LDDR-Niveau I: Statistique Solutions

EXERCICE 1.1 : a) population: les ménages du canton de Neuchâtel
échantillon : 1000 ménages
variable étudiée: le nombre d'enfants par ménage
quantitive discrète

b) population: population Suisse.
variable: la langue parlée le plus souvent.
type: qualitative

c) population: les élèves de 1MG
variable: le taux d'échec au semestre.
type: quantitative continue.

EXERCICE 1.2 . $n = 820$

a)

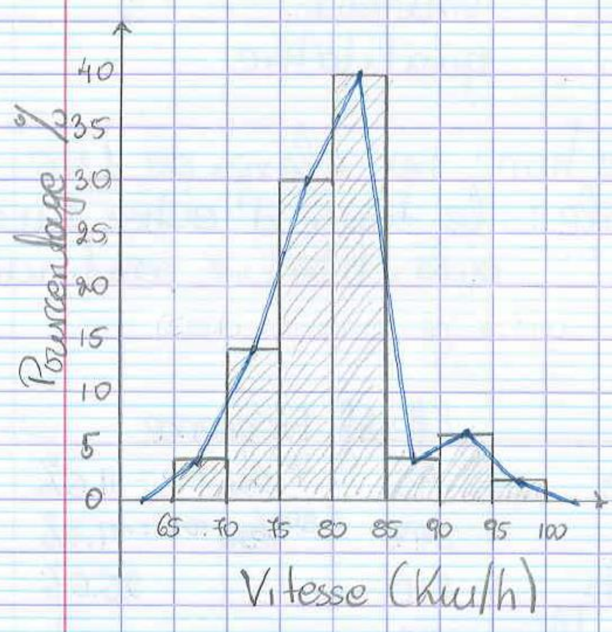
Utilité	Effectif	Fréquence
très utiles	95	$95/820 \cdot 100 = 11.6\%$
Utiles	342	$342/820 \cdot 100 = 41.7\%$
Nuisibles	210	25.6%
très nuisibles	46	5.6%
Sans opinion	127	15.5%
total	n=820	100%

b) $11.6\% + 41.7\% = 53.3\%$
utiles + très utiles

EXERCICE **1.3**

n = 50

Vitesse	Eff	Pourcentage
]65;70]	2	4%
]70;75]	7	14%
]75;80]	15	30%
]80;85]	20	40%
]85;90]	2	4%
]90;95]	3	6%
]95;100]	1	2%
total	50	100%



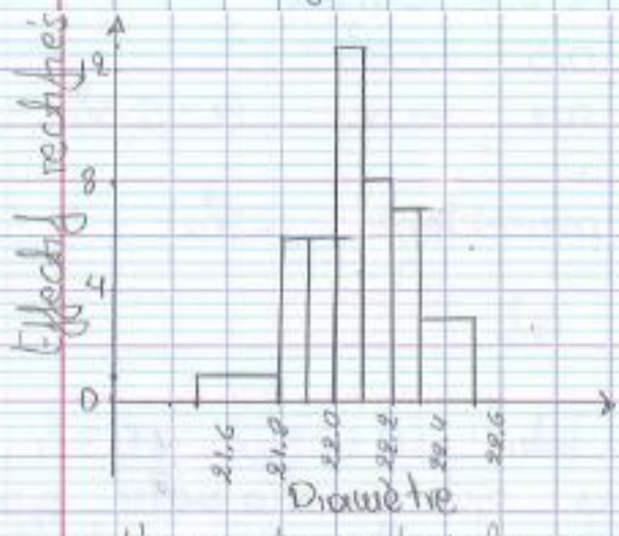
d) "Une petite moitié (48%) des véhicules respectent la limitation de vitesse de 80 Km/h et 40% roulent entre 80 et 85 Km/h. En tenant compte d'une marge de tolérance de 5 Km/h, 12% des véhicules sont amendables

EXERCICE 1.4

Diamètre [mm] (x_i)	Effectifs (n_i)	Pourc %	Freq cumulées %
[21.5; 21.8[	4	8	8
[21.8; 21.9[	6	12	20
[21.9; 22.0[	6	12	32
[22.0; 22.1[	13	26	58
[22.1; 22.2[	8	16	74
[22.2; 22.3[	7	14	88
[22.3; 22.5[	6	12	100
total	50	100	

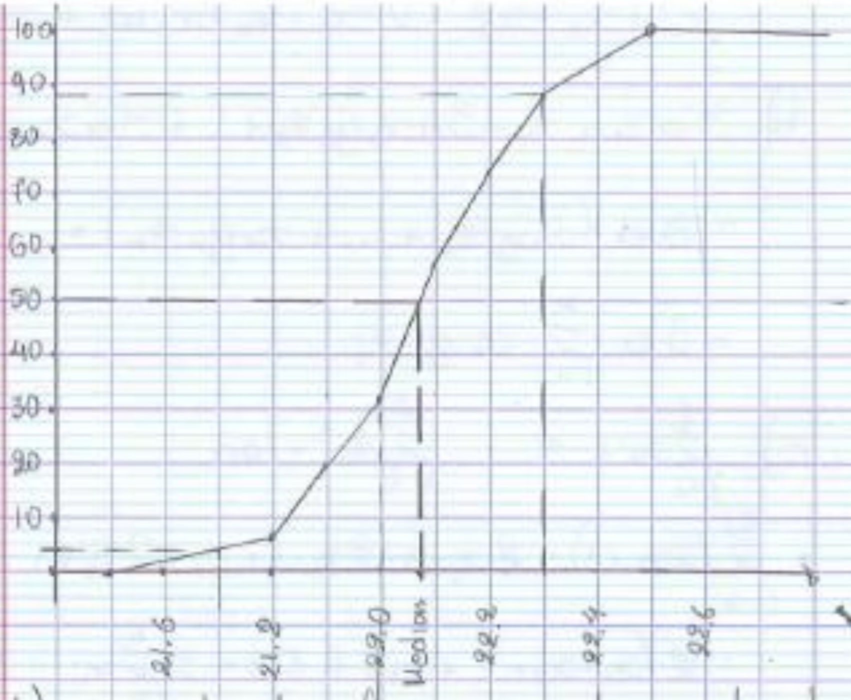
$= \frac{1}{7}\left(18+12\right) = \frac{30}{7} \Rightarrow \sigma = \sqrt{\frac{30}{7}} \approx 2.07$

On observe que les 3 séries ont le même moyen mais $\sigma_1 < \sigma_2 < \sigma_3$



- L'aire des rectangles sont proportionelles à la fréquence des classes qu'ils représentent
Donc pour la première classe on a
$3 \cdot x = 4 \Rightarrow x = \frac{4}{3} \approx 1.33 =$ hauteur

Pour la dernière: $2x = 6 \Rightarrow x = 3 =$ hauteur



b) On cherche le pourcentage des boulons qui ont de diamètre

$d < 21.7$ ou $d > 22.3$

$d < 21.7$: $0.3 \rightarrow 8\%$

$0.2 \rightarrow x$ $\quad x = 5.3\%$

Donc : le pourcentage 5.3%

$d > 22.3$ $\quad 12\%$

Donc au total : $(12+5.3)\% = 17.3\%$ des boulons ont un diamètre qui s'ecarte de plus de 0.3 mm de la valeur nominale

EXERCICE **1.5**

a) $\sum_{i=1}^{3}(x_i-a)^2 = (x_1-a)^2+(x_2-a)^2+(x_3-a)^2 =$

$= x_1^2-2x_1\cdot a+a^2+x_2^2-2x_2a+a^2+x_3^2-2x_3a+a^2=$

$= x_1^2+x_2^2+x_3^2-2(x_1+x_2+x_3)a+3a^2$.

b) $3ax_1y_1z_1+3ax_2y_2z_2+\dots+3ax_ky_kz_k =$

$= 3a(x_1y_1z_1+\dots+x_ky_kz_k) =$

$= 3a\sum_{i=1}^{k} x_iy_iz_i$

c) $\sum_{i=1}^{6} x_i = -4 \qquad \sum_{i=1}^{6} x_i^2 = 100$,

- $\sum_{i=1}^{6}(2x_i+3) = 2x_1+3+2x_2+3+\dots+2x_6+3 =$

$= 2(x_1+x_2+\dots+x_6)+6\cdot3 = 2\sum_{i=1}^{6}x_i+3 = 2(-4)+3$

$= -8+3 = -5$

- $\sum_{i=1}^{6}x_i(x_i-1) = x_1(x_1-1)+x_2(x_2-1)+\dots+x_6(x_6-1) =$

$= x_1^2-x_1+x_2^2-x_2+\dots+x_6^2-x_6 =$

$= \sum_{i=1}^{6}x_i^2-\sum_{i=1}^{6}x_i = 100-(-4) = 104$

- $\sum_{i=1}^{6}(x_i-5)^2 = (x_1-5)^2+(x_2-5)^2+\dots+(x_6-5)^2 =$

$= x_1^2-10x_1+25+\dots+x_6^2-10x_6+25 =$

$= \sum_{i=1}^{6}x_i^2-10\left(\sum_{i=1}^{6}x_i\right)+6\cdot25 =$

$= 100-10(-4)+150 = 100+40+150 = 290$

d)

					$\sum x_i$ / $\sum y_i$
x	2	-5	4	-8	-7
y	-3	-8	10	6	5

1. $\sum_{i=1}^{4}x_iy_i = x_1y_1+\dots+x_4y_4 = -6+40+40-48 = 26$

2. $\sum_{i=1}^{4}x_i\cdot\sum_{i=1}^{4}x_i = (x_1+\ \ +x_4)(x_1+\ \ +x_4) = (-7)^2 = 49$

3. $\sum_{i=1}^{4}(x_i^2+y_i^2) = \sum_{i=1}^{4} x_i^2 + \sum_{i=1}^{4} y_i^2 =$

$= 4+25+16+64+ 9+64+100+36 =$

$= 109 + 209 = 318$

4. $\sum_{i=1}^{4} x_i^2 + \sum_{i=1}^{4} y_i^2 = 109+209 = 318$

EXERCICE **1.6**

a)

x_i	n_i	$x_i n_i$
$\frac{21.8+21.5}{2} = 21.65$	4	86.6
21.85	6	131.1
21.95	6	131.7
22.05	13	286.65
22.15	8	177.2
22.25	7	155.75
22.31	6	134.9
total	50	1103.4

$\bar{x} = \frac{1}{50} \sum_{i=1}^{7} x_i n_i =$

$= \frac{1103.4}{50} \Rightarrow$

$\bar{x} = 22.068$ mm

b) $\tilde{x} \approx 22.07$ mm

c) $\bar{x} = 22.068$ mm

$\tilde{x} \approx 22.069$ mm

classe modale: $[22.0; 22.1[$

$\bar{x}, \tilde{x} \in [22.0; 22.1[$

et aussi $\bar{x} \approx \tilde{x}$

La distribution est normal (symmetrique)

EXERCICE **1.7**

$S_1 = \{3, 4, 4, 4, 4, 4, 5\}$ $\quad n = 7$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{3}(x_i - \bar{x})^2 \cdot n_i$$

x_i	n_i
3	1
4	5
5	1
total	7

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{3} x_i \cdot n_i =$$

$$= \frac{1}{7}(3\cdot 1 + 4\cdot 5 + 5\cdot 1) = \frac{28}{7} \Rightarrow$$

$$\Rightarrow \underline{\bar{x} = 4}$$

$$\sigma^2 = \frac{1}{7}\left[(3-4)^2\cdot 1 + (4-4)^2\cdot 4 + (5-4)^2\cdot 1\right] =$$

$$= \frac{1}{7}(1 + 1) = \frac{2}{7} \approx$$

Donc $\underline{\sigma = \sqrt{\frac{2}{7}}} \approx 0{,}53$

$S_2 = \{1, 3, 4, 4, 5, 5, 6\}$ $\quad n = 7$

x_i	n_i
1	1
3	1
4	2
5	2
6	1

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{5} x_i \cdot n_i =$$

$$= \frac{1}{7}(1\cdot 1 + 3\cdot 1 + 4\cdot 2 + 5\cdot 2 + 6\cdot 1) =$$

$$= \frac{1}{7}(28) = \underline{4 = \bar{x}}$$

$$\sigma^2 = \frac{1}{7}\sum_{i=1}^{5}(x_i - \bar{x})^2 \cdot n_i =$$

$$= \frac{1}{7}\left[(1-4)^2\cdot 1 + (3-4)^2\cdot 1 + (4-4)^2\cdot 2 + (5-4)^2\cdot 2 + (6-4)^2 \cdot 1\right]$$

$$= \frac{1}{7}(9 + 1 + 0 + 2 + 4) = \frac{16}{7} \Rightarrow \underline{\sigma = \sqrt{\frac{16}{7}} \approx 1{,}51}$$

$S_3 = \{1, 1, 4, 4, 6, 6, 6\}$ $\qquad n = 7$

x_i	n_i
1	2
4	2
6	3

$$\bar{x} = \frac{1}{7} \sum_{i=1}^{3} x_i n_i = \frac{1}{7}\left(1 \cdot 2 + 4 \cdot 2 + 6 \cdot 3\right) =$$

$$= \frac{28}{7} = \underline{4 = \bar{x}}$$

$$\sigma^2 = \frac{1}{7} \sum_{i=1}^{3} (x_i - \bar{x})^2 \cdot n_i =$$

$$= \frac{1}{7}\left[(1-4)^2 \cdot 2 + (4-4)^2 \cdot 2 + (6-4)^2 \cdot 3\right] =$$

$$= \frac{1}{7}\left(18 + 12\right) = \frac{30}{7} \Rightarrow \sigma = \sqrt{\frac{30}{7}} \approx 2.07$$

On obrerare que les 3 séries ont le même moyen mais $\sigma_1 < \sigma_2 < \sigma_3$

EXERCICE 1.8

nombre des points	(n_i) Effectifs	x_i	$n_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^2 n_i$
[10, 20[	1	$\frac{20+10}{2} = 15$	15	-40.37	1629.737	1629.74
[20, 30[	4	25	100	-30.37	922.34	
[30, 40[	4	35	140	-20.37	414.94	
[40; 50[	15	45	675	-10.37	107.54	
[50, 60[	9	55	495	-0.37	0,14	
[60; 70[	10	65	650	9,63	92,74	
[70, 80[	6	75	450	19,63	385,34	
[80, 90[	2	85	170	29,63	877,94	
[90, 100[	2	95	190	39,63	1570,54	
[100; 110[	1	105	105	49,63	2463,14	2463,14
total	54		2990			19192.59

$$\bar{x} = \frac{1}{54} \sum_{i=1}^{10} x_i n_i = \frac{1}{54} 2990 = \underline{55.37 = \bar{x}}$$

$$\sigma^2 = \frac{1}{54} \sum_{i=1}^{10} (x_i - \bar{x})^2 n_i = 355.42$$

$$\sigma = \sqrt{355.42} \Rightarrow \underline{\sigma = 18.85}$$

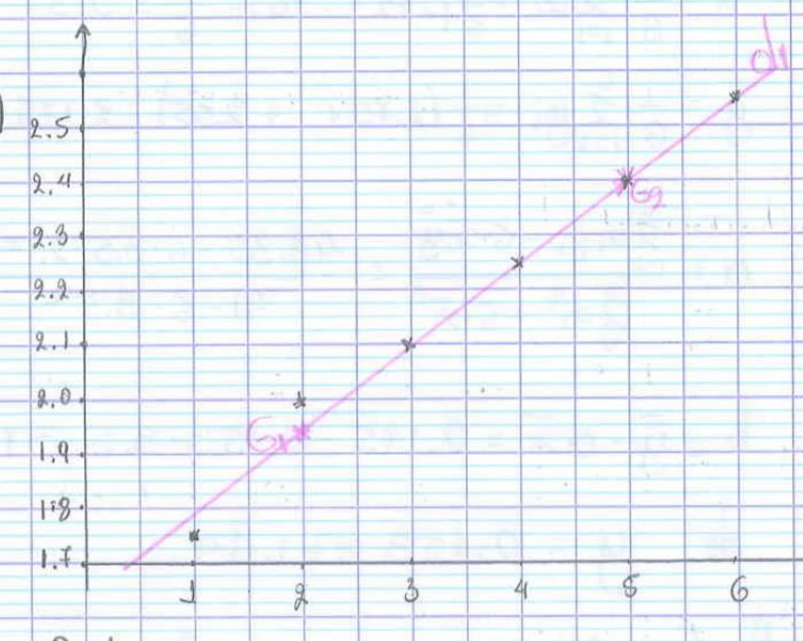
b) $z_{110} = \frac{x_{110} - \bar{x}}{\sigma} = \frac{110 - 55.37}{18.85} \Rightarrow z_{110} \approx 2.9$

$z_x = -2 \Leftrightarrow \frac{x - \bar{x}}{\sigma} = -2 \Leftrightarrow x = -2 \cdot \sigma + \bar{x} \Rightarrow$

$x = -2 \cdot 18.85 + 55.37 \Rightarrow x = 17.67 \approx 18$ pts

Exercice 1.9

a)



Oui

b) $G_1 = \left(\frac{1+2+3}{3} ; \frac{1.75+2+2.1}{3}\right) = (2 ; 1.95)$

$G_2 = \left(\frac{4+5+6}{3} ; \frac{2.25+2.4+2.55}{3}\right) = (5 ; 2.4)$

d_1: $m = \frac{2.4 - 1.95}{5-2} = 0.15$ $\quad y = 0.15x + h$

$1.95 = 0.15 \cdot 2 + h \Leftrightarrow h = 1.65$

d_1: $\underline{y = 0.15x + 1.65}$

c) $\sum_{n=1}^{6} x_i y_i = 1\cdot 1{,}75 + 2\cdot 2 + 3\cdot 2{,}9 + \quad + 6\cdot 2{,}55 = 48{,}35$

$\sum_{i=1}^{6} x_i^2 = 1^2 + 2^2 + \quad + 6^2 = 91$

$\bar{x} = \frac{1}{6}\sum_{i=1}^{6} x_i = \frac{1}{6}(1+2+\ldots+6) = \frac{21}{6} = 3{,}5$

$\bar{y} = \frac{1}{6}\sum_{i=1}^{6} y_i = \frac{1}{6}(1{,}75 + \quad + 2{,}55) = 2{,}175$

$a = \frac{\sum_{i=1}^{6} x_i y_i - 6\cdot\bar{x}\bar{y}}{\sum_{i=1}^{6} x_i^2 - 6\cdot\bar{x}^2} = \frac{48{,}35 - 6\cdot 3{,}5\cdot 2{,}175}{91 - 6\cdot 3{,}5^2} = 0.153$

$b = \bar{y} - a\bar{x} = 2{,}175 - 0{,}153\cdot 3{,}5 = 1{,}64$

d_2: $y = 0.153\,x + 1.64$

d) En 2018 on a $x = 9$

d_1: $y = 3$ $\quad d_2$: $y = 3.017$

EXERCICE 10

$E = \sum_{i=1}^{n} (y_i - b - a x_i)^2$

$= (1-b-a\cdot 1)^2 + (3-b-2a)^2 + (6-b-3a)^2 + (7-b-4a)^2$

$\frac{\partial E}{\partial a} = -2(1-b-a) - 4(3-b-2a) - 6(6-b-3a) - 8(7-b-4a)$

$= 60a + 20b - 106 = 0 \Leftrightarrow$

$\Leftrightarrow 30a + 10b - 53 = 0$

$$\frac{\partial E}{\partial b} = -2(1-b-a)-2(3-b-2a)-2(6-b-3a)-2(7-b-4a)$$

$$= -2+2b+2a-6+2b+4a-12+2b+6a-14+2b+8a$$

$$= 20a+8b-34 = 0$$

(1) $60a+20b-106=0 \Leftrightarrow 30a+10b-53=0$

(2) $20a+8b-34=0 \Leftrightarrow 10a+4b-17=0 \quad *(-3)$

$$+\begin{cases} 30a+10b-53=0 \\ -30a-12b+51=0 \end{cases}$$

$$-2b-2=0 \Leftrightarrow b=-1$$

(2) $\overset{b=-1}{\Longrightarrow} 10a-4-17=0 \Leftrightarrow 10a=21 \Leftrightarrow a=2.1$

$$y = 2.1x-1$$

b) $\sum_{i=1}^{4} x_i y_i = 53 \qquad \bar{x} = \frac{1}{4}\sum_{i=1}^{4} x_i = 2.5$

$\sum_{i=1}^{4} x_i^2 = 30 \qquad \bar{y} = \frac{1}{4}\sum_{i=1}^{4} y_i = \frac{17}{4} = 4.25$

$$a = \frac{53-4 \cdot 2.5 \cdot 4.25}{30-4 \cdot 2.5^2} = 2.1$$

$$b = \bar{y} - a\bar{x} \Rightarrow b = 4.25 - 2.1 \cdot 2.5 \Rightarrow$$

$$b = -1 \qquad y = 2.1x - 1$$

c) $P(\bar{x};\bar{y}) = P(2.5; 4.25)$

$4.25 = 2.1 \cdot 2.5 - 1$ ✓

EXERCICE **1.11**

a) $\sum_{i=1}^{8} x_i y_i = 364$ $\quad \bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = \frac{56}{8} = 7$

$\sum_{i=1}^{8} x_i^2 = 524$ $\quad \bar{y} = \frac{1}{8}\sum_{i=1}^{8} y_i = \frac{1}{8} 40 = 5$

$$a = \frac{364 - 8 \cdot 7 \cdot 5}{524 - 8 \cdot 7^2} = \frac{84}{132} = \frac{7}{11}$$

$$b = \bar{y} - a\bar{x} \Rightarrow b = 5 - \frac{7}{11} \cdot 7 \Rightarrow b = \frac{6}{11}$$

$$y = \frac{7}{11}x + \frac{6}{11}$$

b)

x_i	1	2	4	4	5	7	8	9
y_i	1	3	4	6	8	9	11	14

$\sum_{i=1}^{8} x_i y_i = 364$ $\quad \bar{x} = 5$

$\sum x_i^2 = 256$ $\quad \bar{y} = 7$

$$a = \frac{364 - 8 \cdot 5 \cdot 7}{256 - 8 \cdot 5^2} = \frac{84}{56} = \frac{12}{8} = \frac{3}{2}$$

$$b = \bar{y} - \frac{3}{2}\bar{x} \Rightarrow b = 7 - \frac{3}{2} \cdot 5 = -\frac{1}{2}$$

$$x = \frac{3}{2}y - \frac{1}{2}$$

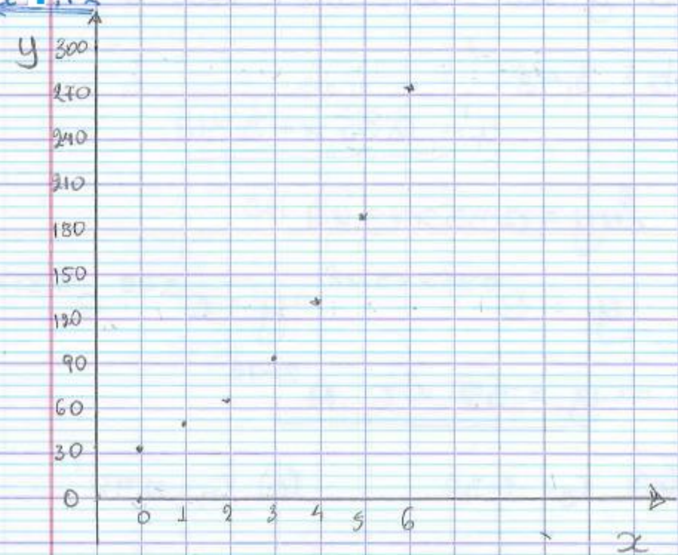
c) (1) $11y = 7x + 6 \Leftrightarrow 7x - 11y = -6 \quad *(-2)$

(2) $2x = 3y - 1 \Leftrightarrow 2x - 3y = -1 \quad *(7)$

$$\begin{array}{r} -\cancel{14x} + 22y = 12 \\ + \cancel{14x} - 21y = -7 \\ \hline y = 7 = \bar{y} \ \checkmark \end{array}$$

(2) $x = \frac{3}{2} \cdot 7 - \frac{1}{2} \Rightarrow x = \frac{20}{2} \Rightarrow \bar{x} = 10 \ \checkmark$

EXERCICE 1.12



b) $y' = \ln(y)$

x_i	0	1	2	3	4	5	6
$\ln(y_i)$	3.46	3.85	4.17	4.52	4.88	5.25	5.62

c) $\sum_{i=1}^{7} x_i y_i' = 105.23$ $\qquad \bar{x} = \frac{21}{7} = 3$

$\sum_{i=1}^{7} x_i^2 = 91$ $\qquad \bar{y} = \frac{31.76}{7} = 4.54$

$$a = \frac{105.23 - 7 \cdot 3 \cdot 4.54}{91 - 7 \cdot 3^2} = \frac{9.89}{28} = 0.35$$

$b = \bar{y} - a\bar{x} \Rightarrow b = 4.54 - 0.35 \cdot 3 \Rightarrow$

$b = 3.48$

$$y' = 0.35x + 3.48$$

d) $\ln y = 0.35x + 3.48 \Leftrightarrow$

$y = e^{0.35x + 3.48} \Rightarrow y = e^{3.48} \cdot e^{0.35x} \Rightarrow$

$\Rightarrow y = 32.46 \cdot e^{3.48}$

Exercice **1.13** (a) $r_3 = -0.36$ (c) $r_4 = 0.98$

(b) $r_2 = -0.91$ (d) $r_1 = 0.44$

Exercice **1.14** a) $r = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$ $\sigma_{xy} = \frac{1}{n} \sum_{i=1}^{n} x_i y_i - \bar{x}\bar{y}$

$\sigma_x = \sqrt{\frac{1}{n} \sum x_i^2 - \bar{x}^2}$ $\sigma_y = \sqrt{\frac{1}{n} \sum y_i^2 - \bar{y}^2}$

$\bar{x} = \frac{1}{9} \sum x_i = \frac{720}{9} = 80 = \bar{x}$ $\sum x_i y_i = 52491.8$

$\bar{y} = \frac{1}{9} 565.6 \Rightarrow \bar{y} = 62.84$

$\sum x_i^2 = 63600$

$\sum y_i^2 = 44471.25$

$\sigma_x = \sqrt{\frac{1}{9} 63600 - 80^2} = 25,82$

$\sigma_y = \sqrt{\frac{1}{9} 44471.25 - 62.84^2} = 31,5$

$\sigma_{xy} = \frac{1}{9} 52491.8 - 80 \cdot 62.84 = 805.2$

$r_{xy} = \frac{805.2}{25.82 \cdot 31.5} \Rightarrow r = 0,99$

$r_{xz} = 1$

b) On trouve la droite de régression

$z = ax + b$

$z = 0.08x + 1.33$

pour $v = 200$ Km/h $\Rightarrow z = 17.33 = \sqrt{d}$

$\Rightarrow d = 300.33$ m

EXERCICE **1.15** $E = \sum_{i=1}^{n} (y_i - b - a x_i)^2$

On obtient le système

$$\begin{cases} (1) \quad 5.327a + 8b = 18.589 \\ 3.993339a + 5.327b = 12.435898 \end{cases}$$

En solvant on a: $b = 0.154025$ $a = 2.22106$.

b) $\ln(t) = 0.154025\, e^{-0.00924x} + 2.22106 \Rightarrow$

$\Rightarrow t = e^{0.154025\, e^{-0.00924x} + 2.22106}$

c) en 2010 on a $x = 110$. En remplaçant

$t = e^{0.154025 \cdot e^{-0.00924 \cdot 110} + 2.22106} \approx 9.75$ sec

d) $\lim_{x \to +\infty} e^{0.154 \cdot e^{-0.00924x} + 2.221} = e^{0 + 2.21} \approx 9.22$ sec

e) Que le record ne pourra jamais être inferieur de 9 sec