

1.12 Exercices

1.1 Find the domain of the following :

1) $y = x^2 + 1$

4) $y = 1 + \sqrt{2x - 5}$

7) $y = |\cos(x)|$

2) $y = \frac{1}{(x - 2)^2}$

5) $y = -\frac{1}{x}$

8) $y = \frac{1}{\sqrt{x^3 - x^2 - 6x}}$

3) $y = \frac{\tan(x)}{x}$

6) $y = \frac{1}{\sqrt{5 - x}}$

9) $y = \frac{1}{\tan(x)}$

1.2 Establish the sign table of the following :

1) $y = 2x + 1$

3) $y = \sqrt{x + 3} - 2$

5) $y = 2 \cos(x) + 1$

2) $y = -x^2 + 7x - 6$

4) $y = x^2 - 3x + 4$

6) $y = x \cdot (x - 3) \cdot (x + 4)$

1.3 By using a sign table, solve $f(x) = \frac{x - 3}{x^2 - 3x + 2} > 0$.

1.4 Analyze the function $f(x) = \frac{3 - 2x}{x + 1}$ that is to say :

1) Domain

2) Intersection with the axes

3) Table of signs

4) Behavior of the function when x tends to the excluded value5) Behavior of the function when x tends to $+\infty$ and $-\infty$?

6) Graph

7) Coherence

1.5 By using the graph of the function $g(x)$, determine, if they exist, the following limits :

1) $\lim_{x \rightarrow 2^-} g(x) =$

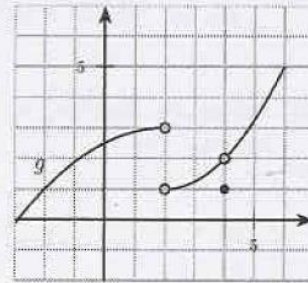
4) $\lim_{x \rightarrow 4^-} g(x) =$

2) $\lim_{x \rightarrow 2^+} g(x) =$

5) $\lim_{x \rightarrow 4^+} g(x) =$

3) $\lim_{x \rightarrow 2} g(x) =$

6) $\lim_{x \rightarrow 4} g(x) =$

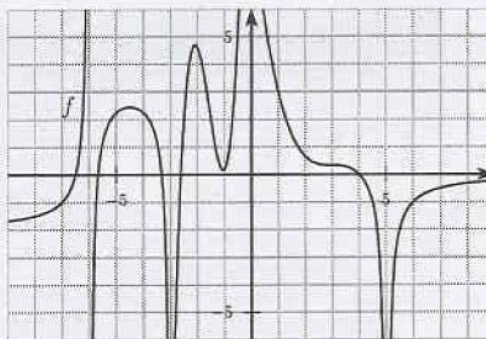
7) What is the value of $g(2)$ and $g(4)$?8) Is the function g continuous?

1.6 By using the graph of $f(x)$, determine, if they exist, the following limits :

$$1) \lim_{x \rightarrow \infty} f(x) = \quad 3) \lim_{x \rightarrow -3} f(x) =$$

$$2) \lim_{x \rightarrow 5} f(x) = \quad 4) \lim_{x \rightarrow -6} f(x) =$$

What is the domain of this function ?
Determine the equation of the vertical and horizontal asymptotes.



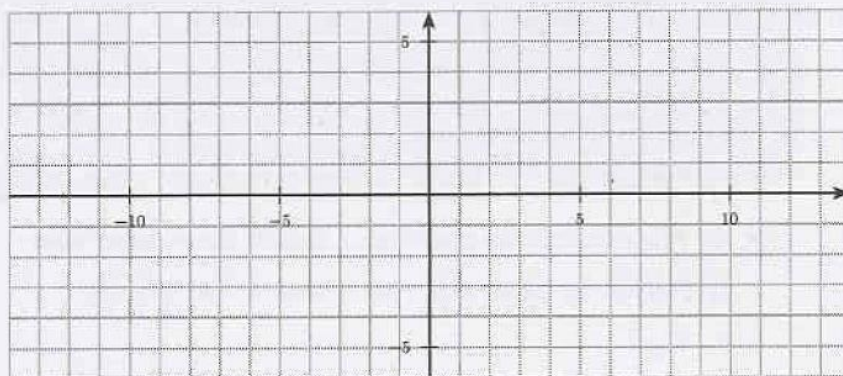
1.7 Sketch the graph of a function h such that :

$$1) \lim_{x \rightarrow +\infty} h(x) = \lim_{x \rightarrow -\infty} h(x) = 3 \quad 4) \lim_{x \rightarrow 4^+} h(x) = +\infty$$

$$2) \lim_{x \rightarrow -7} h(x) = -\infty \quad 5) h(-2) = 1$$

$$3) \lim_{x \rightarrow 4^-} h(x) = -\infty$$

What is the domain of your function h ?



1.8 Compute, if they exist, the following limits :

$$1) \lim_{x \rightarrow 2} 5x = \quad 4) \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 4} = \quad 7) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2} =$$

$$2) \lim_{x \rightarrow 2} (2x + 3) = \quad 5) \lim_{x \rightarrow 4} \sqrt{25 - x^2} = \quad 8) \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{(x - 1)^2} =$$

$$3) \lim_{x \rightarrow 3} \frac{x + 2}{x - 2} = \quad 6) \lim_{x \rightarrow 4} \frac{x - 4}{x^2 - x - 12} = \quad 9) \lim_{x \rightarrow 2} \frac{x^2 - 2x}{x} =$$

1.9 Compute, if they exist, the following limits :

$$\begin{array}{lll} 1) \lim_{x \rightarrow \infty} (-4x^2) = & 4) \lim_{x \rightarrow \infty} \frac{x^2 + x - 2}{4x^3 - 1} = & 7) \lim_{x \rightarrow \infty} \frac{1+x}{x} = \\ 2) \lim_{x \rightarrow -\infty} 5x^3 + 2 = & 5) \lim_{x \rightarrow \infty} \frac{4x - 2}{5x + 7} = & 8) \lim_{x \rightarrow \infty} \frac{(x-1)^2}{x^2 + 3x} = \\ 3) \lim_{x \rightarrow \infty} \frac{3}{x} = & 6) \lim_{x \rightarrow \infty} \frac{2x^3 + 1}{x^2 + 1} = & 9) \lim_{x \rightarrow -\infty} \frac{(x^2 + 6x)(5x - 2)}{-10x^3 + 13x - 7} = \end{array}$$

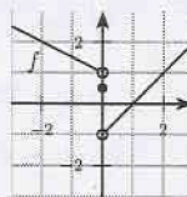
1.10 Given the function $f(x) = \frac{2x-4}{x^2+x-6}$. Calculate, if they exist, the following limits :

$$\begin{array}{lll} 1) \lim_{x \rightarrow 2^-} f(x) = & 4) \lim_{x \rightarrow 1^-} f(x) = & 7) \lim_{x \rightarrow -3^-} f(x) = \\ 2) \lim_{x \rightarrow 2^+} f(x) = & 5) \lim_{x \rightarrow 1^+} f(x) = & 8) \lim_{x \rightarrow -3^+} f(x) = \\ 3) \lim_{x \rightarrow 2} f(x) = & 6) \lim_{x \rightarrow 1} f(x) = & 9) \lim_{x \rightarrow -3} f(x) = \end{array}$$

Determine the value of $f(2)$, $f(1)$ and $f(-3)$?

1.11 Here opposite is the graph of the function $f(x)$. Whenever possible, calculate :

$$\begin{array}{lll} 1) \lim_{x \rightarrow 2^-} f(x) = & 4) f(2) = & 7) \lim_{x \rightarrow 0} f(x) = \\ 2) \lim_{x \rightarrow 2^+} f(x) = & 5) \lim_{x \rightarrow 0^-} f(x) = & \\ 3) \lim_{x \rightarrow 2} f(x) = & 6) \lim_{x \rightarrow 0^+} f(x) = & 8) f(0) = \end{array}$$



Is the function f continuous ? Why ?

1.12 1) Determine the equation of the vertical asymptotes of the functions :

$$\text{a) } f(x) = \frac{x}{x^2 - x - 2} \qquad \text{b) } g(x) = \frac{2 - 2x}{2x^2 - 5x + 3}$$

2) Determine all the asymptotes of the following :

$$\begin{array}{ll} \text{a) } h(x) = \frac{2x-5}{x^2+1} & \text{c) } j(x) = \frac{x^2+1}{2x-5} \\ \text{b) } i(x) = \frac{2x^2+4x+2}{-x^2+4} & \text{d) } k(x) = \frac{-2x^3+9x^2-13x+7}{x^2-4x+4} \end{array}$$

1.13 Determine all the asymptotes of the following :

$$\begin{array}{ll} 1) f(x) = \frac{52.5x - 27.5x^2 - 5x^3}{-7 + 6x + x^2} & 3) h(x) = \frac{2x^3 + 11x^2 - 106x + 165}{x^2 - 4x + 3} \\ 2) g(x) = \frac{-30 + 9x + 3x^2}{-24 + 4x + 4x^2} & \end{array}$$

1.14 Determine the domain of the functions above and then, analyze their behavior close to their excluded values.

$$1) f(x) = \frac{x^2 - 5x + 3}{2x - 6} \quad 2) g(x) = \frac{4x - 3}{x^2 - 4} \quad 3) h(x) = \frac{x^2 - x - 12}{x^2 + 2x - 3}$$

1.15 Analyze the function $f(x) = \frac{-2x^3 - 6x^2 + 20x}{5x^2 - 15x + 10}$

1.16 1) Calculate $\lim_{x \rightarrow 0} \cos(x)$

2) Calculate, numerically, the limit $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$.

3) Thanks to a geometric reflexion, determine $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$.

4) Deduce the value of $\lim_{x \rightarrow 0} \frac{\tan(x)}{x}$

1.17 Calculate, if they exist, the following limits :

$$1) \lim_{x \rightarrow 0} \frac{\sin(2x)}{x} = \quad 4) \lim_{x \rightarrow 0} \frac{\sin(x)}{x^2} = \quad 7) \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin(x)}{\cos^2(x)} =$$

$$2) \lim_{x \rightarrow 0} \frac{\sin(3x)}{2x} = \quad 5) \lim_{x \rightarrow 0} \frac{\tan(5x)}{2x} = \quad 8) \lim_{x \rightarrow 0} x \cdot \sin\left(\frac{1}{x}\right) =$$

$$3) \lim_{x \rightarrow 0} \frac{1 - \cos^2(x)}{x} = \quad 6) \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \quad 9) \lim_{x \rightarrow 0} x^2 \cdot \sin\left(\frac{1}{x}\right) =$$

1.18 Calculate, if they exist, the following limits :

$$1) \lim_{x \rightarrow 0} |x| = \quad 3) \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} = \quad 5) \lim_{x \rightarrow 2} \frac{x^2 + 1}{|x^2 - 4|} =$$

$$2) \lim_{x \rightarrow 0} \frac{|x|}{x} = \quad 4) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \quad 6) \lim_{x \rightarrow 5} \frac{x - 5}{\sqrt{2x - 1} - 3} =$$

1.19 Determine the expression of the rational functions $f_1(x)$ and $f_2(x)$ such that :

1) $f_1(x)$ has a vertical asymptote at $x = 5$, a slant asymptote with equation $y = -x + 5$. Moreover, the curve is over the asymptote when x tends to $+\infty$.

2) $f_2(x)$ has a vertical asymptote at $x = 2$ and a hole with coordinates $(3; 11)$.

1.20 We consider the function $f(x) = \begin{cases} 2x & x < 3 \\ mx+h & 3 \leq x \leq 5 \\ -x & x > 5 \end{cases}$. Given that $f(x)$ is continuous on \mathbb{R} , sketch its graph and find the values of m and h .

- 1. 21**
- 1) Sketch precisely the graph of the parabola $y = f(x) = x^2$. (first quadrant only, 1 unit = 10 squares).
 - 2) We consider on this parabola the fixed point $P(1;1)$ and the moving point $Q(1 + \Delta x; y)$. Calculate the slope of the secant line PQ for $\Delta x \in \{0.5; 0.4; 0.3; 0.2; 0.1; 0.01\}$. Draw one of these secants.
 - 3) To which number tends the quotient $\frac{\Delta y}{\Delta x}$ when $\Delta x \rightarrow 0$? Give a geometric interpretation of this number.
 - 4) Calculate $\lim_{\Delta x \rightarrow 0} \left(\frac{\Delta y}{\Delta x} \right)$ at one point $P_0(x_0; y_0)$ of the parabola.
- 1. 22** By using the definition of the derivative, calculate the derivative of the following functions :
- 1) $f(x) = 2x + 5$
 - 2) $g(x) = mx + h$
 - 3) $h(x) = h$
 - 4) $i(x) = 3x^2 - 4x + 7$
 - 5) $j(x) = x^3$
 - 6) $k(x) = \frac{1}{x}$
 - 7) $l(x) = \frac{1}{2x+1}$
 - 8) $m(x) = \sqrt{x}$
 - 9) $n(x) = \cos(x)$
 - 10) $o(x) = \sin(x)$
- 1. 23** Differentiate the following functions using the product rule and then check your result by using the distributivity first :
- 1) $f(x) = (x+1)(x-1)$
 - 2) $g(x) = (x^3+4)(x^2+3)$
 - 3) $h(x) = x^m x^n$
- 1. 24** By using the rules of derivation, find the derivative of :
- 1) $f(x) = 4x^3 - 7x^2 + 89$
 - 2) $f(x) = -5x^4 + \cos(60^\circ)x^2 - \tan(45^\circ)$
 - 3) $f(x) = (7-5x)^2$
 - 4) $f(x) = 3\sqrt{x} + 8x^2 - 9x$
 - 5) $f(x) = \frac{7}{2x} + 3\sin(x)$
 - 6) $f(x) = \sqrt[5]{x^4}$
- 1. 25** Determine the equation of the tangent to the following curves at the point whose abscissa is given.
- 1) $f(x) = -3x^2 + 5x - 2$ at $x = -2$
 - 2) $g(x) = \frac{-3}{x}$ at $x = 5$
 - 3) $h(x) = 5\sqrt{x} + 2$ at $x = 4$
 - 4) $i(x) = \cos(x)$ at $x = \frac{3\pi}{4}$
- 1. 26** Determine the coordinates of the two points of the curve $y = 2x^3 - 5x^2 + 9x - 1$ where the slope of the tangent is 13.
- 1. 27** At a point P of the curve $y = x^2 + k$, the equation of the tangent is $y = 6x - 7$. Find the value of the constant k as well as the coordinates of the point P .

1.28 Derivate :

1) $y = \frac{x}{x-1}$

6) $y = \frac{x}{\sqrt{x+1}}$

11) $y = \frac{ax+b}{cx+d}$

2) $y = \frac{x}{1+5x}$

7) $y = \frac{\sqrt{x-5}}{x}$

12) $y = x^{\frac{1}{3}} \cdot (7-x^3)$

3) $y = \frac{x^2}{3x-2}$

8) $y = x \cos(x)$

13) $y = \frac{x^2 - 7x + 9}{x}$

4) $y = \frac{x}{1+x^3}$

9) $y = (4-x^2) \sin(x)$

14) $y = x^3(\sin(x) + 1)$

5) $y = \frac{\sin(x)}{x}$

10) $y = \tan(x)$

15) $y = \sin(x) \cdot \cos(x)$

1.29 Determine the equation of the tangent to $y = \frac{x^2+3}{x+3}$ at $x = 1$.

1.30 Determine the gradient of the tangent to $y = \frac{\sin(x)}{x^2}$ at $x = \pi$ (write your answer as a multiple of π).

1.31 For which value of a is the function $f(x) = \frac{2x+a}{x-1}$ tangent to the line $y = -7x+2$? Calculate the coordinates of the contact point.

1.32 By using the chain rule, determine the derivative of the functions :

1) $y = (x^5+1)^4$

3) $y = (6x^3-5)^{-2}$

5) $y = \sqrt{4x+3}$

2) $y = (\sqrt{x}-1)^5$

4) $y = (2x^3-1)^8$

6) $y = \sin\left(\left(\frac{2x-1}{x}\right)^2\right)$

1.33 Determine the equation of the tangent and the normal to the curve :

1) $y = x \sin(x)$ at $x = \pi$

2) $y = x\sqrt{3x+1}$ at $x = 5$

1.34 Given the function $f(x) = \frac{1}{1+x^2}$. Determine

1) $f'(2)$

2) x such that $f'(x) = 0$

1.35 Given the function $y = \sqrt[3]{x^3+8}$. Determine the value of $\frac{dy}{dx}$ at $x = 2$.

1.36 Determine the equation of the tangent to the curve $y = (x^2-5)^3$ at the point with abscissa 2.

1.37 Determine the equation of the tangent to the curve $y = \frac{1}{\sqrt{x}-1}$ at the point with ordinate 1.

1. 38 Derivate the following :

- | | | |
|----------------------------------|--|-------------------------------------|
| 1) $f(x) = (3x + 1)^3$ | 5) $f(x) = (7x^3 - 2x)^4$ | 9) $f(x) = \sqrt[3]{x^4}$ |
| 2) $f(x) = \cos(2x)$ | 6) $f(x) = \cos^3(x)$ | 10) $f(x) = \frac{1}{\sqrt{3x}}$ |
| 3) $f(x) = \sqrt{8x^2 - 2x + 3}$ | 7) $f(x) = \frac{x}{(2x + 5)^2}$ | 11) $f(x) = \sqrt{\frac{1+x}{1-x}}$ |
| 4) $f(x) = -x^2 + \frac{4}{x+3}$ | 8) $f(x) = \sin^4\left(\frac{\pi}{2}\right)$ | 12) $f(x) = \sqrt[3]{\tan(x)}$ |

Challenge : Derivate the function : $f(x) = \left(\frac{\sqrt{2x+1}}{(2x-1)(2x+1)^2}\right)^3$

1. 39 Determine the stationary points and the turning points of :

- | | | |
|-------------------------------|-----------------------------|---------------------------------|
| 1) $y = x^3 - 3x^2 - 45x + 7$ | 3) $y = x^2 + \frac{54}{x}$ | 5) $y = 3x^5 - 20x^3 + 1$ |
| 2) $y = 3x^4 - 8x^3 + 6x^2$ | 4) $y = x - \sqrt{x}$ | 6) $y = x^{\frac{1}{3}}(4 - x)$ |

1. 40 Find the zeroes of the following :

- | | |
|---|--|
| 1) $-2x^{\frac{3}{4}} + x^{\frac{7}{4}} + x^{\frac{11}{4}}$ | 3) $-\frac{0.2}{x^{\frac{4}{3}}} - \frac{2}{x^{\frac{1}{3}}} - 5x^{\frac{2}{3}}$ |
| 2) $-1.8\sqrt{x} + 3x^{\frac{3}{2}}$ | 4) $\frac{8}{x^{\frac{1}{3}}} - x^{\frac{8}{3}}$ |

1. 41 Establish the table of variations of the following functions :

- | | | |
|-----------------------|---|--|
| 1) $y = x^2 - 5x + 6$ | 3) $y = x^{\frac{3}{2}}(x - 1)$ | 5) $y = x + \frac{3}{x}$ |
| 2) $y = x^3 - 12x$ | 4) $y = x^{\frac{3}{4}} - 2x^{\frac{7}{4}}$ | 6) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ |

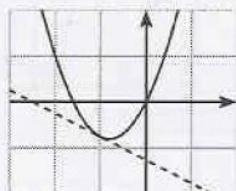
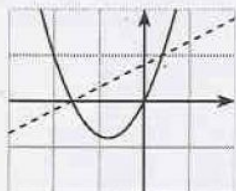
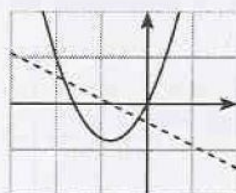
1. 42 Analyse the functions :

- | | |
|---|---|
| 1) $f(x) = x^3 - 3x$ | 5) $f(x) = \frac{x^2}{x^2 - 4}$ |
| 2) $f(x) = \frac{x^2 - x + 2}{x - 2}$ | 6) $f(x) = \frac{3x^2 - 5x + 2}{4x^2 - 5x + 1}$ |
| 3) $f(x) = \sqrt{9 - x^2}$ | 7) $f(x) = \frac{(2x + 1)^3}{8(x - 1)^2}$ |
| 4) $f(x) = \frac{(x + 1)^3}{x^2 - 7x + 10}$ | |

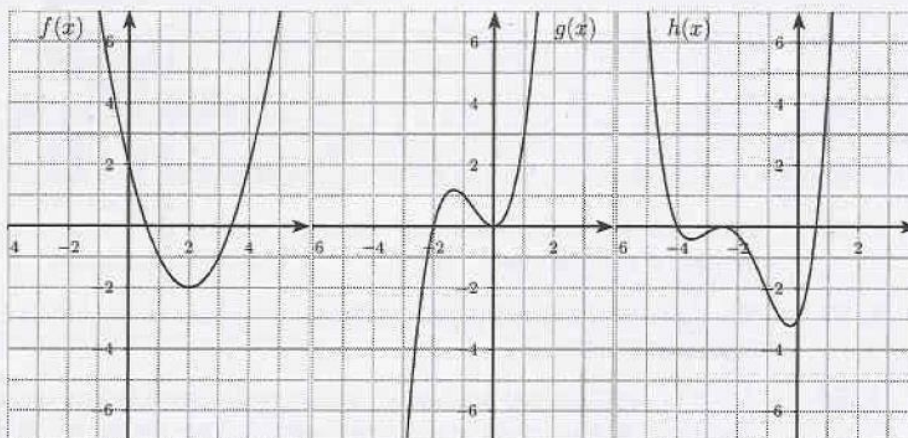
1. 43 Analyse, on the interval $[0; 2\pi]$, the following :

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|---------------------------------|--------------------------------|
| 1) $f(x) = 3\sin(x) + 4\cos(x)$ | 2) $g(x) = \frac{1}{\cos(2x)}$ |
|---------------------------------|--------------------------------|

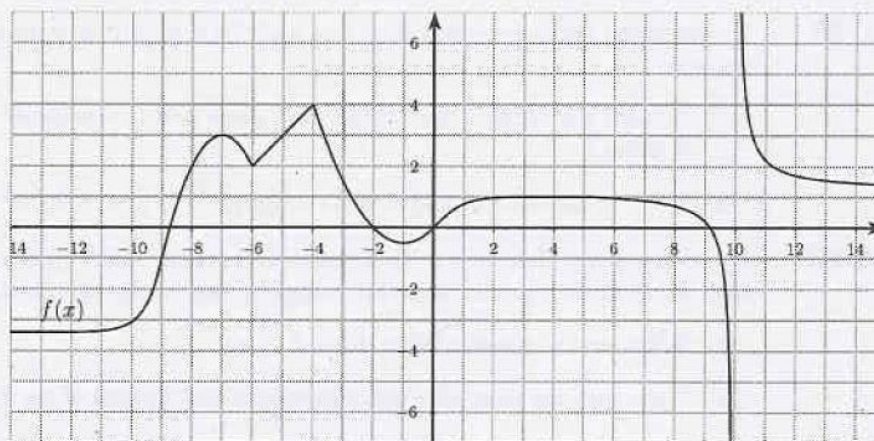
1. 52 Which illustration contains the graph of a function and its derivative ?



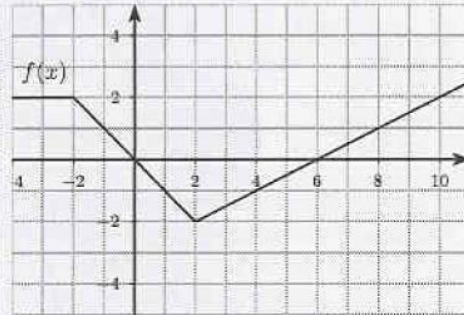
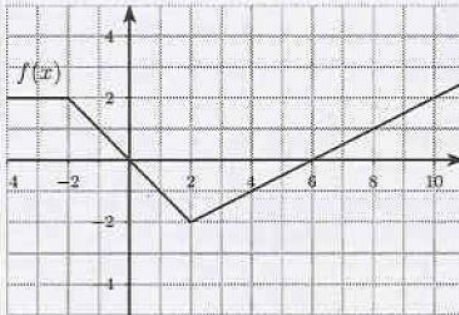
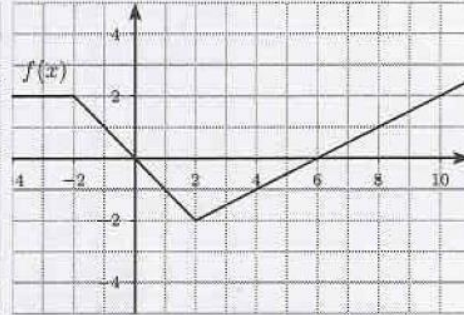
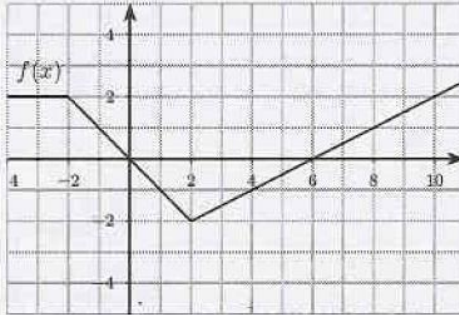
1. 53 Sketch precisely, on the same set of axes, the graph of $y = f'(x)$, $y = g'(x)$ and $y = h'(x)$.



1. 54 Sketch precisely, on the same set of axes, the graph of $y = f'(x)$.



1. 55 Sketch precisely, on the same set of axes, the graph of $y = f'(x)$ as well as the ones of $-f(x)$, $f(x+1) - 2$ and $2|f(x)|$.

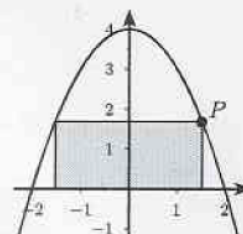


1. 56 Determine two positive numbers x and y such that their sum equals 10 and the product x^3y^2 is maximal.
1. 57 The capacity of a cylindrical box is 500ml. Determine its height and radius so that its surface is minimal.
1. 58 A box (a parallelepiped without lid) is built from a 10cm by 8cm square cardboard. Four identical squares are cut up at each corner. Then the paper is folded and stuck together. Determine the measure of the sides of the squares so that the volume of the box is maximal. Determine the value of the maximal volume.
1. 59 A rectangular enclosure is adjoining to a wall. So it is only enclosed along three of its sides.
- 1) What is the maximal area of this field, given that the enclosure's total length is 200m ?
 - 2) What is the minimal length of its enclosure if its area is 300m² ?
1. 60 We have a metal plate with a 50cm length and a 40cm width. A square with x cm sides is cut at each corner. It is then folded to form a tray with depth x cm.
- 1) What are the possible values for x ?
 - 2) Determine the value of x which maximize the capacity of the tray.

1. 61 From a ring of string with length 1m we build a rectangle with a pair of opposite sides with length x cm. Determine the value of x which maximize the area enclosed by the string.

1. 62 Let's consider the parabola with equation $y = -x^2 + 4$ and one of its point $P(x; y)$ located in the first quadrant. We then draw, under the parabola, a rectangle which has P as a vertex and which has two of its vertices on the x -axis.

- 1) Determine the value of x such that the area of the rectangle is maximal. What is the value of that area ?
- 2) We rotate the rectangle around the y -axis. Calculate the volume of the obtained cylinder. Then, determine the dimensions of the cylinder which maximize its volume. What is the value of that volume ?
- 3) We rotate the rectangle around the x -axis. Calculate the volume of the obtained cylinder. Then, determine the dimensions of the cylinder which maximize its volume. What its the value of that volume ?



1. 63 Find the points of the graph of $f(x) = \frac{4}{x}$ which are the closest to the origin.

1. 64 We consider a parabola $f(x) = 1 - x^2$ and one of its point M located in the first quadrant. The tangent to the parabola at M intersects the O_x axis at A and the O_y axis at B . Determine the coordinates of point M so that the area of the triangle OAB is minimal.