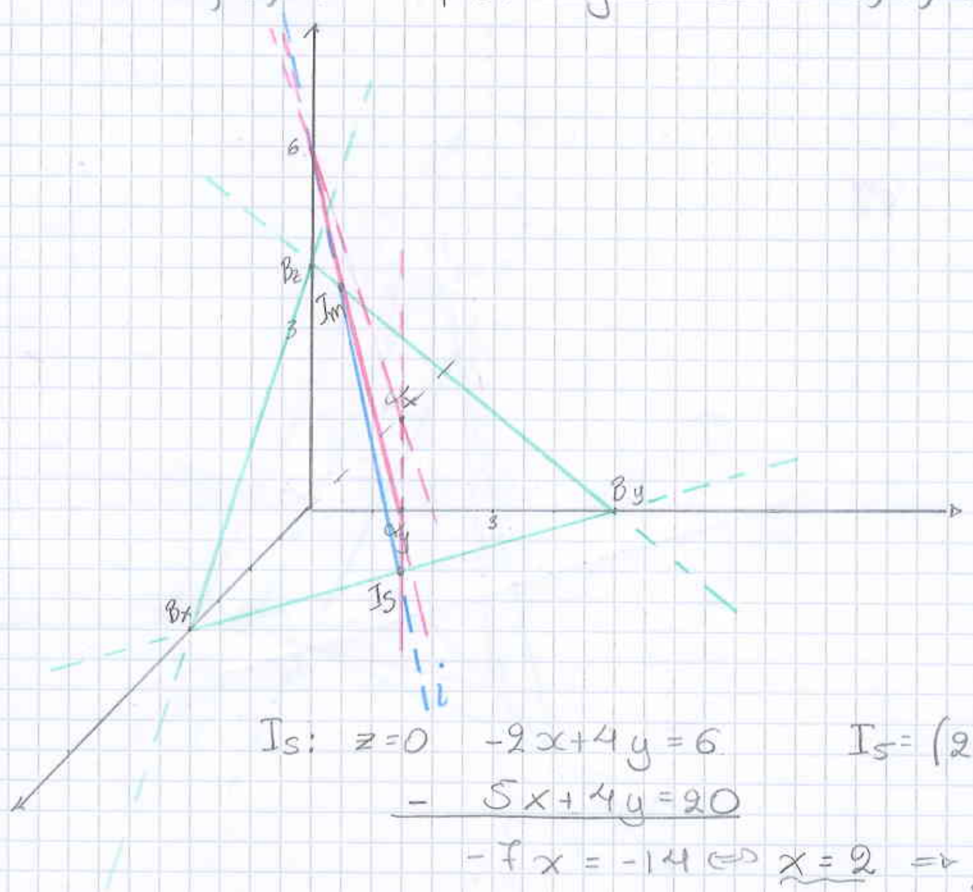
EXERCICE 5.21

a. $\alpha: -2x+4y+z-6=0$ $\beta: 5x+4y+5z-20=0$

α_x : $z=0$, $y=0$ $x=-3$ $(-3,0,0)$ β_x : $z=y=0$ $x=4$ $(4;0;0)$

α_y : $x=z=0$ $y=\frac{3}{2}$ $(0;\frac{3}{2};0)$ β_y : $x=z=0$ $y=5$ $(0;5;0)$

α_z : $x=y=0$ $z=6$ $(0;0;6)$ β_z : $x=y=0$ $z=4$ $(0;0;4)$



Is: $z=0$ $-2x+4y=6$ $I_s=(2;2.5;0)$

$-$ $5x+4y=20$

$-7x=-14 \Leftrightarrow \underline{x=2} \Rightarrow y=2.5$

Im: $x=0$ $4y+z=6$ $I_m(0;\frac{5}{8};\frac{7}{2})$

$-$ $4y+5z=20$

$-4z=-14 \Leftrightarrow z=\frac{14}{4} \Leftrightarrow z=\frac{7}{2}$

$\Rightarrow 4y=6-\frac{7}{2} \Rightarrow 4y=\frac{5}{2} \Leftrightarrow y=\frac{5}{8}$

On trouve $\overrightarrow{I_mI_s}=\begin{pmatrix}2\\15/8\\-7/2\end{pmatrix}$

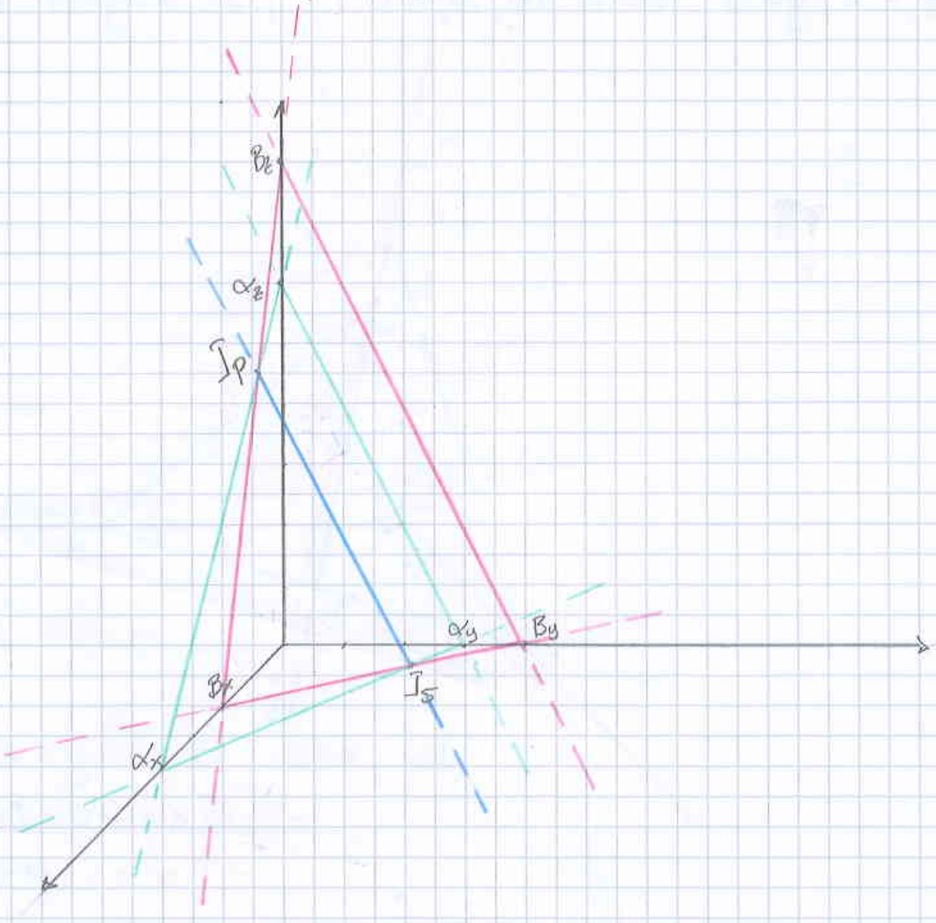
(i) : $\begin{cases} x=2+2t \\ y=\frac{5}{2}+\frac{15}{8}t \\ z=0-\frac{7}{2}t \end{cases}$

b. $\alpha: 3x+4y+2z-12=0$ $\beta: 4x+2y+z-8=0$

$\alpha_x: (4;0;0)$ $\beta_x: (2;0;0)$

$\alpha_y: (0;3;0)$ $\beta_y: (0;4;0)$

$\alpha_z: (0;0;6)$ $\beta_z: (0;0;8)$



$I_s: z=0$ $3x+4y=12$ $3x+4y=12$
$4x+2y=8 \quad \cdot(-2)$ $-8x-4y=-16$
$-5x=-4 \Leftrightarrow x=\frac{4}{5}$

$3\cdot\frac{4}{5}+4y=12 \Leftrightarrow$
$\Leftrightarrow 4y=12-\frac{12}{5} \Leftrightarrow$
$4y=\frac{48}{4} \Leftrightarrow$
$y=\frac{48}{16}=\frac{12}{4}=3$

$I_s\left(\frac{4}{5}; 3; 0\right)$

$I_p: y=0$ $3x+2z=12$ $3x+2z=12$
$4x+z=8$ $-8x-2z=-16$
$-5x=-4 \Leftrightarrow x=\frac{4}{5}\approx 0.8$

$4\cdot\frac{4}{5}+z=8 \Leftrightarrow z=\frac{40-16}{5} \Rightarrow$
$z=\frac{24}{5}\approx 4.8$

$I_p\left(\frac{4}{5}; 0; 4.8\right)$

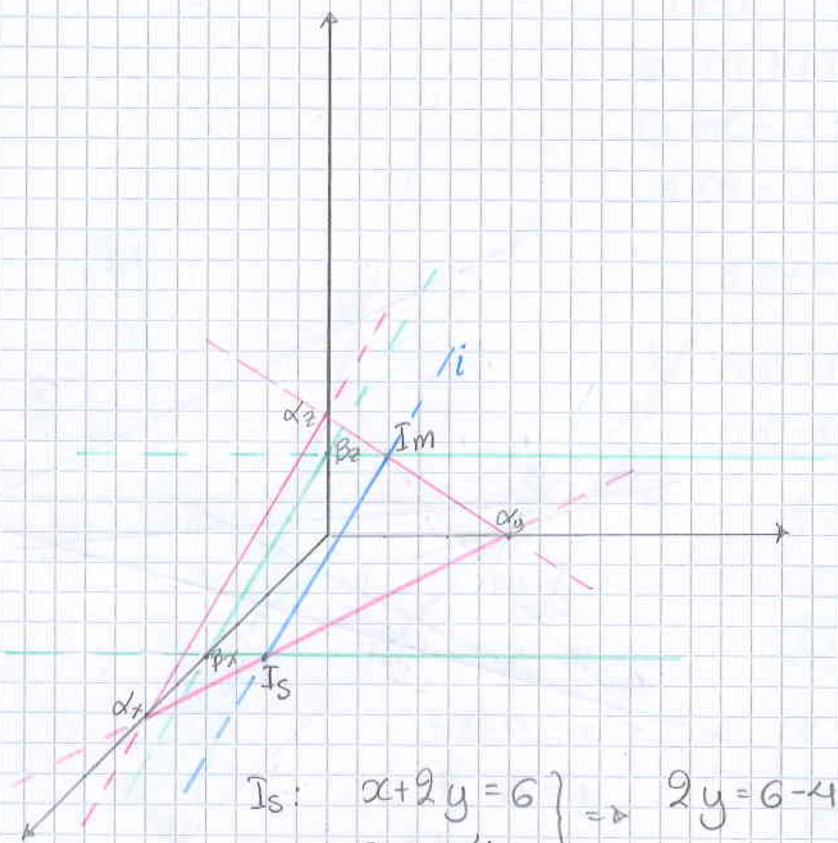
$\overrightarrow{I_pI_s}=\begin{pmatrix}0\\3\\-4.8\end{pmatrix}$ $i=\begin{cases}x=\frac{4}{5}\\ y=3+3t\\ z=-4.8t\end{cases}$

c. $\alpha: x+2y+3z-6=0$ $\qquad$ $\beta: x+3z-4=0$

$\alpha_x: (6;0;0)$ $\qquad$ $\beta_x: (4;0;0)$

$\alpha_y: (0;3;0)$ $\qquad$ β_y: Impossible.

$\alpha_z: (0;0;2)$ $\qquad$ $\beta_z: (0;0;\frac{4}{3})$



$I_s: \left.\begin{array}{l} x+2y=6 \\ x=4 \end{array}\right\} \Rightarrow 2y=6-4 \Leftrightarrow y=1$

$I_s=(4;1;0)$

$I_m: \begin{cases} 2y+3z=6 \\ 3z=4 \end{cases} \Rightarrow z=\frac{4}{3}$ $\qquad$ $I_m=(0;1;\frac{4}{3})$

$-$

$2y=2 \Leftrightarrow y=1$

$\overrightarrow{I_mI_s}=\begin{pmatrix} 4 \\ 0 \\ -4/3 \end{pmatrix}$ $\qquad$ $(i)=\begin{cases} x=4+4t \\ y=1 \\ z=0-\frac{4}{3}t \end{cases}$

EXERCICE 5.22

Π : $A(3;5;1)$ $B=(6;1;3)$ $C(4;2;0)$

d : $D(4;6;0)$ $E(1;5;3)$

$\vec{AB} = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$

$$\Pi : \begin{cases} x = 3 + 3t + m & ① \\ y = 5 - 4t - 3m & ② \\ z = 1 + 2t - m & ③ \end{cases}$$

①+③ $x + z = 4 + 5t$

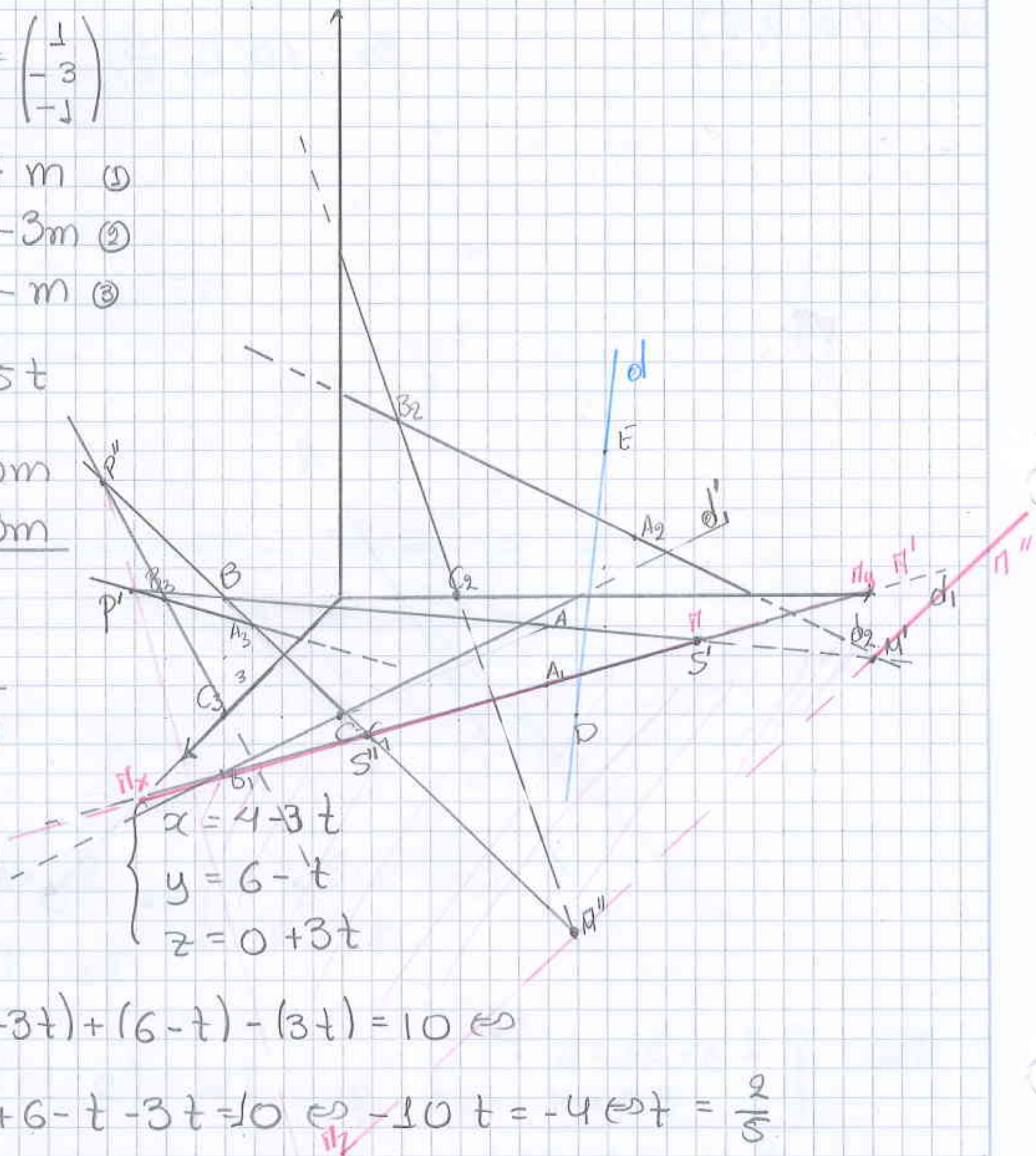
① : $3x = 9 + 9t + 3m$

② $y = 5 - 4t - 3m$

$3x + y = 14 + 5t$

$- \quad x + z = 4 + 5t$

Π $2x + y - z = 10$



d : $\vec{DE} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$ $\begin{cases} x = 4 - 3t \\ y = 6 - t \\ z = 0 + 3t \end{cases}$

$d \to \Pi$: $2(4-3t) + (6-t) - (3t) = 10 \Leftrightarrow$

$\Leftrightarrow 8 - 6t + 6 - t - 3t = 10 \Leftrightarrow -10t = -4 \Leftrightarrow t = \frac{2}{5}$

Donc on a un point d'intersection (sécants)

$x = 4 - 3 \cdot \frac{2}{5} \Rightarrow x = \frac{20-6}{5} \Rightarrow x = \frac{14}{5}$

$y = 6 - \frac{2}{5} \Rightarrow y = \frac{28}{5}$

$z = 3 \cdot \frac{2}{5} \Leftrightarrow z = \frac{6}{5}$

$I\left(\frac{14}{5}; \frac{28}{5}; \frac{6}{5}\right)$

EXERCICE 5.23

a: $d = \begin{cases} x = 2-2\lambda \\ y = 1+\lambda \\ z = 2\lambda \end{cases}$ $\quad$ Π: $A(6;-1;1)$ $B(2;0;2)$ $C(-2;4;0)$

$\vec{AB} = \begin{pmatrix} -4 \\ 1 \\ 1 \end{pmatrix}$ $\quad$ $\vec{AC} = \begin{pmatrix} -8 \\ 5 \\ -1 \end{pmatrix}$

$\Pi: \begin{cases} x = 6-4t-8m & ① \\ y = -1+t+5m & ② \\ z = 1+t-m & ③ \end{cases}$

① − 8*③ $\quad x = 6-4t-8m$

$+ \; -8z = -8-8t+8m$

$x - 8z = -2-12t$

$+ \; 2y+10z = 8+12t$

$\Pi \quad \boxed{x+2y+2z=6}$

② + 5×③ $\quad y = -1+t+5m$

$+ \; 5z = 5+5t-5m$

$y+5z = 4+6t$

$d \to \Pi \quad 2-2\lambda+2(1+\lambda)+2\cdot 2\lambda = 6 \Leftrightarrow$

$\Leftrightarrow 2-\cancel{2\lambda}+2+\cancel{2\lambda}+4\lambda = 6 \Leftrightarrow 4\lambda = 2 \Leftrightarrow \lambda = \frac{1}{2}$

Sécants: $\quad I = d \cap \Pi$ $\quad x = 2-2\cdot\frac{1}{2} \Rightarrow x = 1$

$y = 1+\frac{1}{2} \Rightarrow y = \frac{3}{2}$

$z = 1$

$I\left(1;\frac{3}{2};1\right)$

b) d: $A(3;1;2)$ $B(-1;2;0)$ $\quad$ $\Pi: x+2y-z-3=0$

$\vec{AB} = \begin{pmatrix} -4 \\ 1 \\ -2 \end{pmatrix}$ $\quad d = \begin{cases} x = 3-4t \\ y = 1+t \\ z = 2-2t \end{cases}$

$d \to \Pi: \quad 3-4t+2(1+t)-(2-2t)-3 = 0 \Leftrightarrow$

$\Leftrightarrow \cancel{3}-\cancel{4t}+\cancel{2}+\cancel{2t}-\cancel{2}+\cancel{2t}-\cancel{3} = 0 \Leftrightarrow$

$\Leftrightarrow \quad 0\cdot t = 0 \quad \Rightarrow d \in \Pi$

c) $d : \begin{cases} x = 1 + 5\lambda \\ y = 1 - 3\lambda \\ z = 3 \end{cases}$ $\qquad \pi : \begin{cases} x = 1 - \mu + 2\tau & ① \\ y = 1 + \mu - \tau & ② \\ z = -3 - 2\mu - \tau & ③ \end{cases}$

②, ③

$$\begin{array}{r} y = 1 + \mu - \tau \\ - z = 3 + 2\mu + \tau \\ \hline y - z = 4 + 3\mu \end{array} \qquad \begin{array}{r} x = 1 - \mu + 2\tau \\ + 2y = 2 + 2\mu - 2\tau \\ \hline x + 2y = 3 + \mu \end{array}$$

$$\begin{array}{r} y - z = 4 + 3\mu \\ - \quad 3x + 6y = 9 + 3\mu \\ \hline -3x - 5y - z = -5 \end{array}$$

$-3x - 5y - z = -5 \Leftrightarrow \boxed{3x + 5y + z = 5}$ π

$d \to \pi \quad 3(1 + 5\lambda) + 5(1 - 3\lambda) + 3 = 5 \Leftrightarrow$

$\Leftrightarrow 3 + \cancel{15\lambda} + \cancel{5} - \cancel{15\lambda} + 3 = \cancel{5} \Leftrightarrow 0 \cdot \lambda = -6$ Impossible

$\Longrightarrow d \parallel \pi$

EXERCICE 5.24

a) a. $\tau : \begin{cases} x = \lambda \\ y = 1 + \mu \\ z = 0 \end{cases}$ $\qquad \pi : x + 2y - z - 3 = 0$

$\tau : \ z = 0$

$$\vec{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 - 2 \\ 1 - 0 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{matrix} 0 & & 1 \\ 0 & & 2 \end{matrix}$$

pour $x = 0$ $\quad \begin{cases} z = 0 \\ 2y - z = 3 \end{cases}$ $\quad \begin{matrix} z = 0 \\ y = \frac{3}{2} \end{matrix}$ $\quad A(0; \frac{3}{2}; 0)$

$d = \tau \cap \pi \quad \begin{cases} x = -2t \\ y = \frac{3}{2} + t \\ z = 0 \end{cases}$

b) $\tau: -3x-6y+3z-9=0 \Leftrightarrow x+2y-z+3=0$

$\pi: x+2y-z-3=0$

$\pi \parallel \tau$

b) $\sigma \parallel \pi: x+2y-z-3=0 \quad A(2;4;3)$

$\vec{n}=\begin{pmatrix}1\\2\\-1\end{pmatrix} \quad \Rightarrow \sigma: x+2y-z+c=0$

$A: 2+8-3+c=0 \Leftrightarrow 7+c=0 \Leftrightarrow c=-7$

Donc $\sigma: x+2y-z-7=0$

EXERCICE 5.25

$\vec{a}=\begin{pmatrix}8\\-9\\1\end{pmatrix} \quad \vec{b}=\begin{pmatrix}5\\-2\\3\end{pmatrix} \quad \vec{c}=\begin{pmatrix}1\\0\\-8\end{pmatrix}$

$\vec{a}\cdot\vec{b}=40+18+3=61 \qquad \vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\begin{pmatrix}6\\-2\\-5\end{pmatrix}=$

$\vec{b}\cdot\vec{a}=61 \qquad 48+18-5=61$

$\vec{a}\cdot\vec{c}=8+0-8=0$

$\vec{c}-\vec{a}=\begin{pmatrix}-7\\9\\-9\end{pmatrix} \quad \vec{b}-\vec{a}=\begin{pmatrix}-3\\7\\2\end{pmatrix}$

$(\vec{c}-\vec{a})\cdot(\vec{b}-\vec{a})=21+63-18=84-18=66$

EXERCICE 5.26

$A(-4;7;13) \quad B(8,2;0,7;-9)$

a. $|\vec{OA}|=\sqrt{16+49+169}=15,3$

$|\vec{OB}|=\sqrt{8,2^2+0,7^2+81}=12,2$

$\vec{AB}=\begin{pmatrix}12.2\\-6.3\\-22\end{pmatrix} \qquad |\vec{AB}|=\sqrt{12.2^2+6.3^2+22^2}=25,93$

b. $\vec{OA}\cdot\vec{OB}=(-4)\cdot 8,2+7\times 0.7+13(-9)=$

$=-32,8+4.9-117=-144.9$

$$\vec{u}_{\vec{OA}} = \begin{pmatrix} -4/15.3 \\ 7/15.3 \\ 13/15.3 \end{pmatrix}$$

d. $|\vec{OA}\cdot\vec{OB}| = \|\vec{a}\|\cdot\|\vec{b}'\| \Rightarrow \|\vec{b}'\| = \dfrac{144.9}{15.3} \Rightarrow \|\vec{b}'\| = 9.47$

$|\vec{OA}\cdot\vec{OB}| = \|\vec{a}'\|\cdot\|\vec{b}\| \Rightarrow \|\vec{a}'\| = \dfrac{144.9}{12.2} \Rightarrow \|\vec{a}'\| = 11.877$

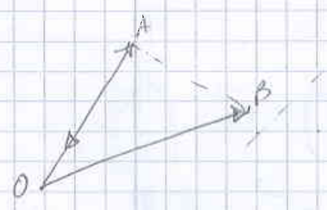
e) $\cos(\hat{AOB}) = \dfrac{\vec{OA}\cdot\vec{OB}}{\|\vec{OA}\|\cdot\|\vec{OB}\|} \Rightarrow \cos(\hat{AOB}) = \dfrac{-144.9}{15.3 \times 12.2} \Rightarrow \cos\alpha_1 = -0.776 \Rightarrow$

$\alpha_1 = 140{,}92°$

$\cos(\hat{OAB}) = \dfrac{|\vec{AO}\cdot\vec{AB}|}{\|\vec{AO}\|\cdot\|\vec{AB}\|} = \dfrac{378.9}{15.3 \times 25.93} = 0.955 \Rightarrow \alpha_2 = 17.24°$

$$\vec{AO} = \begin{pmatrix} 4 \\ -7 \\ -13 \end{pmatrix} \qquad \vec{AB} = \begin{pmatrix} 12.2 \\ -6.3 \\ -22 \end{pmatrix}$$

$\vec{AO}\cdot\vec{AB} = 4\cdot 12{,}2 + (-7)(-6{,}3) + (-13)(-22) = 378.9$



f) $\text{Aire}(OAB) = \frac{1}{2}\|\vec{a}\wedge\vec{b}\| = \frac{1}{2}\left\| \begin{pmatrix} -4 \\ 7 \\ 13 \end{pmatrix} \wedge \begin{pmatrix} 8.2 \\ 0.7 \\ -9 \end{pmatrix} \right\| = \frac{1}{2}\left\| \begin{pmatrix} -63-9.1 \\ 106.6-36 \\ -2.8-57.4 \end{pmatrix} \right\| =$

$$\begin{matrix} 13 & & -9 \\ -4 & & 8.2 \\ 7 & & 0.7 \end{matrix}$$

$= \frac{1}{2}\left\| \begin{matrix} -72.1 \\ 70.6 \\ -60.2 \end{matrix} \right\| = \frac{1}{2}\sqrt{72.1^2 + 70.6^2 + 60.2^2} = 58.75 = \text{Aire}(OAB)$

EXERCICE 5.27

a) $\vec{a}\cdot\vec{b} = 0 \Rightarrow -3 + (2+k)(-6k) + (-0.5)\cdot 5 = 0 \Leftrightarrow$

$\Leftrightarrow -3 - 12k - 6k^2 - 2.5 = 0 \Leftrightarrow -6k^2 - 12k - 5.5 = 0 \Leftrightarrow$

$\Leftrightarrow 6k^2 + 12k + 5.5 = 0$

$\Delta = 12^2 - 4\cdot 6\cdot 5.5 = 12$

$k_{1,2} = \dfrac{-12 \pm \sqrt{12}}{12} = \begin{cases} -0.71 = k_1 \\ -1.289 = k_2 \end{cases}$

b) $\begin{pmatrix}4\\3\\8\end{pmatrix} \wedge \begin{pmatrix}-5\\20\\9\end{pmatrix} = \begin{pmatrix}27-160\\-40-36\\80+15\end{pmatrix} = \begin{pmatrix}-133\\-76\\95\end{pmatrix} = \vec{w}$

$\vec{w}$ doit être $\parallel$ à $\begin{pmatrix}7\\a\\b\end{pmatrix}$

$-133 = \lambda \cdot 7 \Rightarrow \lambda = -19$

$-76 = \lambda \cdot a \Rightarrow a = -76/-19 \Rightarrow a = 4$

$95 = \lambda b \Rightarrow b = 95/-19 \Rightarrow b = -5$

EXERCICE 5.28

a. $\vec{a} = \begin{pmatrix}16\\-2\\8\end{pmatrix}$ $\vec{b} = \begin{pmatrix}-1\\0\\m\end{pmatrix}$ $\hat{\alpha} = 60° \Rightarrow \cos\hat{\alpha} = \frac{1}{2}$

$\cos\hat{\alpha} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|}$ (1)

$\vec{a}\cdot\vec{b} = -16+0+8m = -16+8m$

$\|\vec{a}\| = \sqrt{16^2+4+64} = 18$ $\|\vec{b}\| = \sqrt{1+m^2}$

(1) $\Leftrightarrow \frac{1}{2} = \frac{-16+8m}{18\cdot\sqrt{1+m^2}} \Leftrightarrow 18\sqrt{1+m^2} = 2(-16+8m) \Leftrightarrow$

$\Leftrightarrow 9\sqrt{1+m^2} = (-16+8m) \Rightarrow 81(1+m^2) = (-16+8m)^2 \Leftrightarrow$

$\Leftrightarrow 81(1+m^2) = 256 - 256m + 64m^2 \Leftrightarrow$

$17m^2 + 256m - 175 = 0$

$\Delta = 77436$ $m_{1,2} = \frac{-256 \pm 278.3}{34} = \begin{cases} -15.72 = m_1 \\ 0.656 = m_2 \end{cases}$

b) $\vec{a} = \begin{pmatrix}6\\-3\\4\end{pmatrix}$ $\vec{b} = \begin{pmatrix}1\\n\\-2\end{pmatrix}$ $\vec{a}\cdot\vec{b} = 6-3n-8 = -3n-2$

$\|\vec{a}\| = \sqrt{36+9+16} = \sqrt{61}$

$\|\vec{b}\| = \sqrt{1+n^2+4} = \sqrt{5+n^2}$

$\cos 45 = \frac{\vec{a}\,\vec{b}}{\|\vec{a}\|\|\vec{b}\|} \Rightarrow \frac{\sqrt{2}}{2} = \frac{-3n-2}{\sqrt{61}\cdot\sqrt{5+n^2}} \Leftrightarrow \sqrt{2}\sqrt{61}\sqrt{5+n^2} = 2(-3n-2) \Leftrightarrow$

$\Leftrightarrow 2\cdot 61\cdot(5+n^2) = 4(-3n-2)^2 \Leftrightarrow 122(5+n^2) = 4(9n^2+12n+4) \Leftrightarrow$

$61(5+n^2) = 2(9n^2+12n+4) \Leftrightarrow 61n^2+305 = 18n^2+24n+8 \Leftrightarrow$

$43n^2 - 24n + 297 = 0$

$43n^2 - 24n + 297 = 0$

$\Delta = -50508 < 0$ Pas de solutions

EXERCICE 5.29

d: $A(2;6;0)$ $B(-1;2;5)$

a. $\vec{AB} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$

$d = \begin{cases} x = 2 - 3t \\ y = 6 - 4t \\ z = 5t \end{cases}$

T_1 : $z = 0 \Rightarrow t = 0$ $T_1(2;6;0)$

T_2 : $x = 0 \Leftrightarrow 2 - 3t = 0 \Leftrightarrow t = 2/3$

$y = 6 - 4 \cdot \frac{2}{3} = \frac{18-8}{3} \Rightarrow y = \frac{10}{3}$

$z = 5 \cdot \frac{2}{3} \Leftrightarrow z = \frac{10}{3}$

$T_2 = \left(0; \frac{10}{3}; \frac{10}{3}\right)$

T_3 : $y = 0 \Leftrightarrow 6 - 4t = 0 \Leftrightarrow t = \frac{3}{2}$

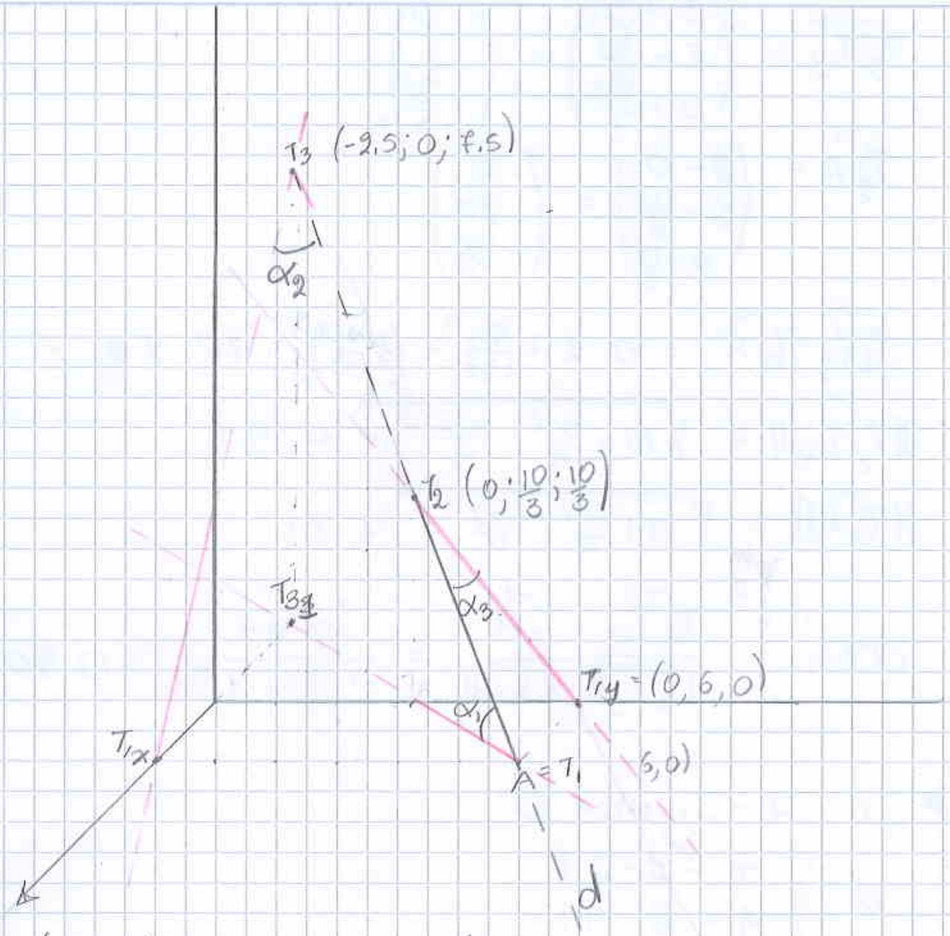
$x = 2 - 3 \cdot \frac{3}{2} = \frac{4-9}{2} = -\frac{5}{2}$ $T_3 = \left(-\frac{5}{2}; 0; \frac{15}{2}\right)$

$z = 5 \cdot \frac{3}{2} \Rightarrow z = \frac{15}{2}$

b) $T_{3_1}\left(-\frac{5}{2}; 0; 0\right)$

$T_{1x}(2; 0; 0)$

$T_{1y}(0; 6; 0$



$$\vec{AT_3} = \begin{pmatrix} -\frac{5}{2} - 2 \\ 0 - 6 \\ \frac{15}{2} - 0 \end{pmatrix} = \begin{pmatrix} -4.5 \\ -6 \\ 7.5 \end{pmatrix} \qquad \vec{AT_{3s}} = \begin{pmatrix} -\frac{5}{2} - 2 \\ 0 - 6 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} -4.5 \\ -6 \\ 0 \end{pmatrix}$$

- $\cos\alpha_1 = \dfrac{\vec{AT_3} \cdot \vec{AT_{3s}}}{\|\vec{AT_3}\| \cdot \|\vec{AT_{3s}}\|} = \dfrac{56.25}{10.61 \times 7.5} = 0.7068 \Rightarrow \alpha_1 \approx 45^\circ$

$\vec{AT_3} \cdot \vec{AT_{3s}} = (-4.5)^2 + 36 = 56.25$

$\|\vec{AT_3}\| = \sqrt{4.5^2 + 36 + 7.5^2} = 10.61$

$\|\vec{AT_{3s}}\| = \sqrt{4.5^2 + 36} = 7.5$

- $\vec{T_3T_{1x}} = \begin{pmatrix} 2 + \frac{5}{2} \\ 0 - 0 \\ 0 - \frac{15}{2} \end{pmatrix} = \begin{pmatrix} 9/2 \\ 0 \\ -15/2 \end{pmatrix} \qquad \vec{T_3A} = \begin{pmatrix} 4.5 \\ 6 \\ -7.5 \end{pmatrix}$

$\|\vec{T_3T_{1x}}\| = \sqrt{4.5^2 + 7.5^2} = 8.75$

$\|\vec{T_3A}\| = 10.61$

$\vec{T_3T_{1x}} \cdot \vec{T_3A} = 4.5^2 + 7.5^2 = 76.5$

$\cos\alpha_2 = \dfrac{76.5}{10.61 \times 8.75} = 0.824 \Rightarrow \alpha_2 = 34.51^\circ$

$$\overrightarrow{T_2T_{1y}} = \begin{pmatrix} 0+0 \\ 6-\frac{10}{3} \\ 0-\frac{10}{3} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{8}{3} \\ -\frac{10}{3} \end{pmatrix}$$

$$\overrightarrow{T_2A} = \begin{pmatrix} 2-0 \\ 6-\frac{10}{3} \\ 0-\frac{10}{3} \end{pmatrix} = \begin{pmatrix} 2 \\ 8/3 \\ -10/3 \end{pmatrix}$$

$$\overrightarrow{T_2T_{1y}} \cdot \overrightarrow{T_2A} = 0 \cdot 2 + \left(\frac{8}{3}\right)^2 + \left(-\frac{10}{3}\right)^2 = 18.22$$

$$\|\overrightarrow{T_2T_{1y}}\| = \sqrt{0+\frac{64}{9}+\frac{100}{9}} = 4,27$$

$$\|\overrightarrow{T_2A}\| = \sqrt{4+\frac{64}{9}+\frac{100}{9}} = 4,71$$

$$\cos\hat{\alpha}_3 = \frac{\overrightarrow{T_2T_{1y}} \cdot \overrightarrow{T_2A}}{\|\overrightarrow{T_2T_{1y}}\| \cdot \|\overrightarrow{T_2A}\|} = \frac{18.22}{4.27 \times 4.71} = 0.905 \Rightarrow \hat{\alpha}_3 = 25,15^\circ$$

- $\Pi : x - 5y + 11 = 0$

$$d : \begin{cases} x = 2-3t \\ y = 6-4t \\ z = 5t \end{cases}$$

$d \to \Pi$: $2-3t-5(6-4t)+11 = 0 \Leftrightarrow 2-3t-30+20t+11=0 \Leftrightarrow$

$\Leftrightarrow 17t - 17 = 0 \Leftrightarrow t = 1$

Donc I : point d'intersection $x = -1$, $y = 2$, $z = 5$

$I(-1;2;5)$

$\vec{d} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$ $\quad \vec{n}$ = vecteur normal du plan Π

vecteur directeur de (d) $\quad \vec{n} = \begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix}$

On trouve l'angle entre $\vec{d}, \vec{n}$

$\vec{d} \cdot \vec{n} = -3 + 20 = 17$

$\|\vec{d}\| = \sqrt{9+16+25} = \sqrt{50} \quad \|\vec{n}\| = \sqrt{1+25} = \sqrt{26}$

donc $\cos(\widehat{\vec{d},\vec{n}}) = \frac{\vec{d}\cdot\vec{n}}{\|\vec{d}\| \cdot \|\vec{n}\|} = \frac{17}{\sqrt{50}\cdot\sqrt{26}} = 0,47 \Rightarrow (\widehat{d,n}) = 61,87$

$\Rightarrow (\widehat{\Pi, d}) = 90 - 61.87 = 28.13^\circ = (\widehat{\Pi,d})$

EXERCICE 5.30

d: $A(4;-1;-2)$ $B(1;3;-3)$

$\vec{AB} = \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix}$

le vecteur $\vec{AB}$ est le vecteur normal $\vec{n}$ du plan

Donc $\pi: -3x+4y-z+c=0$

On remplace les coordonnés de A:

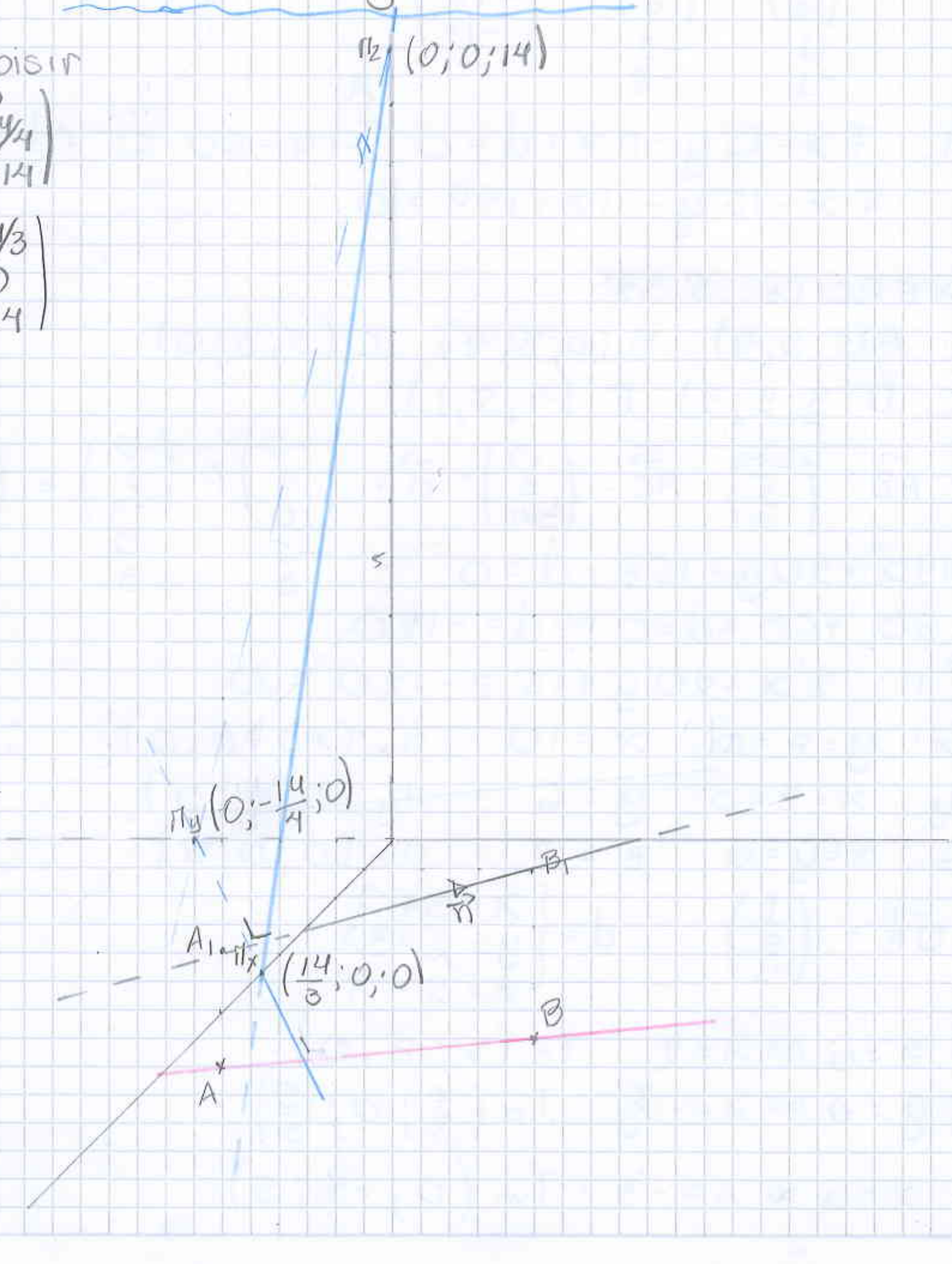
$-3\cdot 4-4+2+c=0 \Leftrightarrow -14+c=0 \Leftrightarrow c=14$

Alors $\pi: -3x+4y-z+14=0$

On peut choisir

$\overrightarrow{\pi_z\pi_y} = \begin{pmatrix} 0 \\ -14/4 \\ -14 \end{pmatrix}$

$\overrightarrow{\pi_z\pi_x} = \begin{pmatrix} 14/3 \\ 0 \\ -14 \end{pmatrix}$



EXERCICE 5.31

a) $\vec{n} = \begin{pmatrix} 1 \\ -3 \\ 6 \end{pmatrix}$ $\quad A\left(\frac{16}{5} ; 9 ; -4\right)$

$x - 3y + 6z + d = 0$ $\quad \overset{A}{\downarrow} \; \frac{16}{5} - 27 - 24 + d = 0 \Leftrightarrow d = \frac{239}{5}$

$\Pi.\; x - 3y + 6z + \frac{239}{5} = 0$

b) $d = \begin{cases} x = 2 + 13\lambda \\ y = -3 - 2\lambda \\ z = 11 + 8\lambda \end{cases}$ $\quad A(-21 ; 17 ; 82)$

$\vec{n} = \begin{pmatrix} 13 \\ -2 \\ 8 \end{pmatrix}$ $\quad \Pi : 13x - 2y + 8z + d = 0$

$A : -273 - 34 + 656 + d = 0 \Leftrightarrow d = -349$

$\Pi : 13x - 2y + 8z - 349 = 0$

c) $\vec{n} = \begin{array}{c} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \\ 1 \\ -1 \end{array} \times \begin{array}{c} \begin{pmatrix} -3 \\ -8 \\ 9 \end{pmatrix} \\ -3 \\ -8 \end{array} = \begin{pmatrix} 7 \\ -15 \\ -11 \end{pmatrix}$ $\quad A(-2 ; 4 ; 5)$

$\Pi : 7x - 15y - 11z + d = 0$ $\quad \overset{A}{\downarrow} -14 - 60 - 55 + d = 0 \Leftrightarrow d = 129$

$7x - 15y - 11z + 129 = 0$

EXERCICE 5.32

$\Pi : A(5 ; 0 ; 4) \quad B(0 ; 3 ; 4) \quad C(5 ; 3 ; 0)$

$d : D(3 ; 2 ; 2) \quad E(4 ; 5 ; 1)$

a. $\vec{AB} = \begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix}$ $\quad \vec{n} = \begin{array}{c} \begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix} \\ -5 \\ 3 \end{array} \wedge \begin{array}{c} \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix} \\ 0 \\ 3 \end{array} = \begin{pmatrix} -12 \\ -20 \\ -15 \end{pmatrix} /\!/ \begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix}$

$12x + 20y + 15z + d = 0$

$A : 60 + 60 + d = 0 \Leftrightarrow d = -120$

$\Pi : 12x + 20y + 15z - 120 = 0$

- $Ox : y = z = 0 \quad x = 10 \quad \Pi_x(10 ; 0 ; 0)$
- $Oy : x = z = 0 \quad y = 6 \quad \Pi_y(0 ; 6 ; 0)$
- $Oz : x = y = 0 \quad z = 8 \quad \Pi_z(0 ; 0 ; 8)$

b. $\vec{DE} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ $\quad d = \begin{cases} x = 3 + \lambda \\ y = 2 + 3\lambda \\ z = 2 - \lambda \end{cases}$

$T_s : z = 0 \Leftrightarrow \lambda = 2 \quad T_s(5 ; 8 ; 0)$

$T_p : y = 0 \Leftrightarrow \lambda = -\frac{2}{3} \quad T_p\left(\frac{7}{3} ; 0 ; \frac{8}{3}\right)$

$T_m : x = 0 \Leftrightarrow \lambda = -3 \quad T_m(0 ; -7 ; 5)$

c. $\hat{A}$: $\cos\hat{A} = \dfrac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|\cdot\|\vec{AC}\|} = \dfrac{9}{\sqrt{34}\cdot 5} = 0.309 \Rightarrow \hat{A} = 72^\circ$

$\vec{AB}\cdot\vec{AC} = 9 \quad \|\vec{AB}\| = \sqrt{25+9} = \sqrt{34} \quad \|\vec{AC}\| = \sqrt{9+16} = 5$

$\vec{BA} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} \quad \vec{BC} = \begin{pmatrix} 5 \\ 0 \\ -4 \end{pmatrix} \quad \vec{BA}\cdot\vec{BC} = 25$

$\|\vec{BA}\| = \sqrt{34} \quad \|\vec{BC}\| = \sqrt{25+16} = \sqrt{41}$

$\cos\hat{B} = \dfrac{\vec{BA}\cdot\vec{BC}}{\|\vec{BA}\|\cdot\|\vec{BC}\|} = \dfrac{25}{\sqrt{34}\cdot\sqrt{41}} = 0.67 \Rightarrow \hat{B} = 47.96^\circ$

Donc $\hat{C} = 180^\circ - 72^\circ - 47.96^\circ \Rightarrow \hat{C} = 60^\circ$

$A_{ABC} = \frac{1}{2}\|\vec{AB}\wedge\vec{AC}\| = \frac{1}{2}\sqrt{25+9} = \frac{1}{2}\sqrt{34}$

$\vec{AB}\wedge\vec{AC} = \begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix}$

d. $d = \begin{cases} x = 3+\lambda \\ y = 2+3\lambda \\ z = 2-\lambda \end{cases} \longrightarrow \Pi : 12x + 20y + 15z - 120 = 0$

$12(3+\lambda) + 20(2+3\lambda) + 15(2-\lambda) - 120 = 0 \Leftrightarrow$

$\Leftrightarrow 36 + 12\lambda + 40 + 60\lambda + 15 - 15\lambda - 120 = 0 \Leftrightarrow$

$57\lambda = 29 \Leftrightarrow \lambda = \dfrac{29}{57} \Rightarrow$ sécants

Le point d'intersection: $I\left(\dfrac{200}{57} ; \dfrac{201}{57} ; \dfrac{85}{57}\right)$

e. • $\cos\alpha = \dfrac{\begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\left\|\begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix}\right\|\cdot 1} = \dfrac{12}{\sqrt{769}} = 0.433 \Rightarrow \alpha = 64.36^\circ$

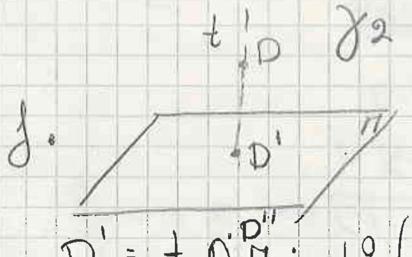
• $\cos\beta_1 = \dfrac{\begin{pmatrix} 12 \\ 20 \\ 15 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}}{\sqrt{769}\cdot\sqrt{11}} = \dfrac{57}{\sqrt{769}\cdot\sqrt{11}} = 0.62 \Rightarrow \beta_1 = 51.7^\circ$

$\Rightarrow \hat{\beta}_2 = 90 - \beta_1 \Rightarrow \beta_2 = 38.3^\circ$

• $\cos\gamma_1 = \dfrac{\left|\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}\cdot\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right|}{\sqrt{11}\cdot\sqrt{1}} = \dfrac{|-1|}{\sqrt{11}} = 0.3 \Rightarrow \gamma_1 = 72,45 \Rightarrow$

$\gamma_2 = 90 - \gamma_1 \Rightarrow \gamma_2 = 17,55^\circ$

f.



$t : \begin{cases} x = 3 + 12\lambda \\ y = 2 + 20\lambda \\ z = 2 + 15\lambda \end{cases}$

$D' = t \cap \Pi : 12(3+12\lambda) + 20(2+20\lambda) + 15(2+15\lambda) - 120 = 0$

$\Leftrightarrow 36 + 144\lambda + 40 + 400\lambda + 30 + 225\lambda - 120 = 0 \Leftrightarrow$

$\Leftrightarrow 769\lambda = 14 \Rightarrow \lambda = \dfrac{14}{769}$

$D'(3.22 ; 2.36 ; 2.3)$

$$\overrightarrow{DD'} = \begin{pmatrix} 0.22 \\ 0.36 \\ 0.3 \end{pmatrix} \quad \overrightarrow{DD''} = 2\overrightarrow{DD'} \Rightarrow \begin{pmatrix} x-3 \\ y-2 \\ z-2 \end{pmatrix} = \begin{pmatrix} 0.44 \\ 0.72 \\ 0.6 \end{pmatrix} \Leftrightarrow$$

$$\Rightarrow x = 3.44 \quad y = 2.72 \quad z = 2.6$$

$$D''(3.44 ; 2.72 ; 2.6)$$

EXERCICE 5.33

$$\vec{a} \wedge \vec{b} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -12 \\ -4 \\ 8 \end{pmatrix}$$

$$\begin{matrix} 2 & 0 \\ 0 & 4 \end{matrix}$$

$$\vec{b} \wedge \vec{a} = -\vec{a} \wedge \vec{b} = \begin{pmatrix} 12 \\ 4 \\ 8 \end{pmatrix}$$

$$\vec{a} \wedge \vec{c} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ -13 \\ 4 \end{pmatrix}$$

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix}$$

$$\vec{b} \wedge \vec{c} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} \wedge \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 16 \\ -2 \\ 4 \end{pmatrix}$$

$$\begin{matrix} 0 & -1 \\ 4 & 2 \end{matrix}$$

EXERCICE 5.34

$$\Pi : 6x + 2y + 3z - 12 = 0 \qquad 6 : x + 2y - 2z - 4 = 0$$

a. $\cos\alpha = \dfrac{|\vec{n}_\Pi \cdot \vec{n}_6|}{\|\vec{n}_\Pi\| \cdot \|\vec{n}_6\|} = \dfrac{4}{7 \cdot 3} = 0.19 \Rightarrow \alpha \approx 79°$

$$\vec{n}_\Pi \cdot \vec{n}_6 = 6 - 4 - 6 = -4 \qquad \|\vec{n}_\Pi\| = \sqrt{49} = 7 \quad \|n_6\| = \sqrt{9} = 3$$

b. sol : $6x + 2y - 12 = 0 \Rightarrow 3x + y - 6 = 0$ $\Leftrightarrow$

$$x + 2y - 4 = 0 \quad * (-3)$$

$$\begin{cases} 3x + y = 6 \\ -3x - 6y = -12 \end{cases} (+) \; -5y = -6 \Leftrightarrow y = 6/5$$

$$x = -\frac{12}{5} + 4 = \frac{8}{5}$$

$$A\left(\frac{8}{5} ; \frac{6}{5} ; 0\right)$$

paroi : $6x + 3z = 12$

$$\left.\begin{matrix} 6x + 3z = 12 \\ x - 2z = 4 \quad (-6) \end{matrix}\right\} \begin{matrix} 6x + 3z = 12 \\ (+) \; -6x + 12z = -24 \end{matrix}$$

$$15z = -12 \Leftrightarrow z = -\frac{4}{5}$$

$$x = 4 - \frac{8}{5} \Rightarrow x = \frac{12}{5}$$

$$B\left(\frac{12}{5} ; 0 ; -\frac{4}{5}\right)$$

$$\overrightarrow{AB} = \begin{pmatrix} 4/5 \\ -6/5 \\ -4/5 \end{pmatrix} /\!/ \begin{pmatrix} 4 \\ -6 \\ -4 \end{pmatrix} /\!/ \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} \qquad i = \begin{cases} x = \frac{8}{5} + 2\lambda \\ y = \frac{6}{5} - 3\lambda \\ z = -2\lambda \end{cases}$$

EXERCICE 5.35

$\Pi : 3x - 6y + 22z - 66 = 0$

$$dis(A;\Pi) = \frac{|3(-\frac{1}{3}) - 6(-2) + 22 \cdot 2.5 - 66|}{\sqrt{3^2+6^2+22^2}} = 0 \Rightarrow A \in \Pi$$

$$dis(B,\Pi) = \frac{|16+48+88-66|}{\sqrt{3^2+6^2+22^2}} = \frac{76}{23} \approx 3.3$$

$$dist(C,\Pi) = \frac{|11.2+12+125.4-66|}{23} = \frac{72.6}{23} \approx 3.16$$

EXERCICE 5.36

$A(0;4;1) \quad B(-3;1;7) \quad \Pi : 7x - 4y + 4z + 11 = 0$

a. $\vec{AB} = \begin{pmatrix} -3 \\ -3 \\ 6 \end{pmatrix} // \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \quad d = \begin{cases} x = \lambda \\ y = 4+\lambda \\ z = 1-2\lambda \end{cases}$

Soit $P \in d$: $P(\lambda; 4+\lambda; 1-2\lambda)$

$$dis(P,\Pi) = 5 \Rightarrow \frac{|7\lambda - 4(4+\lambda) + 4(1-2\lambda) + 11|}{\sqrt{49+16+16}} = 5 \Leftrightarrow$$

$\Leftrightarrow |7\lambda - 16 - 4\lambda + 4 - 8\lambda + 11| = 45 \Leftrightarrow |-5\lambda - 1| = 45 \Leftrightarrow$

$$\begin{cases} -5\lambda - 1 = 45 \Leftrightarrow 5\lambda = -46 \Rightarrow \lambda_1 = -\frac{46}{5} \\ -5\lambda - 1 = -45 \Leftrightarrow 5\lambda = 44 \Leftrightarrow \lambda = \frac{44}{5} \end{cases}$$

$$\left(-\frac{46}{5}; -\frac{26}{5}; \frac{97}{5}\right) \text{ et } \left(\frac{44}{5}; \frac{64}{5}; -\frac{83}{5}\right)$$

b. Soit $Q \in Ox$. $Q(x,0,0)$ $\vec{QA} = \begin{pmatrix} -x \\ 4 \\ 1 \end{pmatrix}$ $\vec{QB} = \begin{pmatrix} -3-x \\ 1 \\ 7 \end{pmatrix}$

$\|\vec{QA}\| = \|\vec{QB}\| \Leftrightarrow \sqrt{x^2+16+1} = \sqrt{(3+x)^2+1+49} \Leftrightarrow$

$\Leftrightarrow \cancel{x^2} + 17 = 9 + 6x + \cancel{x^2} + 50 \Leftrightarrow 6x = -42 \Leftrightarrow x = -7$

$Q(-7;0;0)$

EXERCICE 5.37

a. $A(2;3.5;-1) \quad B(3;8;-5)$

$$\vec{OA} \times \vec{OB} = \begin{pmatrix} 2 \\ 3.5 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} = \begin{pmatrix} -9.5 \\ 7 \\ 5.5 \end{pmatrix}$$

$$\begin{matrix} 2 & 3 \\ 3.5 & 8 \end{matrix}$$

$$Aire = \|\vec{OA} \times \vec{OB}\| = \sqrt{9.5^2 + 7^2 + 5.5^2} \approx 95.42$$

b. $A_1 (1; 3; -2)$ $B_1 (-6; y; 12)$

$$\vec{OA_1} \wedge \vec{OB_1} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \wedge \begin{pmatrix} -6 \\ y \\ 12 \end{pmatrix} = \begin{pmatrix} 36+2y \\ 0 \\ y+18 \end{pmatrix}$$

$$\begin{matrix} 1 & -6 \\ 3 & y \end{matrix}$$

$A = \sqrt{5} \Rightarrow \sqrt{(36+2y)^2 + (y+18)^2} = \sqrt{5} \Leftrightarrow$

$\Leftrightarrow 1296 + 4y^2 + 144y + y^2 + 36y + 324 = 5 \Leftrightarrow$

$\Leftrightarrow 5y^2 + 180y + 1615 = 0 \Leftrightarrow y^2 + 36y + 323 = 0$

$\Delta = 4$ $\quad y_{1,2} = \dfrac{-36 \pm 2}{2} < \begin{matrix} y_1 = -17 \\ y_2 = -19 \end{matrix}$

EXERCICE 5.38

$M_x (3; 0; 0)$ $\quad M_y (0; 5; 0)$ $\quad M_z (0; 0; 8)$

a. $ppmc(3; 5; 8) = 120$

$\frac{120}{3} x + \frac{120}{5} y + \frac{120}{8} z + 120 = 0 \Rightarrow$

$\Pi: 40x + 24y + 15z + 120 = 0$

b: $dis(O; \Pi) = \dfrac{|120|}{\sqrt{40^2 + 24^2 + 15^2}} = \dfrac{120}{49} \simeq 2.45$

c. On calcule l'aire de la base

$\vec{M_x M_y} = \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix}$ $\quad \vec{M_x M_z} = \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix}$ $\quad \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix} = \begin{pmatrix} 40 \\ 24 \\ 15 \end{pmatrix}$

$$\begin{matrix} -3 & -3 \\ 5 & 0 \end{matrix}$$

$Ab = \frac{1}{2} \left\| \begin{pmatrix} 40 \\ 24 \\ 15 \end{pmatrix} \right\| = \frac{49}{2}$

Aire de la pyramide: $\dfrac{Ab \times h}{3} = \dfrac{\frac{49}{2} \cdot 2.45}{3} \simeq 20$

EXERCICE 5.39

$A(2;2;-5)$ $B(4;-1;-4)$ $\vec{AB}=\begin{pmatrix}2\\-3\\1\end{pmatrix}$ $d=\begin{cases}x=2+2\lambda\\y=2-3\lambda\\z=-5+\lambda\end{cases}$

$C(2;1;1)$ $dis(C,d)=\dfrac{\|\vec{AB}\times\vec{AC}\|}{\|\vec{AB}\|}=\dfrac{\sqrt{437}}{\sqrt{14}}$

$\vec{AB}\times\vec{AC}=\begin{pmatrix}2\\-3\\1\end{pmatrix}\times\begin{pmatrix}0\\-1\\6\end{pmatrix}=\begin{pmatrix}-17\\-12\\-2\end{pmatrix}$ $\|\vec{AB}\|=\sqrt{14}$

$\begin{matrix}2 & 0\\-3 & -1\end{matrix}$ $\|\vec{AB}\times\vec{AC}\|=\sqrt{437}$

$D(-6;-3;5)$ $\vec{AD}=\begin{pmatrix}-8\\-5\\10\end{pmatrix}$ $\vec{AD}\times\vec{AB}=\begin{pmatrix}-8\\-5\\10\end{pmatrix}\times\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}25\\28\\34\end{pmatrix}$

$\|\vec{AD}\times\vec{AB}\|=\sqrt{2565}$ $\begin{matrix}-8 & 2\\-5 & -3\end{matrix}$

$dis(D,d)=\dfrac{\|\vec{AD}\times\vec{AB}\|}{\|\vec{AB}\|}=\dfrac{\sqrt{2565}}{\sqrt{14}}\approx 13.54$

$E(6;-4;-3)$ $\vec{AE}=\begin{pmatrix}4\\-6\\2\end{pmatrix}$ $\vec{AE}\times\vec{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}\times\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

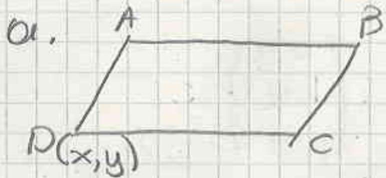
$\|\vec{AE}\times\vec{AB}\|=6$ $\begin{matrix}4 & 2\\-6 & -3\end{matrix}$

$dis(E,d)=\dfrac{\|\vec{AE}\times\vec{AB}\|}{\|\vec{AB}\|}=0\Rightarrow E\in d$

EXERCICE 5.40

$A(3;0;1)$ $B(0;1;3)$ $C(5;4;3)$

a.



$\vec{AB}=\vec{DC}\Leftrightarrow\begin{pmatrix}-3\\1\\2\end{pmatrix}=\begin{pmatrix}5-x\\4-y\\3-z\end{pmatrix}\Leftrightarrow$

$\Leftrightarrow\begin{cases}-3=5-x\Leftrightarrow x=8\\1=4-y\Leftrightarrow y=3\\2=-z+3\Leftrightarrow z=1\end{cases}$ $D(8;3;1)$

b. $\vec{AB}=\begin{pmatrix}-3\\1\\2\end{pmatrix}$ $\vec{AD}=\begin{pmatrix}5\\3\\0\end{pmatrix}$ $\vec{AB}\times\vec{AD}=\begin{pmatrix}-3\\1\\2\end{pmatrix}\wedge\begin{pmatrix}5\\3\\0\end{pmatrix}=\begin{pmatrix}-6\\10\\-14\end{pmatrix}$

$A_{\#}=\|\vec{AB}\times\vec{AD}\|=\sqrt{332}$ $\begin{matrix}-3 & 5\\1 & 3\end{matrix}$

$\vec{AC}=\begin{pmatrix}2\\4\\2\end{pmatrix}$ $\vec{AB}\times\vec{AC}=\begin{pmatrix}-3\\1\\2\end{pmatrix}\times\begin{pmatrix}2\\4\\2\end{pmatrix}=\begin{pmatrix}-6\\10\\-14\end{pmatrix}$

$\begin{matrix}-3 & 2\\1 & 4\end{matrix}$

$dist(C,AB)=\dfrac{\|\vec{AB}\times\vec{AC}\|}{\|\vec{AB}\|}=$

$\|\vec{AB}\times\vec{AC}\|=\dfrac{\sqrt{332}}{\sqrt{14}}\approx 4.87$

c. $\vec{n}_\Pi = \vec{AB} \times \vec{AC} = \begin{pmatrix}-6\\10\\-14\end{pmatrix} \parallel \begin{pmatrix}3\\-5\\7\end{pmatrix}$

$\left.\begin{array}{l}\Pi: 3x-5y+7z+d=0\\ A: 9+7+d=0 \Leftrightarrow d=-16\end{array}\right\} \Pi: 3x-5y+7z-16=0.$

d. $\vec{n}_{\Pi'} = \vec{n}_\Pi \times \vec{AB} = \begin{pmatrix}3\\-5\\7\end{pmatrix} \times \begin{pmatrix}-3\\1\\2\end{pmatrix} = \begin{pmatrix}-17\\-27\\-12\end{pmatrix} \parallel \begin{pmatrix}17\\27\\+12\end{pmatrix}$

$\begin{matrix}3 & & -3\\ -5 & & 1\end{matrix}$

$\left.\begin{array}{l}17x+27y+12z+d=0\\ A: 51+12+d=0 \Leftrightarrow d=-63\end{array}\right\} \Pi': 17x+27y+12z-63=0$

$C(5;4,3)$

e. $dist(C,\Pi') = \dfrac{|85+108+36-63|}{\sqrt{17^2+27^2+12^2}} = \dfrac{166}{\sqrt{1162}} \simeq 4,87 = dis(C,AB)$

EXERCICE 5.41

$d_1 = \begin{cases} x = 3-2\lambda \\ y = 1+\lambda \\ z = -4\lambda \end{cases}$ $d_2: A(0;3;-7)$ $B(-4;5;-15)$

$C(3;1;0)$ $\vec{d_1} = \begin{pmatrix}-2\\1\\-4\end{pmatrix}$ $\vec{AB} = \begin{pmatrix}-4\\2\\-8\end{pmatrix} \parallel \begin{pmatrix}2\\-1\\4\end{pmatrix}$

évidement $d_1 \parallel d_2 \rightarrow dist(d_1,d_2) = dis(A,d_1)$

$dis(A,d_1) = \dfrac{\|\vec{CA} \times \vec{d_1}\|}{\|\vec{d_1}\|} = \dfrac{\sqrt{6}}{\sqrt{21}} \approx 0.53$

$\vec{CA} = \begin{pmatrix}-3\\2\\-7\end{pmatrix}$ $\vec{CA} \times \vec{d_1} = \begin{pmatrix}-3\\2\\-7\end{pmatrix} \times \begin{pmatrix}-2\\1\\-4\end{pmatrix} = \begin{pmatrix}-1\\2\\1\end{pmatrix}$

$\begin{matrix}-3 & & -2\\ 2 & & 1\end{matrix}$

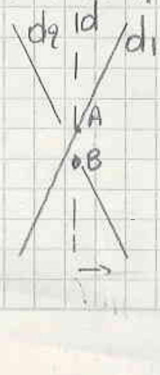
EXERCICE 5.42.

$d_1 = \begin{cases} x = 1+2m_1 \\ y = 2-m_1 \\ z = 3+3m_1 \end{cases}$ $d_2: \begin{cases} x = 3+3m_2 \\ y = 1+m_2 \\ z = -1+2m_2 \end{cases}$

a. $\vec{d_1} = \begin{pmatrix}2\\-1\\3\end{pmatrix} \not\parallel \vec{d_2} = \begin{pmatrix}3\\1\\2\end{pmatrix} \Rightarrow$ sécants / gauches

(1) $1+2m_1 = 3+3m_2$
(2) $2-m_1 = 1+m_2$ $*(2)$ $\left.\begin{array}{l}2m_1 = 2+3m_2\\ -2m_1 = -1+m_2\end{array}\right\}$ (+) $4m_2 = -1 \Leftrightarrow m_2 = -\frac{1}{2}$ $m_1 = \frac{3}{2}$
(3) $3+3m_1 = -1+2m_2$ On remplace en (3): $3+\frac{9}{2} = -2$ imp $\Rightarrow$ gauches

b.



$\vec{d} = \vec{d_1} \wedge \vec{d_2} = \begin{pmatrix}2\\-1\\3\end{pmatrix} \wedge \begin{pmatrix}3\\1\\2\end{pmatrix} = \begin{pmatrix}-5\\5\\5\end{pmatrix} \parallel \begin{pmatrix}1\\-1\\-1\end{pmatrix}$

$\begin{matrix}2 & & 3\\ -1 & & 1\end{matrix}$

On cherche le plan Π: $d, d_2 \in \Pi$

$\vec{n}_\Pi = \vec{d}_1 \wedge \vec{d}_2 = \begin{pmatrix}1\\-1\\-1\end{pmatrix} \wedge \begin{pmatrix}3\\1\\2\end{pmatrix} = \begin{pmatrix}-1\\-5\\4\end{pmatrix} \parallel \begin{pmatrix}1\\5\\-4\end{pmatrix}$

$(3, 1; -1) \in \Pi: +3+5+4+d=0 \Leftrightarrow d=12$

$\Pi: x+5y-4z-12=0$

$A = \Pi \cap d_1: (1+2m_1)+5(2-m_1)-4(3+3m_1)-12=0 \Leftrightarrow$

$\Leftrightarrow 1+2m_1+10-5m_1-12-12m_1-12=0 \Leftrightarrow -15m_1=13 \Rightarrow$

$\Leftrightarrow m_1 = -\frac{13}{15} \Rightarrow A\left(1-\frac{26}{15}; 2+\frac{13}{15}; 3-\frac{39}{15}\right) = \left(-\frac{11}{15}; \frac{43}{15}; \frac{6}{15}\right)$

$d: \begin{cases} x = -\frac{11}{15}+\lambda \\ y = \frac{43}{15}-\lambda \\ z = \frac{6}{15}-\lambda \end{cases}$

$B = d_2 \cap d \quad \left.\begin{array}{l} -\frac{11}{15}+\lambda = 3+3m_2 \\ \frac{43}{15}-\lambda = 1+m_2 \\ \frac{6}{15}-\lambda = -1+2m_2 \end{array}\right\} (+) \quad \frac{32}{15} = 4+4m_2 \Leftrightarrow m_2 = -\frac{7}{15}$

$B\left(\frac{8}{5}; \frac{8}{15}; -2\right)$

c. $dis(d_1, d_2) = \|\vec{AB}\| = 4.08$

$\vec{AB} = \begin{pmatrix}\frac{8}{5}+\frac{11}{15}\\ \frac{8}{15}-\frac{43}{15}\\ -2-\frac{6}{15}\end{pmatrix} = \begin{pmatrix}\frac{35}{15}\\ -\frac{35}{15}\\ -\frac{36}{15}\end{pmatrix} = \begin{pmatrix}\frac{7}{3}\\ -\frac{7}{3}\\ -\frac{12}{5}\end{pmatrix}$

d. C'est le point A.

EXERCICE 5.43

a. $(x-8)^2+(y+2)^2+(z-7)^2 = 12^2$

b. AB diametre: $C\left(\frac{-1+5}{2}; \frac{11+9}{2}; \frac{25-13}{2}\right) = (2; 10; 6)$

$r = \frac{1}{2}\|AB\| \quad \vec{AB} = \begin{pmatrix}6\\-2\\-38\end{pmatrix} \quad \|\vec{AB}\| = 38.523 \Rightarrow r =$

$(x-2)^2+(y-10)^2+(z-6)^2 = 371$

c. tangente au sol $\Rightarrow |z_0| = r \Rightarrow r = 10$

$(x-1)^2+(y+7)^2+(z+10)^2 = 100$

d. $r = dis(\Pi: A) = \frac{|4+2-10-4|}{\sqrt{1+4+4}} = \frac{|-8|}{3} = \frac{8}{3}$

$(x-4)^2+(y-1)^2+(z+5)^2 = \frac{64}{9}$

EXERCICE 5.44

a. $x^2+y^2+z^2-6x+4y-18z+97=0 \Leftrightarrow$

$(x-3)^2-9+(y+2)^2-4+(z-9)^2-81+97=0 \Leftrightarrow$

$(x-3)^2+(y+2)^2+(z-9)^2 = -3$ impossible

b. $x^2+y^2+z^2-2y+3z-5.75=0 \Leftrightarrow x^2+(y-1)^2-1+\left(z+\frac{3}{2}\right)^2-\frac{9}{4}-5.75=0$

$\Leftrightarrow x^2+(y-1)^2+\left(z+\frac{3}{2}\right)^2=9 \quad C\left(0;1;-\frac{3}{2}\right) \quad r=3.$

c. $3x^2+y^2+z^2+66x-8y+10z+378=0$ pas une sphère.

d. $2x^2+2y^2+2z^2-12x+28y-48z+204=0 \Leftrightarrow$

$\Leftrightarrow x^2+y^2+z^2-6x+14y-24z+102=0 \Leftrightarrow$

$\Leftrightarrow (x-3)^2-9+(y+7)^2-49+(z-12)^2-144+102=0$

$\Leftrightarrow (x-3)^2+(y+7)^2+(z-12)^2=100 \quad C(3;-7;12) \quad r=10.$

EXERCICE 5.45

$P(2;3;z) \quad (x-4)^2+y^2+(z+1)^2=64 \quad C(4;0;-1) \quad r=8$

$d(P,C)<r \Rightarrow \sqrt{(2-4)^2+(3)^2+(z+1)^2}<8 \Leftrightarrow 4+9+(z+1)^2<64 \Leftrightarrow$

$(z+1)^2<51 \Leftrightarrow |z+1|<\sqrt{51} \Leftrightarrow -\sqrt{51}<z+1<\sqrt{51} \Leftrightarrow$

$-\sqrt{51}-1<z<\sqrt{51}+1 \Rightarrow -8,14<z<6.14$

EXERCICE 5.46

a. $\Omega(0;4;5) \quad A(-3;4;5) \quad \vec{A\Omega}=\begin{pmatrix}3\\0\\0\end{pmatrix} \quad \|\vec{A\Omega}\|=3=r.$

$x^2+(y-4)^2+(z-5)^2=9$

b. $d_1=\begin{cases}x=3-2\lambda\\y=7-\lambda\\z=2+\lambda\end{cases} \qquad d_2:\begin{cases}x=3+\frac{1}{2}\mu\\y=7-\mu\\z=2+\mu\end{cases}$

$d_1\cap\Sigma: \quad (3-2\lambda)^2+(7-\lambda-4)^2+(2+\lambda-5)^2=9 \Leftrightarrow$

$\Leftrightarrow (3-2\lambda)^2+(3-\lambda)^2+(\lambda-3)^2=9 \Leftrightarrow 9-12\lambda+4\lambda^2+9-6\lambda+\lambda^2+\lambda^2-6\lambda+\cancel{9}=\cancel{9}$

$\Leftrightarrow 6\lambda^2-24\lambda+18=0 \Leftrightarrow \lambda^2-4\lambda+3=0 \Leftrightarrow \lambda_1=3 \quad \lambda_2=1.$

$I_1(-3;4;5) \quad I_2(1;6;3)$

$d_2\cap\Sigma: \quad \left(3+\frac{1}{2}\mu\right)^2+(3-\mu)^2+(\mu-3)^2=9 \Leftrightarrow$

$\Leftrightarrow 9+3\mu+\frac{1}{4}\mu^2+9-6\mu+\mu^2+\mu^2-6\mu+\cancel{9}=\cancel{9} \Leftrightarrow$

$\Leftrightarrow \frac{9}{4}\mu^2-9\mu+18=0 \Leftrightarrow \frac{1}{4}\mu^2-\mu+2=0 \Leftrightarrow \mu^2-4\mu+8=0$

$\Delta=16-32<0$ disjoints

c. $d_3=\begin{cases}x=3+k\tau\\y=7-\tau\\z=2+\tau\end{cases} \qquad d(\Omega;d_3)=r \qquad D(3;7;2)$

$dis(\Omega,d_3)=\dfrac{\|\vec{d_3}\wedge\vec{D\Omega}\|}{\|\vec{d_3}\|}$

$\vec{D\Omega}=\begin{pmatrix}-3\\-3\\3\end{pmatrix} \quad \vec{d_3}\wedge\vec{D\Omega}=\begin{pmatrix}k\\-1\\1\end{pmatrix}\wedge\begin{pmatrix}-3\\-3\\3\end{pmatrix}=\begin{pmatrix}0\\-3-3k\\-3k-3\end{pmatrix}$

$\begin{matrix}k & -3\\-1 & -3\end{matrix}$

$\left.\begin{array}{l}\|\vec{d_3}\wedge\vec{D\Omega}\|=\sqrt{(3+3k)^2+(3k+3)^2}\\ \|\vec{d_3}\|=\sqrt{k^2+1+1}=\sqrt{2+k^2}\end{array}\right\} dis(\Omega,d_3)=\dfrac{\sqrt{2(3+3k)^2}}{\sqrt{k^2+2}}$

$d(\underline{O}; d_3) = r \Rightarrow \sqrt{\frac{2(3+3k)^2}{k^2+2}} = 3 \Leftrightarrow 2(3+3k)^2 = 9k^2+18 \Leftrightarrow$

$\Leftrightarrow 2(9+18k+9k^2) = 9k^2+18 \Leftrightarrow \cancel{18}+36k+18k^2 = 9k^2+\cancel{18} \Leftrightarrow$

$\Leftrightarrow 9k^2+36k=0 \Leftrightarrow k^2+4k=0 \Leftrightarrow k(k+4)=0 \Leftrightarrow k=0 \quad k=-4$

EXERCICE 5.47

a. $x^2+y^2+z^2-6x-8y-17z+25=0$

$\Leftrightarrow (x-3)^2-9+(y-4)^2-16+(z-\frac{17}{2})^2-\frac{17^2}{4}+25=0 \Leftrightarrow$

$\Leftrightarrow (x-3)^2+(y-4)^2+(z-\frac{17}{2})^2 = (\frac{17}{2})^2 \quad \underline{O}(3;4;\frac{17}{2}) \quad \rho=\frac{17}{2}$

b. $dist(C,\Pi) = \frac{|3+8+17+8|}{\sqrt{1+4+4}} = \frac{36}{3} = 12 > \rho \Rightarrow$ disjoints

Comme $|z_0| = \rho \Rightarrow$ tangents

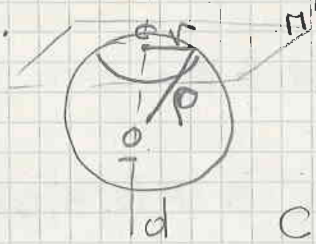
c. $\Pi_1: x+2y+2z+d=0$

$dist(\Pi_1, \underline{O}) = r \Rightarrow \frac{|3+8+17+d|}{3} = \frac{17}{2} \Rightarrow |28+d| = \frac{51}{2} \Rightarrow$

$\Rightarrow 28+d = \frac{51}{2} \Rightarrow d_1 = -\frac{5}{2}$ ou $28+d_2 = -\frac{51}{2} \Rightarrow d_2 = -\frac{107}{2}$

$\Pi_1: x+2y+2z-2.5=0 \qquad \Pi_2: x+2y+2z-53.5=0$

d.



$\Pi': x+2y+2z-10=0$

$\vec{d} = \vec{n}_{\Pi'} = \begin{pmatrix}1\\2\\2\end{pmatrix} \Rightarrow d: \begin{cases} x = 3+\lambda \\ y = 4+2\lambda \\ z = \frac{17}{2}+2\lambda \end{cases}$

$C = \Pi' \cap d: (3+\lambda)+2(4+2\lambda)+2(\frac{17}{2}+2\lambda)-10=0 \Leftrightarrow$

$\Leftrightarrow 3+\lambda+8+4\lambda+17+4\lambda-10=0 \Leftrightarrow 9\lambda=-18 \Rightarrow \lambda=-2$

$C(1;0;4.5)$

$C\underline{O} = dist(\underline{O};\Pi') = \frac{|3+8+17-10|}{3} = \frac{18}{3} = 6.$

$\rho^2 = r^2 + C\underline{O}^2 \Rightarrow r^2 = \frac{17^2}{4} - 6^2 \Rightarrow r^2 = \frac{145}{4} \Rightarrow r = \frac{\sqrt{145}}{2}$

c. $p: \begin{cases} x = 7 + \\ y = 4 \\ z = \lambda \end{cases} \quad // \begin{pmatrix}0\\0\\1\end{pmatrix}$

$p \cap \sigma: \quad 4^2+0+(\lambda-\frac{17}{2})^2 = (\frac{17}{2})^2 \Rightarrow (\lambda-\frac{17}{2})^2 = \frac{225}{4} \Rightarrow$

$\lambda - \frac{17}{2} = \pm\frac{15}{2} \Rightarrow \lambda_1 = \frac{32}{2} = 16 \quad \lambda_2 = 1$

$A(7;4;16) \qquad B(7,4;1)$

$\vec{\underline{O}A} = \begin{pmatrix}4\\0\\7.5\end{pmatrix} = \vec{n}_A \qquad \Pi_A: 4x+7.5z+d=0$

$A: 28+120+d=0 \Rightarrow d=-148$

$\Pi_A: 4x+7.5z-148=0$

$\vec{\underline{O}B} = \begin{pmatrix}4\\0\\-7.5\end{pmatrix} = \vec{n}_B \qquad \Pi_B: 4x+7.5z+d=0$

$B: 28-7.5+d=0 \Rightarrow d=-20.5$

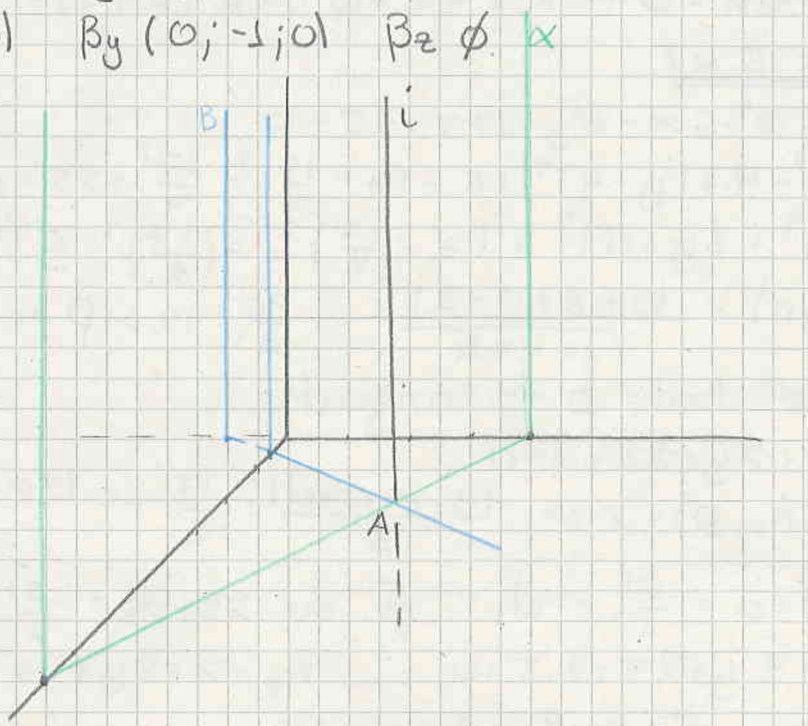
$\Pi_B: 4x-7.5z-20.5=0 \rightarrow 8x-15z-41=0$

EXERCICE 5.48

a. $\alpha : x+2y-8=0 \qquad \beta : 2x-y-1=0$

a) $\alpha_x\ (8;0;0) \quad \alpha_y\ (0;4;0) \quad \alpha_z\ \emptyset$

$\beta_x\ (\frac{1}{2};0;0) \quad \beta_y\ (0;-1;0) \quad \beta_z\ \emptyset$



b) $\vec{n}_\alpha = \begin{pmatrix}1\\2\\0\end{pmatrix} \quad \vec{n}_\beta = \begin{pmatrix}2\\-1\\0\end{pmatrix} \quad \vec{n}_\alpha \cdot \vec{n}_\beta = 0 \Rightarrow \alpha \perp \beta$

c) $\left.\begin{array}{l} x+2y=8 \\ 2x-y=1 \end{array}\right\} \left.\begin{array}{l} x+2y=8 \\ 4x-2y=2 \end{array}\right\} 5x=10 \Leftrightarrow x=2 \Rightarrow y=3 \quad A(2;3;0)$

$i : \begin{cases} x=2 \\ y=3 \\ z=\lambda \end{cases}$ car $i \parallel Oz$

b. e) $O_1\ (1;1;3) \quad dist(O_1, \alpha) = r \Rightarrow r = \dfrac{|1+2-8|}{\sqrt{5}} = \dfrac{5}{\sqrt{5}} = \sqrt{5}$

Donc $\Sigma_1 : (x-1)^2+(y-1)^2+(z-3)^2=5.$

g) 

On trouve la droite O_1T:

$\vec{O_1T} = \vec{n}_\alpha = \begin{pmatrix}1\\2\\0\end{pmatrix} \quad O_1T : \begin{cases} x=1+\lambda \\ y=1+2\lambda \\ z=3 \end{cases}$

$T = \alpha \cap O_1T \quad (1+\lambda)+2(1+2\lambda)-8=0 \Leftrightarrow 1+\lambda+2+4\lambda-8=0$

$\Leftrightarrow 5\lambda=5 \Leftrightarrow \lambda=1 \Rightarrow T(2;3;3)$

h) $\vec{O_1R} = \begin{pmatrix}1\\2\\-3\end{pmatrix} \quad OR : \begin{cases} x=2+\lambda \\ y=3+2\lambda \\ z=-3\lambda \end{cases} \quad \vec{OR} = \begin{pmatrix}2\\3\\0\end{pmatrix}$

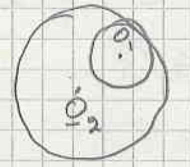
$dis(O, O_1R) = \dfrac{\|\vec{OR} \wedge \vec{O_1R}\|}{\|\vec{O_1R}\|} = \dfrac{\sqrt{118}}{\sqrt{14}} \approx 2.9$

$\vec{OR} \wedge \vec{O_1R} = \begin{pmatrix}2\\3\\0\end{pmatrix} \wedge \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}9\\-6\\1\end{pmatrix}$

$\begin{matrix}2 & & 1\\3 & & 2\end{matrix}$

i) $\text{Aire} = \frac{1}{2}\|\vec{RO} \wedge \vec{RO_1}\| = \frac{1}{2}\sqrt{9^2+36+1} \approx 5.43$

c. $\Sigma_2 : \Omega_2(1;1;z) \quad r=6$

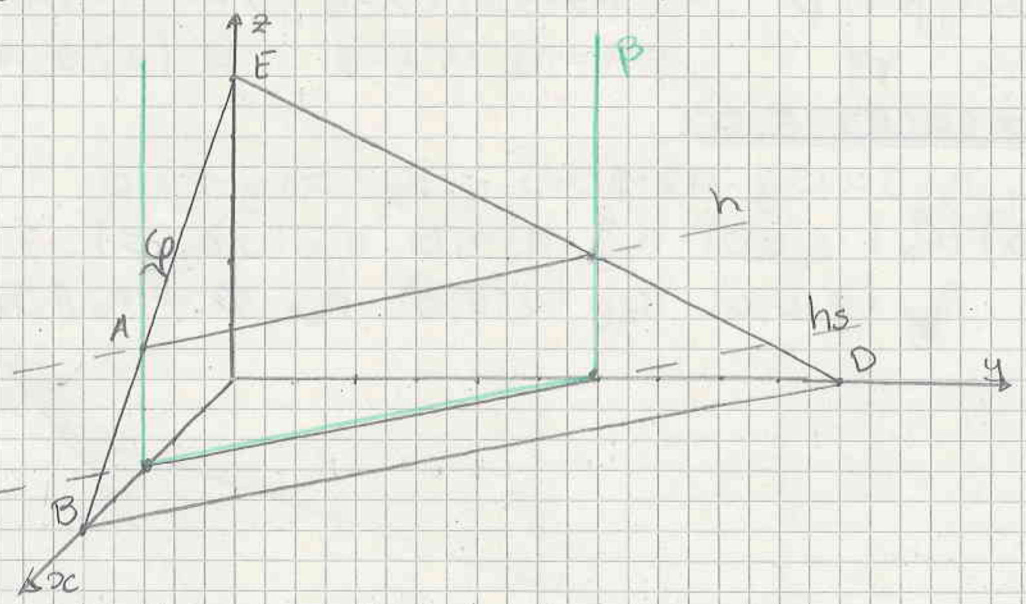


$\|\vec{\Omega_1\Omega_2}\| = r_2 - r_1 \Rightarrow \left\|\begin{pmatrix}0\\0\\z-3\end{pmatrix}\right\| = 6-\sqrt{5} \Rightarrow$

$\Rightarrow |z-3| = 6-\sqrt{5} \Rightarrow \begin{cases} z-3 = 6-\sqrt{5} \Rightarrow z_1 = 9-\sqrt{5} \\ z-3 = -6+\sqrt{5} \Rightarrow z_2 = -3+\sqrt{5} \end{cases}$

EXERCICE 5.49

a. $\alpha: 2x+y+2z-10=0 \quad \overset{B}{\alpha_x}(5;0;0) \quad \overset{D}{\alpha_y}(0,10;0) \quad \overset{E}{\alpha_z}(0;0;5)$



b. $h: A(3;0;2) \quad h$ horizontale $\Rightarrow h \parallel BD$

$\vec{BD} = \begin{pmatrix}-5\\10\\0\end{pmatrix} \parallel \begin{pmatrix}1\\-2\\0\end{pmatrix} \qquad h: \begin{cases} x = 3+\lambda \\ y = -2\lambda \\ z = 2 \end{cases}$

c. $S: C(-2;0;7) \qquad r = \|\vec{AC}\| = \sqrt{(-2-3)^2+(0)^2+(7-2)^2} = \sqrt{50} = 5\sqrt{2}$

a) $(x+2)^2+y^2+(z-7)^2 = 50$

b) $F = h \cap s. \quad (3+\lambda+2)^2+(-2\lambda)^2+(2-7)^2 = 50 \Leftrightarrow (5+\lambda)^2+4\lambda^2+25 = 50$

$\Leftrightarrow 25+10\lambda+\lambda^2+4\lambda^2-25 = 0 \Leftrightarrow 5\lambda^2+10\lambda = 0 \Leftrightarrow 5\lambda(\lambda+2) = 0 \Leftrightarrow$

$\lambda = 0 \quad \lambda = -2. \qquad F(1;4;2)$

d. $\vec{d} = \vec{h} \wedge \vec{AC} = \begin{pmatrix}1\\-2\\0\end{pmatrix} \wedge \begin{pmatrix}-5\\0\\5\end{pmatrix} = \begin{pmatrix}-10\\-5\\-10\end{pmatrix} \parallel \begin{pmatrix}2\\1\\2\end{pmatrix}$

$d: \begin{cases} x = 3+2\mu \\ y = \quad +\mu \\ z = 2+2\mu \end{cases}$

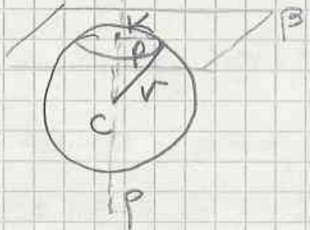
e. $\beta: 2x+y+k=0 \quad \Rightarrow \beta: 2x+y-6=0$

a) $h \in \beta: \; 2(3+\lambda)+(-2\lambda)+k=0 \Leftrightarrow 6+2\lambda-2\lambda+k=0 \Leftrightarrow k=-6$

b) $\beta_x(3;0;0) \quad \beta_y(0;6;0) \quad \beta_z \; \emptyset \Rightarrow \beta \parallel Oz$

c) $\cos\varphi = \dfrac{\vec{n}_\alpha \cdot \vec{n}_\beta}{\|\vec{n}_\alpha\| \cdot \|\vec{n}_\beta\|} = \dfrac{\begin{pmatrix}2\\1\\2\end{pmatrix}\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}}{\sqrt{9}\cdot\sqrt{5}} = \dfrac{5}{3\sqrt{5}} = \dfrac{\sqrt{5}}{3} \Rightarrow \varphi = 41.8^\circ$

d)



$r = 5\sqrt{2}$ $\quad \beta: 2x+y+6=0 \quad C(-2;0;7)$

$\text{dist}(\beta;C) = \dfrac{|-4-6|}{\sqrt{5}} = \dfrac{10}{\sqrt{5}} = 2\sqrt{5}$

$\rho^2 = r^2 - KC^2 \Rightarrow \rho^2 = 50-20 \Rightarrow \rho = \sqrt{30}$

$p: \vec{p} = \vec{n}_\beta = \begin{pmatrix}2\\1\\0\end{pmatrix} \quad p: \begin{cases} x = -2+2\lambda \\ y = \lambda \\ z = 7 \end{cases}$

$K = \beta \cap p : \; 2(-2+2\lambda)+\lambda-6 = 0 \Leftrightarrow -4+4\lambda+\lambda-6=0 \Leftrightarrow$

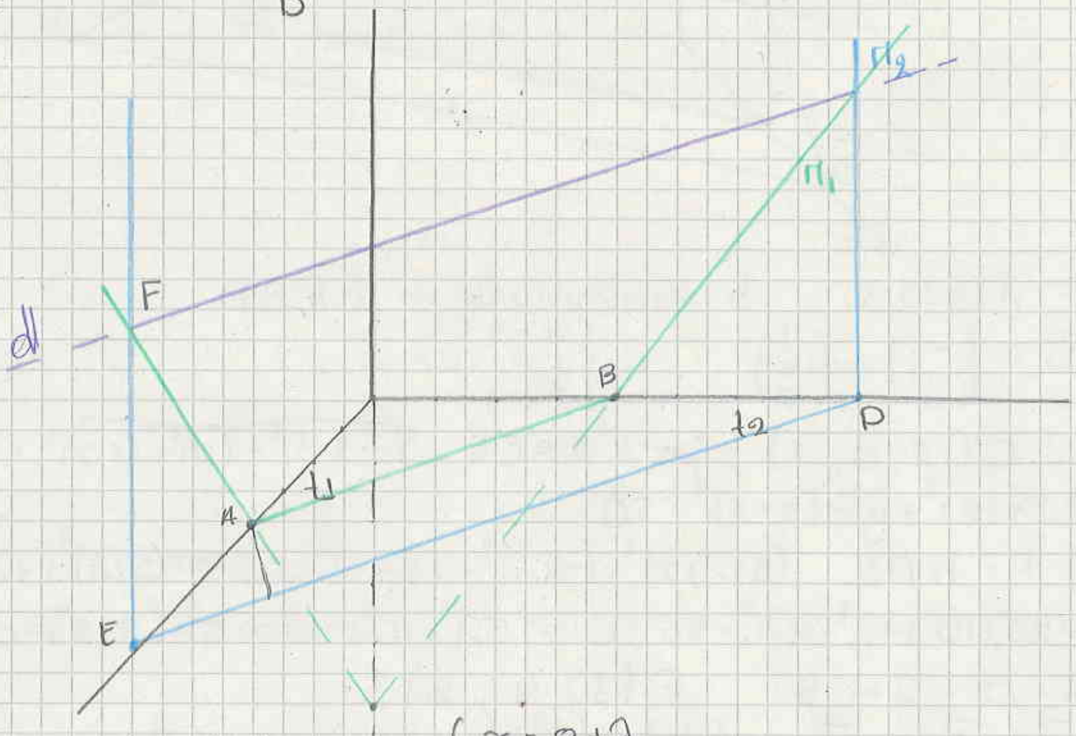
$5\lambda = 10 \Rightarrow \lambda = 2. \quad K(2;2;7)$

EXERCICE 5.50

a. $\pi_1: 5x+5y-4z-20=0 \quad \pi_2: x+y-8=0$

a) π_{1x} A $(4;0;0)$ $\quad \pi_{1y}$ B $(0;4;0)$ $\quad \pi_{1z}\,(0;0;-5)$

π_{2x} E $(8;0;0)$ $\quad \pi_{2y}$ D $(0;8;0)$ $\quad \pi_{2z}\;\emptyset \Rightarrow \pi_2 \,//\, Oz$



b) $\vec{t}_2 = \begin{pmatrix}-8\\8\\0\end{pmatrix} // \begin{pmatrix}1\\-1\\0\end{pmatrix} \quad t_2: \begin{cases} x = 8+\lambda \\ y = -\lambda \\ z = 0 \end{cases}$

$\text{dist}(t_1,t_2) = \text{dist}(A,t_2) = \dfrac{\|\vec{EA} \wedge \vec{t}_2\|}{\|\vec{t}_2\|} = \dfrac{4}{\sqrt{2}} = 2\sqrt{2}$

$\vec{EA} \wedge \vec{t}_2 = \begin{pmatrix}-4\\0\\0\end{pmatrix} \wedge \begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\4\end{pmatrix} \quad \|\vec{t}_2\| = \sqrt{2}$

$\begin{matrix}-4 & & 1\\ 0 & & -1\end{matrix}$

c) $\cos\varphi = \dfrac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|} = \dfrac{\begin{pmatrix}5\\5\\-4\end{pmatrix}\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}}{\sqrt{66}\;\sqrt{2}} = 0.87 \Rightarrow \varphi = 29.5^\circ$

e) $\vec{d} \parallel \vec{t_2} \Rightarrow \vec{d} = \begin{pmatrix}1\\-1\\0\end{pmatrix}$

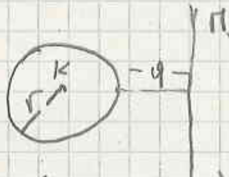
F: $y=0$: $\left.\begin{matrix}5x-4z=20\\ x=8\end{matrix}\right\} \Rightarrow \left.\begin{matrix}40-20=4z \Leftrightarrow z=5\\ x=8\end{matrix}\right\} \Rightarrow F(8;0;5)$

$d: \begin{cases} x = 8+\lambda \\ y = -\lambda \\ z = 5. \end{cases}$

b. $S: (x-3)^2+(y+2)^2+(z-1)^2=100 \quad K(3;-2;1) \quad r=10$

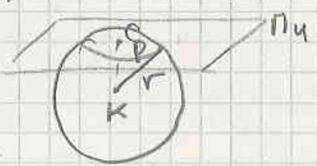
$\Pi_3: x+2y-2z+40=0$

f) $dis(K,\Pi_3) = \dfrac{\|3-4-2+40\|}{\sqrt{1+4+4}} = \dfrac{|37|}{\sqrt{9}} = \dfrac{37}{3} = 12.\bar{3} > r \Rightarrow$ disjoints.



$q = dis(K,\Pi_3) - r = 12.3 - 10 \approx 2.3$

g) $C(1;-6;5) \quad \vec{CK} = \begin{pmatrix}2\\4\\-4\end{pmatrix} \parallel \begin{pmatrix}1\\2\\-2\end{pmatrix} = \vec{n_4}$



$\Pi_4: x+2y-2z+d=0$

$C \in \Pi_4: 1-12-10+d=0 \Rightarrow d=-21$

$\Pi_4: x+2y-2z-21=0$

$\|\vec{CK}\| = 3 \quad \rho^2 = r^2 - CK^2 \Rightarrow \rho^2 = 100-9 \Rightarrow \rho = \sqrt{91} \approx 9.54$

h) $t: \begin{cases} x = 13+4\lambda \\ y = 3-3\lambda \\ z = 3+2\lambda \end{cases} \longrightarrow S$

$(13+4\lambda-3)^2+(3-3\lambda+2)^2+(3+2\lambda-1)^2=100 \Leftrightarrow$

$\Leftrightarrow (10+4\lambda)^2+(5-3\lambda)^2+(2+2\lambda)^2=100 \Leftrightarrow$

$\Leftrightarrow 100+80\lambda+16\lambda^2+25-30\lambda+9\lambda^2+4+8\lambda+4\lambda^2=100 \Leftrightarrow$

$\Leftrightarrow 29\lambda^2+58\lambda+29=0 \Leftrightarrow \lambda^2+2\lambda+1=0 \Leftrightarrow (\lambda+1)^2=0 \Leftrightarrow \lambda=-1$

$\Rightarrow$ t est tangente à la sphère. $T(9;6;1)$

i) $T(9;6;1): (9-3)^2+(6+2)^2+(1-1)^2=100 \checkmark \Rightarrow T \in S.$

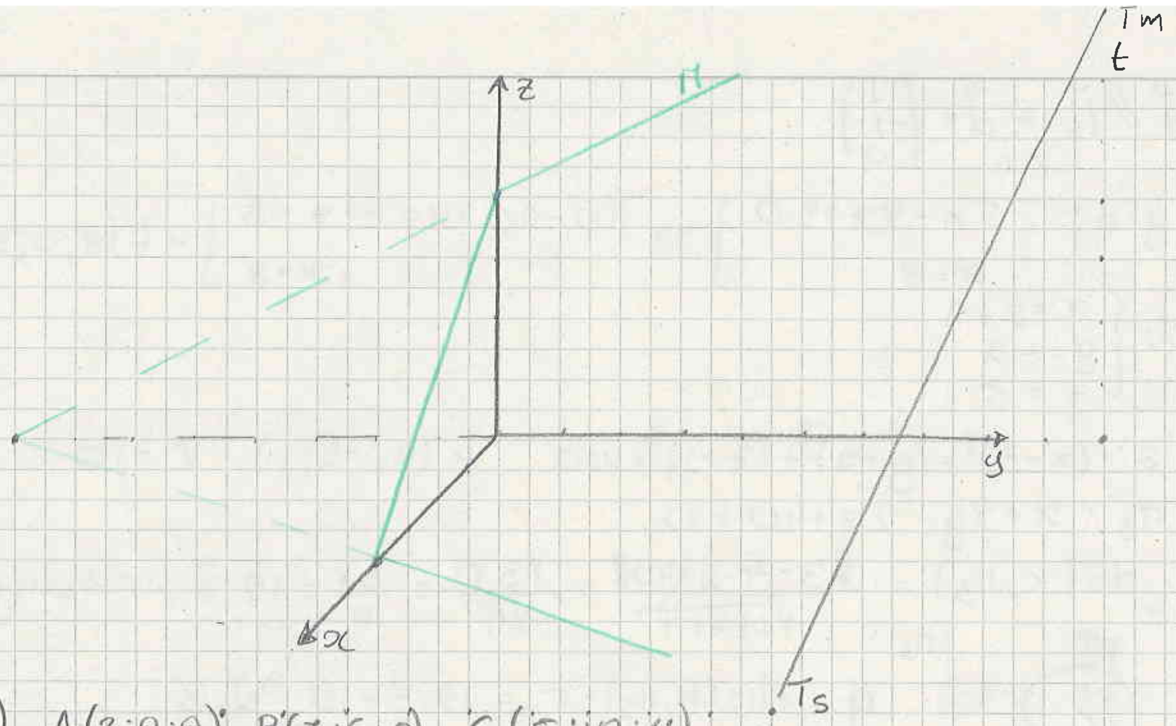
j) $\vec{t'} = \vec{CT} \wedge \vec{t} = \begin{pmatrix}8\\12\\-4\end{pmatrix} \wedge \begin{pmatrix}4\\-3\\2\end{pmatrix} = \begin{pmatrix}12\\-32\\-72\end{pmatrix} \parallel \begin{pmatrix}3\\-8\\-18\end{pmatrix} = \vec{t'}$

$\begin{matrix}8 & & 4\\12 & & -3\end{matrix}$

Exercice 5.51

a. $\Pi. 2x-y+2z-8=0 \quad A(3;2;2) \quad B(7,6;0) \quad C(5;10;4)$

$\Pi_x(4;0;0) \quad \Pi_y(0;-8;0) \quad \Pi_z(0;0;4)$



b) $A(3;2;2)$ $B(7;6;0)$ $C(5;10;4)$

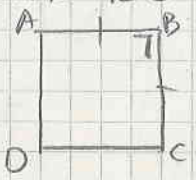
$\vec{AB} = \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix}$ $\|\vec{AB}\| = 6$

$\vec{AC} = \begin{pmatrix} 2 \\ 8 \\ 2 \end{pmatrix}$ $\|\vec{AC}\| = \sqrt{44}$

$\vec{BC} = \begin{pmatrix} -2 \\ 4 \\ 4 \end{pmatrix}$ $\|\vec{BC}\| = 6$

$\|\vec{AB}\| = \|\vec{BC}\|$

$\vec{AB} \cdot \vec{BC} = -8 + 16 - 8 = 0 \Rightarrow \vec{AB} \perp \vec{BC}$



Il souffit de trouver D tel que ABCD est un parallelogramme

$\vec{AB} = \vec{DC} \Rightarrow \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 5-x \\ 10-y \\ 4-z \end{pmatrix} \Leftrightarrow \begin{cases} x = 1 \\ y = 6 \\ z = 2. \end{cases}$ $D(1;6;2)$

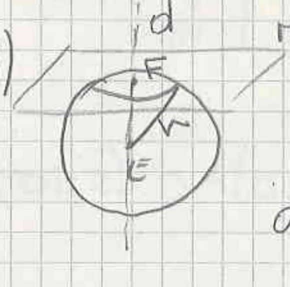
b.c) S: $E(8;4;7)$ $A(3;2;2)$ $\Pi: 2x - y + 2z - 8 = 0$

$r = \|\vec{EA}\| = \sqrt{(-5)^2 + (-2)^2 + (-5)^2} = \sqrt{54}$

$\|\vec{EB}\| = \sqrt{1 + 2^2 + (-7)^2} = \sqrt{54} \Rightarrow B \in S$

$\|\vec{EC}\| = \sqrt{3^2 + 6^2 + 3^2} = \sqrt{54} \Rightarrow C \in S$

d)



$EF = dist(E, \Pi) = \dfrac{|16 - 4 + 14 - 8|}{\sqrt{4+1+4}} = \dfrac{18}{3} = 6$

$\rho^2 = r^2 - EF^2 \Rightarrow \rho = \sqrt{54 - 36} = \sqrt{18}$

d: $\vec{d} = \vec{n}_\Pi = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ d: $\begin{cases} x = 8 + 2\lambda \\ y = 4 - \lambda \\ z = 7 + 2\lambda \end{cases}$

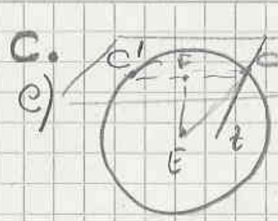
$E \in d$

$F = d \cap \Pi$: $2(8+2\lambda) - (4 - \lambda) + 2(7 + 2\lambda) - 8 = 0 \Leftrightarrow 16 + 4\lambda - 4 + \lambda + 14 + 4\lambda - 8 = 0$

$\Leftrightarrow 9\lambda = -18 \Rightarrow \lambda = -2$ $F(4;6;3)$

C.

e)



$\vec{t} = \vec{EF} \wedge \vec{CE} = \vec{n}_\pi \wedge \vec{CE}$

$\vec{n}_\pi = \begin{pmatrix}2\\-1\\2\end{pmatrix} \wedge \vec{CE} = \begin{pmatrix}3\\-6\\3\end{pmatrix} = \begin{pmatrix}9\\0\\-9\end{pmatrix} \parallel \begin{pmatrix}1\\0\\-1\end{pmatrix} = \vec{t}$

$2 \qquad 3$

$-1 \qquad -6$

$C \in t$: $\quad t: \begin{cases} x = 5 + \lambda \\ y = 10 + \\ z = 4 - \lambda \end{cases}$

f) T_S: $z = 0 \Rightarrow \lambda = 4$. $\quad T_S(9;10;0)$

T_m: $x = 0 \Rightarrow \lambda = -5 \quad T_m(0;10;9)$

g) $\vec{t'} = \vec{t} = \begin{pmatrix}1\\0\\-1\end{pmatrix}$ tangente à S à C'

$C(5;10;4) \quad F(4;6;3)$

$S: (x-8)^2 + (y-4)^2 + (z-7)^2 = 54$

$\vec{CF} = \vec{FC'} \Rightarrow \begin{pmatrix}-1\\-4\\-1\end{pmatrix} = \begin{pmatrix}x-4\\y-6\\z-3\end{pmatrix} \Rightarrow \begin{cases} x = 3 \\ y = 2 \\ z = 2 \end{cases} \quad C'(3;2;2)$

$C' \in t'$: $\quad t': \begin{cases} x = 3 + \lambda \\ y = 2 \\ z = 2 - \lambda \end{cases}$

d. $d: \begin{cases} x = 6 + \lambda \\ y = 2\lambda \\ z = 0 \end{cases}$

h) $(6+\lambda-8)^2 + (2\lambda-4)^2 + (-7)^2 = 54 \Leftrightarrow (\lambda-2)^2 + (2\lambda-4)^2 + 49 = 54$

$\Leftrightarrow \lambda^2 - 4\lambda + 4 + 4\lambda^2 - 16\lambda + 16 + 5 = 0 \Leftrightarrow$

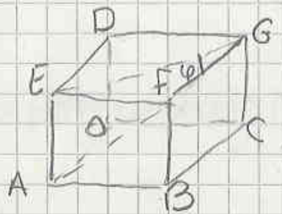
$\Leftrightarrow 5\lambda^2 - 20\lambda + 25 = 0 \Leftrightarrow \lambda^2 - 4\lambda + 5 = 0 \Leftrightarrow \begin{matrix}\lambda_1 = 5 \\ \lambda_2 = -1\end{matrix}$

$G(11;10;0) \quad H(5;-2;0)$

EXERCICE 5.59

$a = 5 \quad O(0,0,0) \quad A(a;0;0) \quad B(a;a;0) \quad D(0;0;a) \quad E(a;0;a) \quad F(a;0;a)$

$G(0,a;a)$



a. $\vec{GA} = \begin{pmatrix}a\\-a\\-a\end{pmatrix} \quad \vec{GE} = \begin{pmatrix}a\\-a\\0\end{pmatrix}$

$\cos\varphi = \dfrac{\vec{GA} \cdot \vec{GE}}{\|\vec{GA}\| \|\vec{GE}\|} = \dfrac{a^2 + a^2}{\sqrt{3a^2}\sqrt{2a^2}} \overset{a=5}{=} \dfrac{50}{25\sqrt{6}} = \dfrac{2}{\sqrt{6}}$

$\Rightarrow \varphi = 35.26$

b: $\vec{OD} = \begin{pmatrix}0\\0\\a\end{pmatrix} = \begin{pmatrix}0\\0\\5\end{pmatrix} \quad \vec{OF} = \begin{pmatrix}5\\5\\5\end{pmatrix}$

$\cos\varphi = \dfrac{\vec{OD} \cdot \vec{OF}}{\|\vec{OD}\| \cdot \|\vec{OF}\|} = \dfrac{25}{5 \cdot \sqrt{75}} = \dfrac{25}{5 \cdot 5\sqrt{3}} \Rightarrow \varphi = 54.73$

$\hat{A}$: $\vec{AD} = \begin{pmatrix}-5\\0\\5\end{pmatrix}$ $\vec{AF} = \begin{pmatrix}0\\5\\5\end{pmatrix}$

$\cos\hat{A} = \dfrac{\vec{AD}\cdot\vec{AF}}{\|\vec{AD}\|\cdot\|\vec{AF}\|} = \dfrac{25}{\sqrt{50}\cdot\sqrt{50}} \Rightarrow \hat{A} = 60^\circ$

C. $\vec{AG} = \begin{pmatrix}-5\\5\\5\end{pmatrix}$ $\vec{DF} = \begin{pmatrix}+5\\5\\0\end{pmatrix}$ $\vec{AG}\cdot\vec{DF} = -25+25 = 0 \Rightarrow \vec{AG} \perp \vec{DF}$

$\vec{u} = \vec{AG} \wedge \vec{DF} = \begin{pmatrix}-5\\5\\5\end{pmatrix} \wedge \begin{pmatrix}5\\5\\0\end{pmatrix} = \begin{pmatrix}-25\\25\\-50\end{pmatrix} \parallel \begin{pmatrix}-1\\1\\-2\end{pmatrix}$

$\begin{matrix}-5\\5\end{matrix}$ $\begin{matrix}5\\5\end{matrix}$

d $H(0,0,z)$ $\vec{HA} = \begin{pmatrix}5\\0\\-z\end{pmatrix}$ $\vec{HC} = \begin{pmatrix}0\\5\\-z\end{pmatrix}$

$\cos\alpha = \cos 45^\circ \Leftrightarrow \dfrac{\vec{HA}\cdot\vec{HC}}{\|\vec{HA}\|\cdot\|\vec{HC}\|} = \dfrac{\sqrt{2}}{2} \Leftrightarrow \dfrac{z^2}{\sqrt{25+z^2}\,\sqrt{25+z^2}} = \dfrac{\sqrt{2}}{2} \Leftrightarrow$

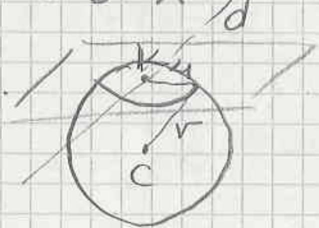
$\Leftrightarrow \dfrac{z^2}{25+z^2} = \dfrac{\sqrt{2}}{2} \Leftrightarrow 2z^2 = \sqrt{2}(25+z^2) \Leftrightarrow 2z^2 = 25\sqrt{2} + z^2\sqrt{2} \Leftrightarrow$

$\Leftrightarrow (2-\sqrt{2})z^2 = 25\sqrt{2} \Leftrightarrow z^2 = \dfrac{25\sqrt{2}}{2-\sqrt{2}} \Rightarrow z = \pm\sqrt{\dfrac{25\sqrt{2}}{2-\sqrt{2}}}$

EXERCICE 5.53

$d: \begin{cases} x = 3 \\ y = 5 \\ z = \lambda \end{cases}$ $S: (x-4)^2 + (y-7)^2 + (z-5)^2 = 25$ $C(4,7,5)$

$r = 5$

a. 

$K \in d \Rightarrow K(3;5;\lambda)$

$KC = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$. $\vec{KC} = \begin{pmatrix}4-3\\7-5\\5-\lambda\end{pmatrix} = \begin{pmatrix}1\\2\\5-\lambda\end{pmatrix}$

$\|\vec{KC}\| = 3 \Rightarrow \sqrt{1+2^2+(5-\lambda)^2} = 9 \Leftrightarrow$

$\Leftrightarrow 5 + (5-\lambda)^2 = 9 \Leftrightarrow (5-\lambda)^2 = 4 \Leftrightarrow \begin{cases} 5-\lambda = 2 \Leftrightarrow 3 = \lambda_1 \\ 5-\lambda = -2 \Leftrightarrow 7 = \lambda_2 \end{cases}$

λ_1: $K_1(3;5;3)$ λ_2: $K_2(3;5;7)$

b. $\underline{K_1}$ $\vec{K_1C} = \begin{pmatrix}1\\2\\2\end{pmatrix} = \vec{n}_{\Pi_1}$ $K_1 \in \Pi_1$: $3+10+6+d_1 = 0 \Leftrightarrow d_1 = -19$

Π_1: $x + 2y + 2z - 19 = 0$

$\underline{K_2}$: $\vec{K_2C} = \begin{pmatrix}1\\2\\-2\end{pmatrix} = \vec{n}_{\Pi_2}$ $K_2 \in \Pi_2$: $3+10-14+d_2 = 0 \Leftrightarrow d_2 = 1$

Π_2: $x + 2y - 2z + 1 = 0$

EXERCICE 5.54

$d: \begin{cases} x = 1+m \\ y = -2m \\ z = 5+m \end{cases}$ $\Pi_1: 3x + 4y - 12 = 0$ $\Pi_2: 3y - 4z + 6 = 0$

$C \in d \Rightarrow C(1+m, -2m, 5+m)$

$dist(M_1, C) = dist(M_2, C) \Rightarrow \frac{|3(1+m)+4(-2m)-12|}{\sqrt{9+16}} = \frac{|3(-2m)-4(5+m)+6|}{\sqrt{9+16}}$

$\Leftrightarrow |3+3m+8m-12| = |-6m-20+4m+6| \Leftrightarrow$

$\Leftrightarrow |-5m-9| = |-10m-14| = 0 \Leftrightarrow \begin{cases} -5m-9 = -10m-14 \Leftrightarrow m_1 = -1 \\ -5m-9 = 10m+14 \Leftrightarrow m = -\frac{23}{15} \end{cases}$

$C_1(0; 2; 4) \quad C_2\left(-\frac{8}{15}; \frac{46}{15}; \frac{52}{15}\right)$

$r_1 = dist(M_1, C_1) = \frac{|0+8-12|}{\sqrt{9+16}} = \frac{4}{5}$

$r_2 = dis(M_2, C_2) = \frac{4}{15}$

$\left.\right\} \begin{array}{l} S_1: x^2+(y-2)^2+(z-4)^2 = \frac{16}{25} \\ S_2: \left(x+\frac{8}{15}\right)^2+\left(y-\frac{46}{15}\right)^2+\left(z-\frac{52}{15}\right)^2 = \frac{16}{225} \end{array}$

EXERCICE 5.55

a. $\begin{vmatrix} -2 & 5 \\ 3 & 7 \end{vmatrix} = -14-15 = -29$

b. $\begin{vmatrix} k & 3 \\ k-2 & -k \end{vmatrix} = -k^2-3(k-2) = 0 \Leftrightarrow +k^2+3k-6 = 0 \Leftrightarrow k_{1,2} = \frac{3 \pm \sqrt{33}}{-2}$

c. $\begin{vmatrix} 2 & -1 & -2 \\ 6 & -1 & 1 \\ 4 & 5 & 3 \end{vmatrix} = 2\begin{vmatrix} -1 & 1 \\ 5 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 1 \\ 4 & 3 \end{vmatrix} - 2\begin{vmatrix} 6 & -1 \\ 4 & 5 \end{vmatrix} =$

$= 2(-3-5) + (18-4) - 2(30+4) = 2\cdot(-8) + 14 - 2\cdot 34 = -70$

$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 3 & 2 & 6 \\ 1 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 6\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 12$

$\begin{vmatrix} 3 & 2 & -1 \\ 1 & -2 & 5 \\ 4 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 2 & -1 \\ 4 & 0 & 4 \\ 4 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 2 & -1 \\ 0 & 0 & 4 \\ 2 & 1 & 2 \end{vmatrix} = -4\begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} =$

$= -4(4-4) = 0$

EXERCICE 5.56

$[a, b, c] = \det(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} 4 & -2 & k \\ 2 & -1 & 1 \\ -3 & 6 & -2 \end{vmatrix} = 0 \Leftrightarrow$

$\Leftrightarrow 4\begin{vmatrix} -1 & 1 \\ 6 & -2 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ -3 & -2 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ -3 & 6 \end{vmatrix} = 0 \Leftrightarrow$

$\Leftrightarrow 4(2-6) + 2(-4+3) + k(12-3) = 0 \Leftrightarrow$

$\Leftrightarrow -16-2+k9 = 0 \Leftrightarrow 9k-18 = 0 \Leftrightarrow k = 2$

Exercice 5.57

$A(2;-1;1)$ $B(5;5;4)$ $C(3;2;-1)$ $D(4;1;3)$

$\vec{AB} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ $\vec{AD} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$

$$\det(\vec{AB}, \vec{AC}, \vec{AC}) = \begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 2 \\ 3 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 2 \\ 3 & 2 & 0 \\ 0 & -3 & 0 \end{vmatrix} = 2 \begin{vmatrix} 3 & 2 \\ 0 & -3 \end{vmatrix} = -18$$

$$Vol = \frac{1}{6} |-18| = 3$$

Exercice 5.58

$A(2;1;-1)$ $B(3;0;1)$ $C(2;-1;3)$ $D(0;y;0)$

$\vec{AB} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$ $\vec{AD} = \begin{pmatrix} -2 \\ y-1 \\ 1 \end{pmatrix}$

$$\det(\vec{AB}, \vec{AC}, \vec{AD}) = \begin{vmatrix} 1 & 0 & -2 \\ -1 & -2 & y-1 \\ 2 & 4 & 1 \end{vmatrix} = \begin{vmatrix} -2 & y-1 \\ 4 & 1 \end{vmatrix} - 2 \begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix} =$$

$$= -2 - 4(y-1) - 2(-4+4) = -2 - 4y + 4 + \cancel{8} - \cancel{8} = -4y + 2$$

$$V = \frac{1}{6} |-4y+2| = 5 \Leftrightarrow |-4y+2| = 30 \Leftrightarrow \begin{cases} -4y+2 = 30 \Rightarrow y_1 = -7 \\ -4y+2 = -30 \Rightarrow y_2 = 8 \end{cases}$$

$D_1(0;-7;0)$ $D_2(0;8;0)$

Exercice 5.59

$$A: \begin{cases} \frac{x}{5} + \frac{y}{6} = 18 \\ \frac{x}{2} - \frac{y}{4} = 21 \end{cases} \Rightarrow \begin{matrix} 6x + 5y = 540 \\ 2x - y = 84 \end{matrix}$$

$$D = \begin{vmatrix} 6 & 5 \\ 2 & -1 \end{vmatrix} = -6 - 10 = -16 \neq 0 \Rightarrow \text{1 seule solution}$$

$$D_x = \begin{vmatrix} 540 & 5 \\ 84 & -1 \end{vmatrix} = -540 - 420 = -960 \quad x = \frac{-960}{-16} = 60$$

$$D_y = \begin{vmatrix} 6 & 540 \\ 2 & 84 \end{vmatrix} = 504 - 1080 = -576 \Rightarrow y = \frac{-576}{-16} = 36$$

Donc $(x, y) = (60; 36)$

B : $\begin{cases} x+3y+2z=-13 \\ 2x-6y+3z=32 \\ 3x-4y-z=12 \end{cases}$

$D = \begin{vmatrix} 1 & 3 & 2 \\ 2 & -6 & 3 \\ 3 & -4 & -1 \end{vmatrix} = 71 \neq 0 \Rightarrow$ 1 seule solution

$D_x = \begin{vmatrix} -13 & 3 & 2 \\ 32 & -6 & 3 \\ 12 & -4 & -1 \end{vmatrix} = -142 \Rightarrow x = \frac{-142}{71} \Rightarrow x = -2$.

$D_y = \begin{vmatrix} 1 & -13 & 2 \\ 2 & 32 & 3 \\ 3 & 12 & -1 \end{vmatrix} = -355 \Rightarrow y = \frac{-355}{-142} \Rightarrow y = 2.5$

$D_z = \begin{vmatrix} 1 & 3 & -13 \\ 2 & -6 & 32 \\ 3 & -4 & 12 \end{vmatrix} = 142 \Rightarrow z = \frac{142}{-142} \Rightarrow z = -1$

$(x, y, z) = (-2; 2.5; -1)$

C: $\begin{cases} 2x-3y+z=0 \\ x+5y-3z=3 \\ 5x+12y-8z=9 \end{cases}$

$D = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 5 & -3 \\ 5 & 12 & -8 \end{vmatrix} = 0$

$D_x = \begin{vmatrix} 0 & -3 & 1 \\ 3 & 5 & -3 \\ 9 & 12 & -8 \end{vmatrix} = 0 \quad D_y = \begin{vmatrix} 2 & 0 & 1 \\ 1 & 3 & -3 \\ 5 & 9 & -8 \end{vmatrix} = 0 \quad D_z = 0$

$\Rightarrow$ une infinité de solutions

$D = \begin{cases} 2x+3y-4z=1 \\ 3x-y+2z=-2 \\ 5x-9y+14z=3 \end{cases}$ Impossible

EXERCICE 5.60

$\begin{cases} \lambda x+y+z=1 \\ x+\lambda y+z=1 \\ x+y+\lambda z=1 \end{cases}$ $D = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} \lambda+2 & \lambda+2 & \lambda+2 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} =$

$= \begin{vmatrix} \lambda+2 & 0 & 0 \\ 1 & \lambda-1 & 0 \\ 1 & 0 & \lambda-1 \end{vmatrix} = (\lambda+2)(\lambda-1)^2$

- $\lambda \neq -2 \quad \lambda \neq 1$ 1 seule solution
- $\lambda = -2 \quad D = 0 \quad D_x = 9 \Rightarrow$ impossible (aucune solution)
- $\lambda = 1 \quad D = 0 \quad D_x = 0 \quad D_y = 0 \quad D_z = 0 \Rightarrow$ 1 infinité de solutions